REG-EOC-2324-ASM-SET 5-MATH
Suggested solutions

Conventional Questions

1. (a) $10^2 + (-2)^2 + 5k(10) - k(-2) = 0$ 1M

$$k = -2$$ 1A

(b) The equation of C is $x^2 + y^2 - 10x + 2y = 0$.

The coordinates of M are $(5,\ -1)$. 1A

(c) Slope of $MA = \frac{-1+2}{5-10} = -\frac{1}{5}$. 1M

The slope of tangent to C at $A = 5$.

The equation of tangent to C at A is

$$y + 2 = 5(x - 10)$$ 1M

$$5x - y - 52 = 0$$ 1A

2. (a) Substitute L into C,

$$x^2 + (mx+2)^2 = 2$$ 1M

$$(1+m^2)x^2 + 4mx + 2 = 0$$

If L touches C,

$$(4m)^2 - 4(1+m^2)(2) = 0$$ 1M

$$8m^2 - 8 = 0$$

$$m = \pm 1$$ 1A

(b) Substitute $x = -2y - \frac{1}{2}$ into C,

$$\left(-2y - \frac{1}{2}\right)^2 + y^2 = 2$$ 1M

$$5y^2 + 2y - \frac{7}{4} = 0$$

$\Delta = 2^2 - 4(5)\left(-\frac{7}{4}\right) = 39 > 0$ 1M

They intersect each other. 1A

3. (a) Slope of $L = \tan 45° = 1$ 1A

The equation of L is $y = x + 1$. 1A

(b) $x^2 + (x+1)^2 - 8x - 6(x+1) + k = 0$ 1M

$$2x^2 - 12x - 5 + k = 0$$

$$\Delta = (-12)^2 - 4(2)(-5+k) = 0$$ 1M+1A

$$k = 23$$ 1A

(c) Put $k = 23$ into the equation in (b),

$$2x^2 - 12x + 18 = 0$$

$$x = 3$$

When $x = 3$, $y = 4$. The coordinates of B are $(3,\ 4)$. 1A

Since B is the mid-point of the centres of two circles, the coordinates of centre of S_1 are $(2,\ 5)$. 1A

4. (a) $P\,(8,\ 6)$ 1A

Radius = 6 1A

(b) (i) $y = mx$ 1A

(ii) $x^2 + (mx)^2 - 16x - 12(mx) + 64 = 0$ 1M

$(1+m^2)x^2 + (-16-12m)x + 64 = 0$ 1A

$\Delta = (-16-12m)^2 - 4(1+m^2)(64) = 0$ 1M

$$7m^2 - 24m = 0$$

$$m = \frac{24}{7} \quad \text{or} \quad 0 \text{ (rejected)}$$ 1A

(c) $OQ = \sqrt{8^2 + 6^2} + 6 = 16$ 1A

Let $Q\,(p,\ q)$.

$$\frac{p}{8} = \frac{16}{10} \quad \text{and} \quad \frac{q}{6} = \frac{16}{10}$$ 1M

$$p = \frac{64}{5} \qquad q = \frac{48}{5}$$

$Q\left(\frac{64}{5},\ \frac{48}{5}\right)$ 1A

Slope of $OQ = \frac{3}{4}$

Slope of $AB = -\frac{4}{3}$

Equation of AB is

$$y - \frac{48}{5} = -\frac{4}{3}\left(x - \frac{64}{5}\right)$$ 1M

$$4x + 3y - 80 = 0$$ 1A

5. (a) $Q\,(8,\ 6)$ 1A

Coordinates of $C = \left(\dfrac{0+8}{2},\ \dfrac{0+6}{2}\right)$ 1M

$= (4,\ 3)$ 1A

(b) Slope of $RC = \dfrac{6-3}{0-4} = -\dfrac{3}{4}$

Slope of tangent to the circle at $R = \dfrac{4}{3}$ 1M

The equation of tangent to the circle at P is

$\dfrac{y-0}{x-8} = \dfrac{4}{3}$ 1M

$4x - 3y - 32 = 0$ 1A

The equation of tangent to the circle at R is $y = \dfrac{4}{3}x + 6$ 1A

(c) Slope of $OC = \dfrac{3-0}{4-0} = \dfrac{3}{4}$

Slope of tangent to the circle at $O = -\dfrac{4}{3}$ 1M

Product of slopes of two tangents $= -\dfrac{4}{3} \times \dfrac{4}{3} = -\dfrac{16}{9} \neq -1$ 1M

So, the tangents to the circle at R and at O are not perpendicular.

