REG-EOC-2324-ASM-SET 6-MATH

Suggested solutions

Multiple Choice Questions

1. A	2. D	3. C	4. A	5. C
6. D	7. A	8. A	9. D	10. A
11. A	12. B	13. B	14. B	15. C
16. A	17. A	18. D	19. B	20. B
21. A	22. B	23. A	24. C	25. C
26. D	27. C	28. D	29. B	30. B

1. $\boxed{A}$

   Solve $\begin{cases} 4x - 3y = 0 \\ x^2 + y^2 - 4x - 22y + 75 = 0 \end{cases}$, we have $(x,\ y) = (3,\ 4)$ or $(9,\ 12)$.

   mid-point of MN is at $(6,\ 8)$.

   Required equation is

   $$(x - 6)^2 + (y - 8)^2 = (3 - 6)^2 + (4 - 8)^2$$
   $$(x - 6)^2 + (y - 8)^2 = 25$$

2. $\boxed{D}$

   Solve the system $\begin{cases} x - y + 9 = 0 \\ x^2 + y^2 - 6x + cy - 7 = 0 \end{cases}$ using the calculator program.
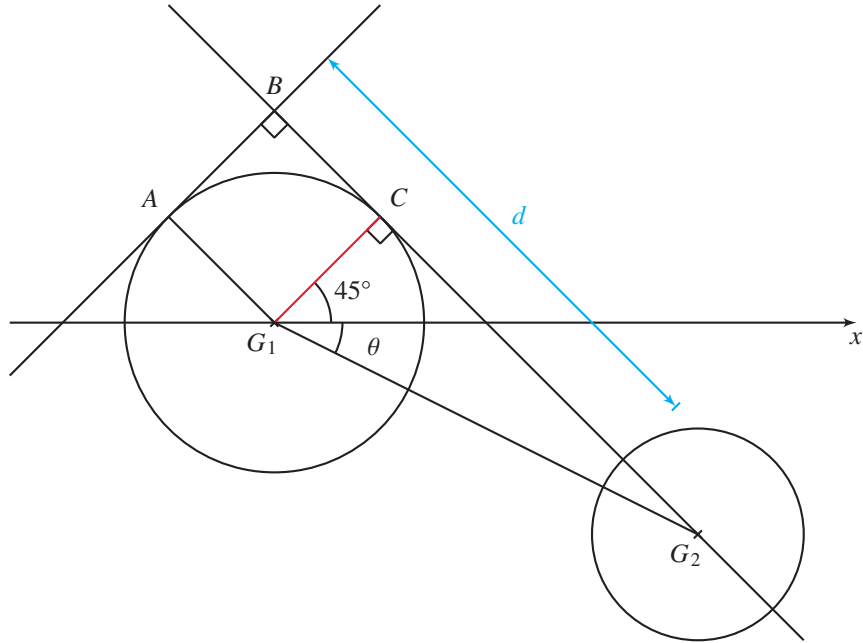
Value of c	Number of intersections	Sign of Δ
-56	2	$+$

   Required range contains -56 and -56 is not a boundary value of the range.

   The answer is D.

3. $\boxed{\text{C}}$

Let G_1 and G_2 be the centres of circles C_1 and C_2 respectively.

Suppose L touches C_1 at A, B is a point on L such that $BG_2 \perp L$.

Let C be a point on BG_2 such that $AB /\!/ CG_1$.

Note that C does not necessarily lie on C_1.

Since slope of $L = 1$, the inclination of $CG_1 = 45°$.

$G_1G_2 = \sqrt{4^2 + 2^2} = \sqrt{20}$

slope of $G_1G_2 = -\tan\theta = \dfrac{-2}{4}$

$$\theta = \tan^{-1}\dfrac{1}{2}$$

In $\triangle CG_1G_2$, $G_1G_2 = \sqrt{20}$ and $CG_2 = G_1G_2 \sin(45° + \theta) = \sqrt{18}$

Required distance $= BG_2 - 1$

$$= (AG_1 + CG_2) - 1$$
$$= (\sqrt{2} + 3\sqrt{2}) - 1$$
$$= 4\sqrt{2} - 1$$

4. $\boxed{\text{A}}$

Solve the system $\begin{cases} 3x - y - 2 = 0 \\ x^2 + y^2 + \dfrac{k}{5}x + \dfrac{4}{5} - 4 = 0 \end{cases}$ using calculator program.

