REG-CP2A-2324-ASM-SET 4-MATH
Suggested solutions

Multiple Choice Questions

1. B	2. B	3. B	4. A	5. D
6. B	7. A	8. C	9. A	10. A
11. C	12. C	13. C	14. D	15. B
16. B	17. D	18. C	19. D	20. A
21. B	22. A	23. D	24. D	25. C
26. C	27. B	28. A	29. D	30. D
31. C	32. A	33. B	34. C	35. D
36. C	37. C	38. C	39. C	40. D
41. B	42. B	43. B	44. D	45. A
46. B	47. D	48. D	49. C	50. B
51. C	52. A	53. D	54. A	55. A
56. C	57. B	58. D	59. C	60. B
61. C	62. C	63. D	64. D	65. C
66. A	67. C	68. B	69. A	70. D
71. C	72. D	73. A	74. C	75. D

1. $\boxed{\text{B}}$

Upper quartile = \$40
Angle of sector \$10 = $360° - 72° - 36° - 90° - 144° = 18°$
Lower quartile = $\frac{20+30}{2}$ = \$25
Inter-quartile range = $40 - 25$ = \$15

2. $\boxed{\text{B}}$

Inter-quartile range $= 31 - 26$
$= 5$

3. $\boxed{\text{B}}$

The data are more concentrated on the higher values.
The maximum, upper quartile and median should be closed to each other.
Options A and D should be wrong.
The minimum, lower quartile and the median are 22, 40 and 47 respectively.
Lower quartile should appear closer to the median than the minimum.
The answer is B.

4. $\boxed{\text{A}}$

The class marks are 22 cm, 27 cm, . . . , 42 cm.
Standard deviation $\approx$ 6.28 cm

5. $\boxed{\text{D}}$

Since the mode is 58, at least two of the unknowns are 58.
Take $j = k = 58$.

$$51 = \frac{25 + 32 + 32 + \ldots + i}{10}$$

$$i = 56$$

Arrange the numbers in ascending order:

25	32	32	50	56	58	58	58	63	78

Median is 57.

6. $\boxed{\text{B}}$

$$\begin{aligned}\text{Inter-quartile range} &= 25 - 10 \\ &= 15\end{aligned}$$

7. $\boxed{\text{A}}$

Arrange the 7 known integers in ascending order.

7	8	10	12	17	19	23

Median is the average of the 4th datum and the 5th datum.
The value of a should lie between 12 and 17.

$$\frac{12 + a}{2} = 13$$

$$a = 14$$

$$\begin{aligned}\text{Inter-quartile range} &= \frac{17 + 19}{2} - \frac{8 + 10}{2} \\ &= 9\end{aligned}$$

8. $\boxed{\text{C}}$

I. ✗. Median is the 7th datum. We have $14 \le b \le 16$.
Note that when $b = 14$, the median is also 14.

II. ✓. Mode is 16.

$$\text{Mean} = \frac{6 + 7 + 11 + \ldots + b}{13}$$

$$= \frac{166 + b}{13}$$

Since $14 \le b \le 16$, we have $\frac{180}{13} \le \text{mean} \le 14 < 16$.
Thus, we have $a < c$.

III. ✓. Note that $\frac{180}{13} \le a \le 14$ and $14 \le b \le 16$.

9. $\boxed{\text{A}}$

I. ✗. It may happen that the 58 kg person gains weight after training.

II. ✓. The new maximum is 3 kg less than the original upper quartile.
At least 25% of the members have lost 3 kg or more.

III. ✗. It may happen that the 90 kg person becomes 53 kg after training.

10. $\boxed{\text{A}}$

Range is 14. There are two possible cases:

- One of the unknown is 14, while the values of the others are between 14 and 28 inclusively.
Take $a = 14$, $14 \le b \le 28$ and $14 \le c \le 28$ for simplicity.
- One of the unknown is 29, while the values of the others are between 15 and 29 inclusively.
Take $a = 29$, $15 \le b \le 29$ and $15 \le c \le 29$ for simplicity.

