

REG-CP2B-2324-ASM-SET 3-MATH**Suggested solutions****Multiple Choice Questions**

- | | | | | |
|-------|-------|-------|-------|-------|
| 1. C | 2. A | 3. B | 4. A | 5. D |
| 6. C | 7. C | 8. D | 9. C | 10. D |
| 11. A | 12. C | 13. B | 14. A | 15. A |
| 16. B | 17. C | 18. C | 19. C | 20. B |
| 21. B | 22. C | 23. C | 24. A | 25. A |
| 26. C | 27. C | 28. D | 29. D | 30. D |
| 31. D | 32. A | 33. A | 34. B | 35. D |
| 36. A | 37. C | 38. C | 39. B | 40. D |
| 41. A | | | | |

1. C

$$\angle AOC + \angle OAC + \angle OCA = 180^\circ$$

$$\angle AOC + 2(27^\circ) = 180^\circ$$

$$\angle AOC = 126^\circ$$

$$\angle BOC = \angle AOC \times \frac{2}{1+2} = 84^\circ$$

$$\angle BAC = \frac{\angle BOC}{2} = 42^\circ$$

$$\angle BCD = \angle BAC = 42^\circ$$

2. A

$$\angle PBA = \angle PAB = \frac{180^\circ - 42^\circ}{2} = 69^\circ$$

$$\angle ABD = 180^\circ - 69^\circ - 35^\circ = 76^\circ$$

$$\angle ACD = 180^\circ - 76^\circ = 104^\circ$$

3. B

Let O be the centre of the semi-circle $ABCD$.

We have $OC \perp EF$ and $OB \perp BE$.

$$\text{Thus, } \angle BOC = 360^\circ - 90^\circ - 90^\circ - 90^\circ = 90^\circ.$$

$$\angle AOC = \angle AFE + 90^\circ = 132^\circ$$

$$\angle AOB = 132^\circ - 90^\circ = 42^\circ$$

$$\angle ACB = \frac{\angle AOB}{2} = 21^\circ$$

4. A

$$\angle AEB = \angle SAB = 60^\circ$$

$$\angle BCD = \angle AEB = 60^\circ$$

$$\angle BCE : \angle ECD = 15^\circ : 60^\circ - 15^\circ = 1 : 3$$

5. D

$$\begin{aligned}
 EB &= EC \\
 \angle EBC &= \frac{180^\circ - 72^\circ}{2} = 54^\circ \\
 \angle BAC &= \angle EBC = 54^\circ \\
 \angle ACF &= \angle ABC = 63^\circ \\
 AB &= AC \\
 \angle ABC &= \frac{180^\circ - 54^\circ}{2} = 63^\circ \\
 \angle DBE &= 90^\circ \\
 \angle CBD &= 90^\circ - 54^\circ = 36^\circ \\
 \angle CAF &= \angle CBD = 36^\circ \\
 \angle AFC &= 180^\circ - 36^\circ - 63^\circ = 81^\circ
 \end{aligned}$$

6. C

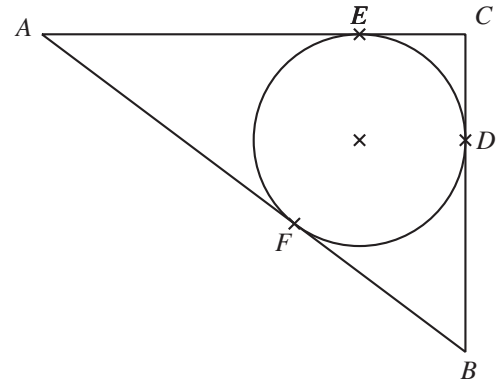
Let D, E, F be the point of contact of the inscribed circle and BC, AC, AB respectively.

Let $CE = x$. Then $AE = 12 - x$, $CD = x$ and $BD = 5 - x$.

By tangent properties, $AF = 12 - x$ and $BF = 5 - x$.