The coordinates in the calculator are saved as $(\mathbf{A}, \mathbf{B})$ and $(\mathbf{X}, \mathbf{Y})$.

x-coordinate of mid-point $= \dfrac{\mathbf{A} + \mathbf{X}}{2}$

Value of k	Number of intersections	$\dfrac{\mathbf{A} + \mathbf{X}}{2}$
-152	2	2
-52	2	1
148	2	-1
248	2	-2

The answer is A.

5. $\boxed{\text{C}}$

Solve the simultaneous equations $\begin{cases} mx - y - 5 = 0 \\ x^2 + y^2 - 11x + 7y + 20 = 0 \end{cases}$ using the calculator program.

Value of m	Number of intersections	Sign of Δ
-3	0	$-$

Required range contains -3 and -3 is not a boundary value.

The answer is C.

6. $\boxed{\text{D}}$

Solve the system $\begin{cases} x - 2y + 1 = 0 \\ x^2 + y^2 - 6x + k = 0 \end{cases}$ using the calculator program.

Value of k	Number of intersections	Sign of Δ
0	2	$+$

Required range does not contain $k = 0$ and 0 is not a boundary value of the required range.

The answer is D.

7. $\boxed{\text{A}}$

Solve $\begin{cases} x - 3y + k = 0 \\ x^2 + y^2 + 6x - 8y + 15 = 0 \end{cases}$ by the calculator program.

Value of k	Number of intersections	Sign of Δ
0	0	−

Required range does not contain 0.

Note that the required range should contain the boundary value as $\Delta = 0$ also satisfies the condition.

The answer is A.

We have $x = 3y - k$.

$$(3y - k)^2 + y^2 + 6(3y - k) - 8y + 15 = 0$$

$$10y^2 + (10 - 6k)y + (k^2 - 6k + 15) = 0$$

The quadratic equation has real root(s).

$$\Delta = (10 - 6k)^2 - 4(10)(k^2 - 6k + 15) \geq 0$$

$$-4k^2 + 120k - 500 \geq 0$$

$$5 \leq k \leq 25$$

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8. $\boxed{\text{A}}$

Solve the system $\begin{cases} 3x + 4y + k = 0 \\ x^2 + y^4 - \dfrac{9}{4} = 0 \end{cases}$ using the calculator program.

Value of k	Number of intersections	Sign of Δ
$-\dfrac{15}{2}$	1	0
0	2	+

Required range has $-\dfrac{15}{2}$ as a boundary value (not equal to) and includes 0.

The answer is A.

9. $\boxed{\text{D}}$

A. ✗. Using the calculator program, the system $\begin{cases} x + y - 9 = 0 \\ x^2 + y^2 + 6x + 6y + 9 = 0 \end{cases}$ does not have real solutions.

B. ✗. $x + y + 9 = 0$ does not pass through $(3, \ 6)$.

C. ✗. Same reason as A.

D. ✓.

10. $\boxed{A}$

Solve the system $\begin{cases} kx - y + 2 = 0 \\ x^2 + y^2 - 5x - 9y + 24 = 0 \end{cases}$ using calculator program.

Value of k	Number of intersection	Sign of Δ
$\dfrac{1}{3}$	1	0
-3	0	$-$

Required range has $\dfrac{1}{3}$ as one of the boundary value, and it contains $k = -3$.

The answer is A.

11. $\boxed{A}$

Solve the system $\begin{cases} x + 2y + k = 0 \\ x^2 + y^2 + 2y - 4 = 0 \end{cases}$ using the calculator program.

Value of k	Number of intersections	Sign of Δ
7	1	0
0	2	$+$

Required range contains "7" as a boundary value, and does not include 0.

The answer is A.

12. $\boxed{B}$

Solve the system $\begin{cases} 2x - y + k = 0 \\ x^2 + y^2 - 8x - 10y - 39 = 0 \end{cases}$ using the calculator program.