I. ✗. Take $a = b = c = 14$.

$$\text{Mean} = \frac{15 + 16 + \ldots + 14}{11} = \frac{194}{11} \ne 19$$

II. ✓. One of the unknowns has to be 14 or 29.
In either case, frequency of the datum '19' remains the highest among all.

III. ✗. Take $a = b = c = 14$.
Median $= 17 \ne 19$

11. $\boxed{\text{C}}$

Range = $(40 + b) - (10 + a) \leq 36$

$$b - a \leq 6$$

I. ✓. Note that $a \leq 2$ and $b \geq 5$.

$$\begin{aligned} \text{Range} &= (40 + b) - (10 + a) \\ &= 30 + (b - a) \\ &\geq 30 + (5 - 2) \\ &= 33 \end{aligned}$$

II. ✗. Take $b = 8$ and $a = 2$, the range of the above distribution is 36.

III. ✓. Median = $\dfrac{30 + 32}{2} = 31$

12. $\boxed{\text{C}}$

	$k = 7$	$k = 8$	$k = 9$
Mean p	8.13	8.2	8.27
Median q	7	8	8
Mode r	7	7	7

I. ✓.

II. ✗. When $k = 7$, we have $q = r$.

III. ✓.

13. $\boxed{\text{C}}$

$$x = \frac{1+1+2+\ldots+a}{10}$$
$$= \frac{40+a}{10}$$

The median is the average of the 5th datum and 6th datum.

We have $y = \dfrac{a+6}{2}$.

Since $3 \le a \le 6$, the datum with the highest frequency must be 6.

We have $z = 6$.

I. ✓. $x - z = \dfrac{40+a}{10} - 6$

$= \dfrac{a-20}{10}$

Since $3 \le a \le 6$, we have $x - z < 0$.

Thus, $x < z$.

II. ✗. $y - z = \dfrac{a+6}{2} - 6$

$= \dfrac{a-6}{2}$

Take $a = 6$, then $y = z$.

III. ✓. $x - y = \dfrac{40+a}{10} - \dfrac{a+6}{2}$

$= \dfrac{5-2a}{5}$

Since $3 \le a \le 6$, we have $x - y < 0$.

Thus, $x < y$.

14. $\boxed{\text{D}}$

A. ✗. Mode of the distribution is 8.

B. ✗. Mean $= \dfrac{5(3)+6(4)+7(23)+8(50)+9(40)}{3+4+23+50+40}$

$= 8$

C. ✗. Median of the distribution is 8.

D. ✓. Inter-quartile range $= 9 - 7.5$

$= 1.5$

15. $\boxed{\text{B}}$

The upper quartile of the distribution is 210 g.

Required probability $= \dfrac{7}{24}$

16. $\boxed{\text{B}}$

Angle of meals sector

$= (360° - 90°) \times \dfrac{5}{2+3+5}$

$= 135°$

Required expenditure $= 1350 \times \dfrac{135°}{90°}$

$= \$2025$

17. $\boxed{\text{D}}$

We have $x = 5$ or $x + 2 = 5$.

When $x + 2 = 5$, $x = 3$ and the mode is 3 and 5, which should be rejected.

Thus, we have $x = 5$, and the numbers are 2, 3, 5, 5, 7, 8.

I. ✓. Mean $= \dfrac{2+3+5+5+7+8}{6} = 5$

II. ✓. Range $= 8 - 2 = 6$

III. ✓. Inter-quartile range $= 7 - 3 = 4$

18. $\boxed{\text{C}}$

A. ✗. 25% of the passengers wait for more than 12 min.

B. ✗. 75% of the passengers wait for 2 to 12 min.

C. ✓.

D. ✗. 25% of the passengers wait for 8 to 12 min.

19. $\boxed{\text{D}}$

Mean $= 26 \Rightarrow 25 + 32 + \ldots + y = 26 \times 10 \Rightarrow x + y = 59$

Mode $= 20 \Rightarrow x = 20$ or $y = 20$. Without lost of generality, set $x = 0$. Then $y = 39$.