Since $AB = \sqrt{5^2 + 12^2} = 13$, we have $x = 2$.

Thus, the coordinates of incentre are $(10, 3)$.



7. C

$$\begin{aligned}
 \angle OBC + \angle OCB + \angle BOC &= 180^\circ \\
 \angle BOC &= 180^\circ - 2 \times 55^\circ \\
 &= 70^\circ \\
 \angle OCT &= 90^\circ \\
 \angle OTC + \angle OCT + \angle COT &= 180^\circ \\
 \angle OTC &= 180^\circ - 70^\circ - 90^\circ \\
 \angle ATC &= 20^\circ
 \end{aligned}$$

8. D

$$\begin{aligned}
 \angle BCE &= \angle CEG = 70^\circ \\
 \angle CBE &= \angle CEG = 70^\circ \\
 \angle BEC &= 180^\circ - 70^\circ - 70^\circ = 40^\circ \\
 \angle ABE &= \angle BEC = 40^\circ \\
 \angle ADE &= \angle ABE = 40^\circ
 \end{aligned}$$

9. C

Join OE and let the radius of circle be r .

$$OD = 8 - r.$$

$$DE = \sqrt{(8-r)^2 - r^2} = \sqrt{64 - 16r}$$

As $\triangle OED \sim \triangle ABD$ (AA),

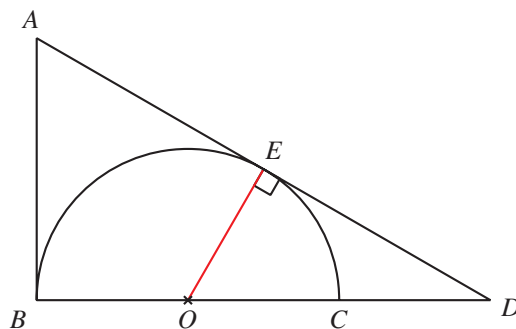
$$\frac{AB}{BD} = \frac{6}{8} = \frac{r}{\sqrt{64 - 16r}} = \frac{OE}{ED}$$

$$\frac{9}{16} = \frac{r^2}{64 - 16r}$$

$$9 = \frac{r^2}{4 - r}$$

$$0 = r^2 + 9r - 36$$

$$r = 3 \text{ or } -12 \text{ (rejected)}$$



10. D

Denote the intersections of BC and the circle by C and F .

$$\angle CAF = \angle ECB = 30^\circ$$

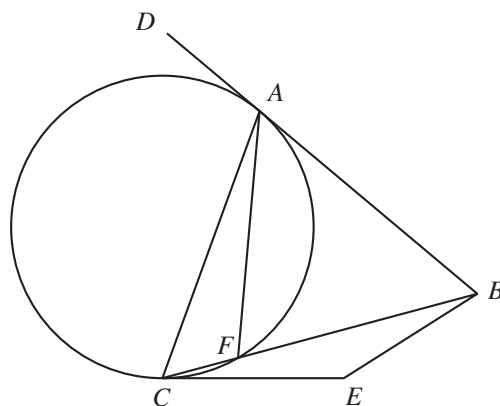
$$\angle BAF = \angle FCA$$

$$\angle FCA + \angle BAF + 30^\circ + 40^\circ = 180^\circ$$

$$2\angle BAF = 110^\circ$$

$$\angle BAF = 55^\circ$$

$$\angle CAD = 180^\circ - 30^\circ - 55^\circ = 95^\circ$$



11. A

Note that $\angle ABC = 90^\circ$.

Let the radius be r cm.

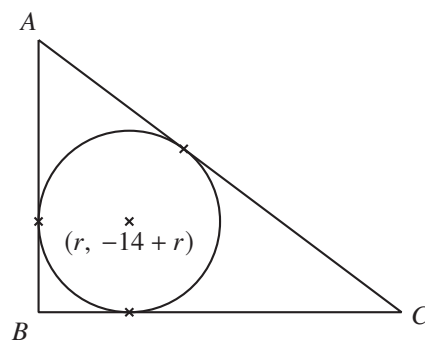
The coordinates of centre are $(r, -14 + r)$.