    A. ✗. 2 distinct intersections.

    B. ✓. 1 intersection: (4, 9).

    C. ✗. 2 distinct intersections.

    D. ✗. 2 distinct intersections.

13. $\boxed{B}$

Solve the system $\begin{cases} x - y + k = 0 \\ x^2 + y^2 + 2x - 4y - 13 = 0 \end{cases}$ using the calculator program.

Value of k	Number of intersections	Sign of Δ
-9	0	$-$

Required range does not contain -9 and -9 is not a boundary value.

The answer is B.

14. $\boxed{\text{B}}$

Solve the system $\begin{cases} x + y + 4 = 0 \\ x^2 + y^2 + 2x - 6y + k = 0 \end{cases}$ using the calculator program.

    A. ✗. 2 distinct intersections: $(-2, -2)$ and $(-6, 2)$.

    B. ✓.

    C. ✗. No intersections.

    D. ✗. No intersections.

15. $\boxed{\text{C}}$

When the straight line passes through centre $(-1, 2)$, $k = -1 - 2 = -3$.

mid-point of AB is then centre of the circle.

When $k = -3$, y-coordinate of mid-point $= 2$.

Only option C satisfies this.

16. $\boxed{\text{A}}$

Solve the system $\begin{cases} 3x + 4y + k = 0 \\ x^2 + y^2 - 12x - 14y + 60 = 0 \end{cases}$ using the calculator program.

Value of k	Number of intersections	Sign of Δ
-71	1	0
0	0	$-$

Required range has -71 as a boundary value and excludes 0.

The answer is A.

17. A

    I. ✓. G lies inside $\triangle OAB$, which is in the second quadrant. The x- and y-coordinates are not equal (one positive and one negative).

    II. ✓. Let the radius of inscribed circle be r. Then the coordinates of G are $(-r,\ r)$.

$$4r + (-r) = 3kb$$
$$r = kb$$

Using tangent properties, OB is divided into two segments with lengths $b - r$ and r.
OA is divided into two segments with lengths $10 - r$ and r.

$$(10 - r) + (b - r) = \sqrt{10^2 + b^2}$$
$$[10 + b(1 - 2k)]^2 = b^2 + 100$$
$$100 + 20b(1 - 2k) + b^2(1 - 2k)^2 = b^2 + 100$$
$$b^2(4k^2 - 4k) + 20b(1 - 2k) = 0$$
$$b = -\frac{20(1 - 2k)}{4k^2 - 4k}$$
$$= \frac{5(1 - 2k)}{k(1 - k)}$$

Required distance $= r = kb = \dfrac{5(1 - 2k)}{1 - k}$

    III. ✗. When $k = \dfrac{1}{6}$, $r = \dfrac{5(1 - 2k)}{1 - k} = 4$.

Equation of inscribed circle is $(x + 4)^2 + (y - 4)^2 = 4^2$.

$$(x + 4)^2 + (5 - 3x - 4)^2 = 16$$
$$10x^2 + 2x + 1 = 0$$

$\Delta = 2^2 - 4(10)(1) = -36 < 0$.

The straight line $3x + y = 5$ does not cut the inscribed circle of $\triangle OAB$ and hence is not a tangent.

18. D

Solve the system $\begin{cases} x - y + m = 0 \\ x^2 + y^2 + 2x - 4y - 13 = 0 \end{cases}$ using the calculator program.

Value of m	Number of intersections	Sign of Δ
−9	0	−

Required range does not contain $m = -9$ and -9 is not a boundary value of the required range. The answer is D.

19. B

Solve the system $\begin{cases} 2x + y - 5 = 0 \\ x^2 + y^2 - kx + 6y - 10 = 0 \end{cases}$ using the calculator program.

Value of k	Number of intersections	Sign of Δ
2	2	+

Required range does not contain 2 and 2 is not a boundary value of the range.

The answer is B.

20. B

Solve the system $\begin{cases} 3x - 4y + k = 0 \\ x^2 + y^2 + 2x - 2y - 7 = 0 \end{cases}$ using the calculator program.

Value of k	Number of intersections	Sign of Δ
-8	1	0

Required range has -8 as a boundary value (can equal to).

The answer is B.

21. A

The mid-point of MN lies on $hx + ky = 6$.

$$h(1) + k(0) = 6$$
$$h = 6$$

The coordinates of the centre are $(4,\ 2)$.