I. ✓. Median $= \dfrac{25+26}{2} = 25.5$

II. ✓.

III. ✓. IQR $= 32 - 20 = 12$

20. $\boxed{\text{A}}$

I. ✗. Mean is not obtainable through box-and-whisker diagram.

II. ✓.

III. ✗. Range $= 90 - 45 = 45$ kg

21. $\boxed{\text{B}}$

Simple calculator work.

22. $\boxed{\text{A}}$

Median $= \dfrac{(20+n)+25}{2} \leq 24$ and Interquartile range $= (30+n)-(10+m) \geq 18$

$n \leq 3$

$n - m \geq -2$

$m - n \leq 2$

I. ✓. $m \leq n+2 \leq 3+2 = 5$ and $m \geq 0$.

II. ✓. From the stem-and-leaf diagram, $n \geq 1$. Combine with $n \leq 3$, we have $1 \leq n \leq 3$.

III. ✗. It is possible that $m = n = 1$ such that all conditions are satisfied.

23. $\boxed{\text{D}}$

A. ✗. Mode = 3

B. ✗. Median = 3

C. ✗. Lower quartile = 2.5

D. ✓.

24. $\boxed{\text{D}}$

In the cumulative frequency curve, steeper $\Rightarrow$ more data in the corresponding class.
So, the data is more concentrated in the lower part.
Minimum, lower quartile, median and upper quartile will be closed to each other.

25. $\boxed{\text{C}}$

Let $\angle ADB = x$. Then $\angle CAD = 2x$.

$$\angle CED = \angle ADB + \angle CAD$$
$$87° = x + 2x$$
$$x = 29°$$

26. $\boxed{\text{C}}$

$\angle DCA = 180° - 120° = 60°$
$\angle BDC = \angle ABD \times \dfrac{3+4}{4+6} = 84°$
$\angle EDB = 180° - 84° = 96°$
$\angle AEC = 120° - 96° = 24°$

27. $\boxed{\text{B}}$

Join OC (to use the property from equal arcs)

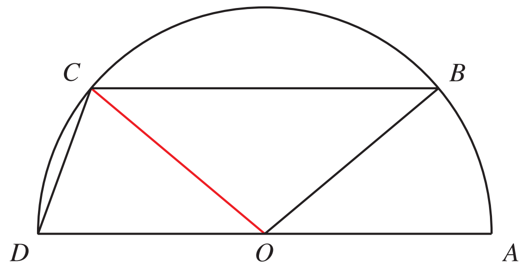
Reflex $\angle BOD = 2 \times 110° = 220°$

$\angle AOB = 220° - 180° = 40°$

$\angle COD = \angle AOB = 40°$

$\angle COB = 180° - 2 \times 40° = 100°$

$\angle OBC = \angle OCB = \dfrac{180° - 100°}{2} = 40°$



28. $\boxed{\text{A}}$

Reflex $\angle AOD = 70° \times \dfrac{3}{1} = 210°$

$\angle AOD = 360° - 210° = 150°$

$\angle ACD = \dfrac{150°}{2} = 75°$

29. $\boxed{\text{D}}$

$\angle P + \angle R = 180°$

$\angle P = 180° \times \dfrac{3}{3+5}$

$= 67.5°$

$\angle Q = \angle P \times \dfrac{4}{3}$

$= 90°$

$\angle S = 180° - 90° = 90°$

30. $\boxed{\text{D}}$

Note that $\triangle EBC \sim \triangle EDA$.

$$\frac{CE}{AE} = \frac{BE}{DE}$$

$$\frac{CE}{4+8} = \frac{8}{10+CE}$$

$$(CE)^2 + 10(CE) - 96 = 0$$

$$CE = 6\,\text{cm} \quad \text{or} \quad -16\,\text{cm (rejected)}$$

31. $\boxed{\text{C}}$

$\angle ABE = 90°$ and $\angle AEB = \angle ACB = 30°$

$\angle EAB = 180° - 90° - 30° = 60°$

32. $\boxed{\text{A}}$

Let O be the centre of the circle.