Consider the length of AC .

$$(24 - r) + (18 - r) = \sqrt{18^2 + 24^2}$$

$$r = 6$$

The coordinates of centre are $(6, -8)$.



12. C

Let $\angle CBD = \angle CDB = x$ and $\angle DAS = y$.

$\angle BCD = 180^\circ - 2x$ and $\angle BAD = 2x$.

$\angle ABD = \angle DAS = y$.

$$y + (2x + y) + 40^\circ = 180^\circ$$

$$x + y = 70^\circ$$

$$\angle ABC = x + y = 70^\circ.$$

13. B

Let $\angle CAP = x$. Then $\angle ABC = \angle CAP = x$.

$$\angle DBA + \angle BAP = 180^\circ$$

$$40^\circ + x + x + 90^\circ = 180^\circ$$

$$x = 25^\circ$$

14. A

BC is a diameter, $\angle BDC = 90^\circ$.

Since $\triangle BDC \sim \triangle CDA$, $\frac{BD}{BC} = \frac{CD}{AC}$

$$\frac{BD}{CD} = \frac{3}{4}$$

$$\angle BCD = \tan^{-1} \frac{3}{4} \text{ and } BD = 9 \sin \angle BCD = 5.4 \text{ cm.}$$

15. A

$$\angle CBP = \angle BCP = \angle BAC = 56^\circ$$

$$\angle BPC = 180^\circ - 56^\circ - 56^\circ = 68^\circ$$

$$\angle PBD = 90^\circ$$

$$\angle BQP = 180^\circ - 90^\circ - 68^\circ$$

$$= 22^\circ$$

16. B

$$EA = ED$$

$$\angle EDA = \frac{180^\circ - 86^\circ}{2} = 47^\circ$$

$$\angle ACD = \angle EDA = 47^\circ$$

$$\angle FGH = 180^\circ - \angle FEH = 94^\circ$$

$$\angle GBC = \frac{180^\circ - 94^\circ}{2} = 43^\circ$$

$$\angle BDC = \angle GBC = 43^\circ$$

$$\angle AID = \angle BDC + \angle ACD = 90^\circ$$

17. C

RT is the angle bisector of $\angle SRP$. So, $\angle SRP = 24^\circ$.

$\angle QPS = \angle QSR$.

$$\angle QPS + (\angle QSR + 70^\circ) + 24^\circ = 180^\circ$$

$$\angle QPS = 43^\circ$$

18. C

$$\angle ADC = 180^\circ - 100^\circ = 80^\circ$$

$$\angle ADB = 80^\circ \times \frac{2}{2+3} = 32^\circ$$

$$\angle BCA = \angle ADB = 32^\circ$$

$$\angle DAT = \angle ADC - \angle ATD = 50^\circ$$

$$\angle ACT = \angle DAT = 50^\circ$$

$$\angle BCT = 32^\circ + 50^\circ = 82^\circ$$

19. C

The graph passes through $(0, 3)$.

A. ✗. When $x = 0$, $y = \sin 30^\circ + 3 = 3.5 \neq 3$

B. ✗. When $x = 0$, $y = 2 \sin 30^\circ + 1 = 2 \neq 3$

C. ✓. When $x = 0$, $y = 2 \sin 30^\circ + 2 = 3$

D. ✗. When $x = 0$, $y = 3 \sin 30^\circ + 4 = 5.5 \neq 3$

20. B

$$4 + \cos(90^\circ + \theta) = 4 \cos^2 \theta$$

$$4 - \sin \theta = 4(1 - \sin^2 \theta)$$

$$4 \sin^2 \theta - \sin \theta = 0$$

$$\sin \theta = 0 \quad \text{or} \quad \frac{1}{4}$$

The equation $\sin \theta = 0$ has one root.