The line joining centre and the mid-point of MN is perpendicular to MN.

$$\frac{2-0}{4-1} \times \left(-\frac{h}{k}\right) = -1$$
$$k = 4$$

We have $h = 6$.
Use the values of k given in each option.

Value of k	Intersections	Mid-point
4	(3.77, −4.16) and (−1.77, 4.16)	(1, 0)
6	(5.07, −4.07) and (−2.07, 3.07)	$\left(\dfrac{3}{2}, -\dfrac{1}{2}\right)$
9	(6.47, −3.65) and (−2.16, 2.11)	$\left(\dfrac{28}{13}, -\dfrac{10}{13}\right)$
12	(7.35, −3.17) and (−2.15, 1.57)	$\left(\dfrac{13}{5}, -\dfrac{4}{5}\right)$

The answer is A.

22. B

C cuts x-axis at two points $\Rightarrow$ when $y = 0$, x has two distinct real values.

Check by calculator formula 01, only options A and B are possible.

Equation of L is $y = -\dfrac{\sqrt{3}}{3}x \rightarrow \sqrt{3}x + 3y = 0$. Using the program to check the intersections:

    A. Distinct coordinates $\rightarrow$ ✗

    B. Identical coordinates $\rightarrow$ ✓

23. A

Solve the system $\begin{cases} mx - y = 0 \\ x^2 + y^2 - 6x + 4 = 0 \end{cases}$ using the calculator program.

    A. 1 intersection $\rightarrow$ tangent

    B. No intersections

    C. No intersections

    D. No intersections

24. C

Consider the system

$$\begin{cases} x - y + 13 = 0 \\ x^2 + y^2 - 14x + cy - 223 = 0 \end{cases}$$

Put $y = x + 13$ into equation of circle gives a quadratic equation.

Discriminant of the quadratic equation should be positive (2 distinct real roots) $\Rightarrow$ options A or C

When $c = 0$, by calculator program, there are two intersections $\Rightarrow$ required range contains 0

$\Rightarrow$ the answer is C.

25. C

When the straight line passes through centre $(-1,\ 2)$, $k = -1 - 2 = -3$.
mid-point of AB is then centre of circle.
When $k = -3$, x-coordinate of mid-point $= -1$.
Only option C satisfies this.

26. D

Solve the system $\begin{cases} mx - y - 1 = 0 \\ x^2 + y^2 - 16x - 2y + 31 = 0 \end{cases}$ using the calculator program.

The system has repeated solutions when $m = \dfrac{5}{3}$ and when $m = -\dfrac{3}{5}$.

Thus, $m = \dfrac{5}{3}$ or $-\dfrac{3}{5}$.

27. C

Solve the system $\begin{cases} 2x - y - 6 = 0 \\ x^2 + y^2 - 8y - 14 = 0 \end{cases}$ using the calculator program.

The coordinates in the calculator are saved as $(\mathbf{A},\ \mathbf{B})$ and $(\mathbf{X},\ \mathbf{Y})$.

y-coordinate of mid-point $= \dfrac{\mathbf{B} + \mathbf{Y}}{2} = 2$

28. D

Solve the system $\begin{cases} x - y + k = 0 \\ x^2 + y^2 - 2x + 4y - 3 = 0 \end{cases}$ using the calculator program.

Value of k	Number of intersections	Sign of Δ
-7	1	0
0	2	+

Required range has -7 as a boundary value and excludes 0.
The answer is D.

29. B

Solve the system $\begin{cases} 4x - 3y = 0 \\ x^2 + y^2 - 3kx + ky + 1 = 0 \end{cases}$ using the calculator program.

  I. ✓. 1 intersection: $\left(-\dfrac{3}{5},\ -\dfrac{4}{5}\right)$.

  II. ✓. 1 intersection: $\left(\dfrac{3}{5},\ \dfrac{4}{5}\right)$.

  III. ✗. 2 intersections.

30. B

Solve the system $\begin{cases} 2x - y + b = 0 \\ x^2 + y^2 - 2x - y + \dfrac{5}{4} = 0 \end{cases}$ using the calculator program.

    A. ✗. No intersections

    B. ✓. 1 intersection: $\left(1, \dfrac{1}{2}\right)$

    C. ✗. No intersections

    D. ✗. No intersections