Then $\angle AOC = 2\angle ABC = 56°$.

$$\frac{56°}{360°} = \frac{\overset{\frown}{AC}}{\text{circumference}}$$

Circumference = 45 cm

33. $\boxed{\text{B}}$

Let D, E and F be mid-points of PQ, RS and TU respectively.

$\triangle BOD \cong \triangle BOE$ and $\triangle COF \cong \triangle COE$.

Let $\angle OBD = a$ and $\angle OCE = b$.

Then $\angle OBE = a$ and $\angle OCF = b$.

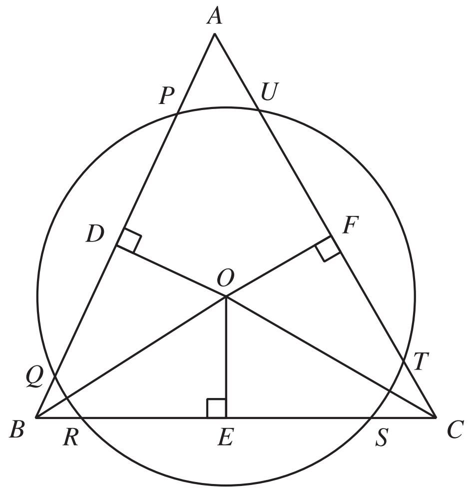
$\angle BOC + a + b = 180°$

$a + b = 72°$

$\angle BAC = 180° - 2a - 2b$

$= 180° - 2(72°)$

$= 36°$



34. $\boxed{\text{C}}$

$\angle POS = \angle ROS = \dfrac{136°}{2} = 68°$

$\angle SPO = \angle PSO = \dfrac{180° - 68°}{2} = 56°$

35. $\boxed{\text{D}}$

Reflex $\angle AOC = 2x$. So, $y = 360° - 2x$.

36. $\boxed{\text{C}}$

Let $\angle PRQ = \theta$.

Since $PQ = QR = RS$, we have $\angle QSR = \angle SQR = \theta$.

Since $\overset{\frown}{PS} : \overset{\frown}{RS} = 2 : 1$, we have $\angle PRS = 2\theta$.

In $\triangle QRS$,

$$\theta + (\theta + 2\theta) + \theta = 180°$$

$$\theta = 36°$$

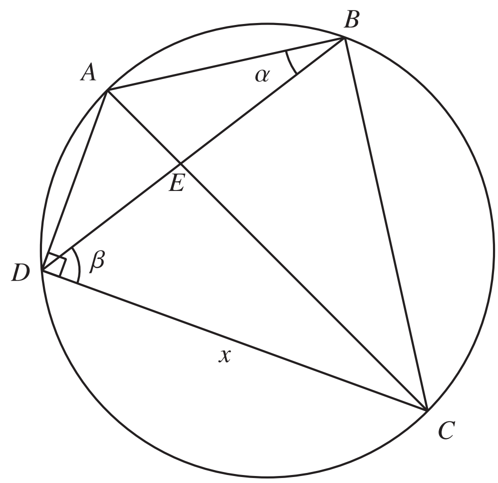
$\angle PES = 2\theta + \theta = 108°$

37. $\boxed{\text{C}}$

$\angle EBC = \angle EDF = 76°$
$\angle AEC = \angle EBC \times \dfrac{1+1}{3+1} = 38°$
$\angle ABC = 180° - \angle AEC = 142°$

38. $\boxed{\text{C}}$

Draw circle $ABCD$.