The equation $\sin \theta = \frac{1}{4}$ has two roots.

Thus, the equation $4 + \cos(90^\circ + \theta) = 4 \cos^2 \theta$ has 3 roots.

21. B

We have $2 \sin^2 x = 1$ or $\sin x = 0$.

When $\sin x = 0$, $x = 180^\circ$

When $2 \sin^2 x = 1$, $\sin x = \pm \frac{1}{\sqrt{2}}$ and there are four roots within the required range.

Thus, there are 5 roots.

22. C

The curve passes through $(0, 1)$. Only option C satisfies this.

23. C

The graph passes through $(0^\circ, 1)$ and $(90^\circ, 3)$.

	<u>$(0^\circ, 1)$</u>	<u>$(90^\circ, 3)$</u>
A.	✓	✗
B.	✗	
C.	✓	✓
D.	✗	

24. A

If p is positive, then the maximum value and the minimum value of $y = p \sin 3x^\circ + q$ are $p + q$ and $-p + q$ respectively.

Solve $\begin{cases} p + q = 1 \\ -p + q = -3 \end{cases}$, we have $p = 2$ and $q = -1$.

25. A

$$3 \tan x = 2 \sin(90^\circ - x)$$

$$\frac{3 \sin x}{\cos x} = 2 \cos x$$

$$3 \sin x = 2 \cos^2 x$$

$$3 \sin x = 2(1 - \sin^2 x)$$

$$2 \sin^2 x + 3 \sin x - 2 = 0$$

$$\sin x = 0.5 \quad \text{or} \quad -2 \text{ (rejected)}$$

$$x = 30^\circ \quad \text{or} \quad 150^\circ$$

There are 2 roots.

26. C

A. 3 roots

B. 3 roots

C. 4 roots

D. 2 roots

27. C

$$4 - \sin \theta = 4 \cos^2 \theta$$

$$4 - \sin \theta = 4(1 - \sin^2 \theta)$$

$$4 \sin^2 \theta - \sin \theta = 0$$

$$\sin \theta = 0 \quad \text{or} \quad \frac{1}{4}$$

The equation $\sin \theta = 0$ has one root.

The equation $\sin \theta = \frac{1}{4}$ has two roots.

Thus, there are 3 roots.

28. D

$$3 \tan^2 x = \tan x$$

$$\tan x(3 \tan x - 1) = 0$$

$$\tan x = 0 \quad \text{or} \quad \frac{1}{3}$$

When $\tan x = 0$, $x = 0^\circ$ or 180° or 360°

When $\tan x = \frac{1}{3}$, there are 2 roots (in quadrants I and III)

Altogether 5 roots.

29. D

$$\sin \theta = \sin^3 \theta$$

$$0 = \sin \theta(\sin^2 \theta - 1)$$

$$\sin \theta = 0 \quad \text{or} \quad \pm 1$$

With reference to the graph of sine, there are 5 roots.

30. D

$$\sin^2 \theta = \cos^2 \theta$$

$$\tan^2 \theta = 1$$

$$\tan \theta = \pm 1$$

$$\theta = 45^\circ \text{ or } 135^\circ \text{ or } 225^\circ \text{ or } 315^\circ$$

There are 4 roots.

31. D

G is the centre of circle ABC .

$$\text{Radius} = GB = \sqrt{(18 - 10)^2 + (3 + 3)^2} = 10$$

Since A is a point on the circle, the y -coordinate of A lies between $3 + 10$ and $3 - 10$ inclusively.

Thus, -8 is not a possible y -coordinate of A .

32. A

Note that $\angle ABC = 90^\circ$.

Let the radius be r cm.