Thus, the quadrilateral formed is not a rectangle. The claim is not correct. 1A

6. (a) (i) $\dfrac{1}{x-1} - \dfrac{x+2}{x^2-1} = \dfrac{x+1-(x+2)}{x^2-1}$

$= \dfrac{-1}{x^2-1}$ 1A

Thus, $A = 0$ and $B = -1$. 1A

(ii) $f(x) = -4x^2 - 16x - 19$

$= -4[x^2 + 2(2)(x) + 2^2] - 3$ 1M

$= -4(x+2)^2 - 3$ 1A

The maximum value of $f(x)$ is -3. 1A

(b) $x^2 + [(k+2)x]^2 + \dfrac{1}{k-1}x - \dfrac{1}{k^2-1}(k+2)x + \dfrac{1}{(k^2-1)^2} = 0$ 1M

$(k^2+4k+5)x^2 + \left(\dfrac{1}{k-1} - \dfrac{k+2}{k^2-1}\right)x + \dfrac{1}{(k^2-1)^2} = 0$

$(k^2+4k+5)x^2 - \dfrac{1}{k^2-1}x + \dfrac{1}{(k^2-1)^2} = 0$ 1A

$\Delta = \left(\dfrac{1}{k^2-1}\right)^2 - 4(k^2+4k+5)\left(\dfrac{1}{(k^2-1)^2}\right)$ 1M

$= \dfrac{1}{(k^2-1)^2}(-4k^2 - 16k - 19)$

$= \dfrac{1}{(k^2-1)^2}[-4(k+2)^2 - 3]$ 1M

$\leq -3\left(\dfrac{1}{(k^2-1)^2}\right) < 0$

C and L do not intersect.

The claim is agreed. 1A

7. (a) $\angle ABC = \angle AOC$

$\angle ADC = 2\angle ABC = 2\angle AOC$ 1M

$\angle OCD = \angle OAD = 90°$ 1M

In quadrilateral $OACD$,

$$\angle AOC + 2 \times 90° + \angle ADC = 360° \quad \text{1M}$$

$$3\angle AOC = 180°$$

$$\angle AOC = 60° \quad \text{1A}$$

(b) (i) $\angle AOD = \frac{1}{2}\angle AOC = 30°$ 1M

In $\triangle AOD$,

$$\sin 30° = \frac{AD}{OD} = \frac{BD}{OD}$$

$$OD = 2BD \quad \text{1M}$$

Therefore, the coordinates of D are $\left(\frac{0+2\times 9}{3}, \frac{0+2\times 9}{3}\right) = (6, 6)$. 1A

Required equation is

$$(x-6)^2 + (y-6)^2 = (9-6)^2 + (9-6)^2$$

$$(x-6)^2 + (y-6)^2 = 18 \quad \text{1A}$$

(ii) Let $y = mx$ be the equation of tangent from O.

$$(x-6)^2 + (mx-6)^2 = 18 \quad \text{1M}$$

$$(m^2+1)x^2 - 12(m+1)x + 54 = 0$$

Since $y = mx$ is a tangent,

$$\Delta = [-12(m+1)]^2 - 4(m^2+1)(54) = 0 \quad \text{1M}$$

$$-72m^2 + 288m - 72 = 0 \quad \text{1A}$$

$$m = \frac{4 \pm \sqrt{4^2 - 4}}{2} = 2 \pm \sqrt{3}$$

The equation of OA and OC are $y = (2-\sqrt{3})x$ and $y = (2+\sqrt{3})x$ respectively. 1A+1A

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