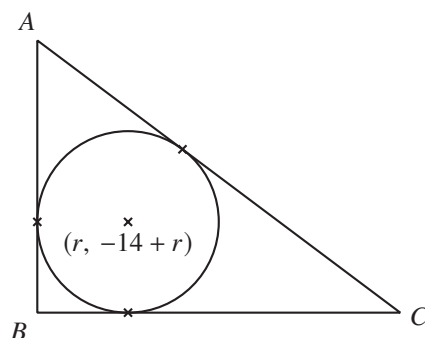
The coordinates of centre are $(r, -14 + r)$.

Consider the length of AC .

$$(24 - r) + (18 - r) = \sqrt{18^2 + 24^2}$$

$$r = 6$$

The coordinates of centre are $(6, -8)$.



33. A

Note that $\angle POQ = 90^\circ$.

The orthocentre of $\triangle OPQ$ is O .

The circumcentre of $\triangle OPQ$ is the mid-point of PQ .

The coordinates of the circumcentre of $\triangle OPQ$ are $\left(\frac{p}{2}, \frac{q}{2}\right)$.

The straight line $2x - y = 3k$ passes through O and $\left(\frac{p}{2}, \frac{q}{2}\right)$.

$$\begin{cases} 2(0) - 0 = 3k \\ 2\left(\frac{p}{2}\right) - \frac{q}{2} = 3k \end{cases}$$

Solving, we have $k = 0$ and $2p = q$.

Thus, $p : q = 1 : 2$.

34. B

Denote the mid-point of AB by M .

Let G be the centroid of $\triangle ABP$.

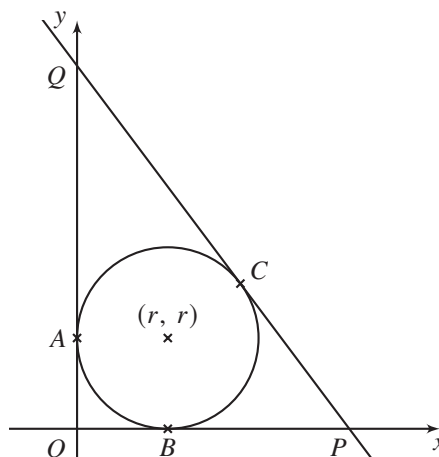
Note that $MG : GP = 1 : 2$.

The y -coordinate of G is therefore constant.

The locus of centroid of $\triangle ABP$ is a straight line parallel to L .

35. D

The coordinates of P and Q are $(6, 0)$ and $(0, 8)$ respectively.



Let the radius of the inscribed circle be r .

Consider the area of $\triangle OPQ$.

$$\frac{(6)(8)}{2} = \frac{(6)(r)}{2} + \frac{(8)(r)}{2} + \frac{(\sqrt{6^2 + 8^2})(r)}{2}$$

$$r = 2$$

Required coordinates are $(2, 2)$.

Refer to the figure.

$$OA = OB = r$$

$$BP = CP = 6 - r$$

$$AQ = CQ = 8 - r$$

Consider the length of PQ .

$$(6 - r) + (8 - r) = \sqrt{6^2 + 8^2}$$

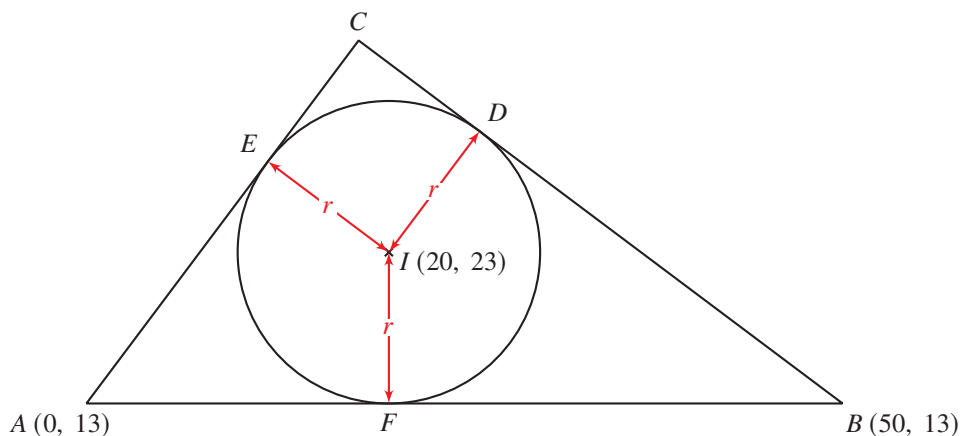
$$r = 2$$

Required coordinates are $(2, 2)$.

36. A

Since C is the orthocentre of $\triangle ABC$, $\angle ACB = 90^\circ$.

Let D, E and F be the points of contact as shown below. Denote the incentre of $\triangle ABC$ by I .



Consider the vertical distance from $(20, 23)$ to AB .

$$r = 23 - 13$$

$$= 10$$

Note that $CDIE$ is a square.

We have $AC = AE + r = AF + r = 20 + 10 = 30$.

Consider the horizontal distance between A and C .

Horizontal distance $= AC \cos \angle BAC$

$$= 30 \cos \angle BAC$$

$$= 30 \left(\frac{AC}{AB} \right) \quad (\text{Note that } \angle ABC \text{ is a right-angled triangle.})$$

$$= 30 \times \frac{30}{50}$$

$$= 18$$

The x -coordinate of C is $0 + 18 = 18$.

37. C

The coordinates of A and B are $\left(\frac{k}{6}, 0\right)$ and $\left(0, -\frac{k}{3}\right)$ respectively.

Denote the mid-point of AB by M .

The coordinates of M are $\left(\frac{k}{12}, -\frac{k}{6}\right)$.

Let the coordinates of C be $(c, 0)$.

We have $CG : GM = 2 : 1$, where G is the centroid of $\triangle ABC$.

$$\frac{CG}{GM} = \frac{c - 0}{0 - \frac{k}{12}}$$

$$2 = -\frac{12c}{k}$$

$$c = -\frac{k}{6}$$

The coordinates of A and B are $\left(\frac{k}{6}, 0\right)$ and $\left(0, -\frac{k}{3}\right)$ respectively.

Let the coordinates of C be $(c, 0)$.

Note that the x -coordinate of the centroid of $\triangle ABC$ is 0.

$$\frac{\frac{k}{6} + 0 + c}{3} = 0$$

$$c = -\frac{k}{6}$$

38. C

I. ✓. y -coordinate of the mid-point of BC is 0.

y -coordinate of centroid of $\triangle ABC$

$$= \frac{1(6) + 2(0)}{1 + 2}$$

$$= 2$$

The centroid of $\triangle ABC$ lies on $y = 2$.

II. ✓. Note that the y -axis passes through A and is perpendicular to BC .

Thus, the orthocentre of $\triangle ABC$ lies on the y -axis.

39. B

$$\angle BAC + 2\angle CBD + 2\angle BCD = 180^\circ$$

$$\angle CBD + \angle BCD = 55^\circ$$

$$\angle BDC + \angle CBD + \angle BCD = 180^\circ$$

$$\angle BDC = 125^\circ$$

40. D

Note that $AB = AD = AE$.

A is the centre of the circumcircle of BDE .

Since BE is a diameter of circle BDE , we have $\angle BDE = 90^\circ$.

$$\angle ADE = \angle DEA = 35^\circ$$

$$\angle ADB = 90^\circ - 35^\circ = 55^\circ$$

$$\angle CBD = \angle ADB = 55^\circ$$

41. A

AB is the angle bisector of $\angle CAD$.

$$\angle BAC = \angle BAD = 48^\circ$$

$$\angle ACD = \angle BAD = 48^\circ$$

$$\angle ADC + \angle ACD + \angle CAD = 180^\circ$$

$$\angle ADC + 48^\circ + (48^\circ + 48^\circ) = 180^\circ$$

$$\angle ADC = 36^\circ$$