Note that AC is a diameter of the circle and $\angle ABC = 90°$.
We have $\angle ACD = \angle ABD = \alpha$ and $\angle BAC = \angle BDC = \beta$.

$$\begin{aligned} BC &= AC \sin\beta \\ &= \left(\frac{x}{\cos\alpha}\right)\sin\beta \\ &= \frac{x\sin\beta}{\cos\alpha} \end{aligned}$$

39. $\boxed{\text{C}}$

$\angle ABE = \angle ADE = 28°$ and $\angle ABC = 90°$.
So, $\angle CBE = 28° + 90° = 118°$.

40. $\boxed{\text{D}}$

$\angle ADC = 90°$. $\angle ADE = 90° - 48° = 42°$.
$\angle BCE = \angle ADE = 42°$ and $\angle BEC = 180° - 42° - 56° = 82°$.

41. $\boxed{\text{B}}$

Since $\overset{\frown}{PS} = \overset{\frown}{SR}$, we have $\angle POS = \angle ROS = \dfrac{136°}{2} = 68°$.
Since $OP = OS$, we have $\angle SPO = \angle PSO = \dfrac{180° - 68°}{2} = 56°$.

42. $\boxed{\text{B}}$

$$\angle ADE = \angle DAC + \angle ACD$$
$$= 36^\circ + 24^\circ$$
$$= 60^\circ$$
$$\angle ABE = \angle ADE$$
$$= 60^\circ$$
$$\angle ABD = \angle ABE + \angle DBE$$
$$90^\circ = 60^\circ + \angle DBE$$
$$\angle DBE = 30^\circ$$

43. $\boxed{\text{B}}$

$$\angle ABC = 140^\circ$$
$$\angle AEC = \frac{360^\circ - \angle ABC}{2}$$
$$= 110^\circ$$
$$\angle DAE = \angle ADC - \angle AED$$
$$= 140^\circ - 110^\circ$$
$$= 30^\circ$$

44. $\boxed{\text{D}}$

$\angle CAB = \angle CDB = \angle DBA = \angle DCA = 20^\circ$ and $\angle ACB = 90^\circ$.

$\angle CBD = 180^\circ - 20^\circ - 90^\circ - 20^\circ = 50^\circ$.

45. $\boxed{\text{A}}$

$\angle BAD = 180^\circ - \angle ABC = 38^\circ$. $\angle BOD = 2 \times 38^\circ = 76^\circ$

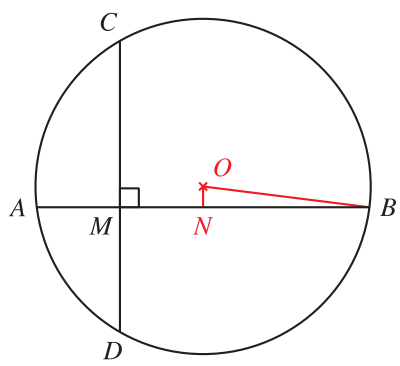
$\angle ADC = \angle ABC = 142^\circ$. $\angle ODE = 180^\circ - 142^\circ = 38^\circ$.

$\angle BED = 76^\circ + 38^\circ = 114^\circ$

46. $\boxed{\text{B}}$

Let O be the centre of the circle, and N be the mid-point of AB such that $ON \perp AB$.

$$\triangle AMD \sim \triangle CMB$$
$$\frac{AM}{MD} = \frac{CM}{MB}$$
$$CM = 4$$
$$NB = \frac{2+6}{2} = 4 \text{ and } ON = 4 - \frac{4+3}{2} = 0.5$$
$$\text{Radius} = \sqrt{4^2 + 0.5^2} \approx 4.03$$



47. $\boxed{\text{D}}$

Reflex $\angle SOP = 360° - 100° = 260°$

$\angle STP = \dfrac{260°}{2} = 130°$

$\angle RTP = 180° - 135° = 45°$

$\angle STR = 130° - 45° = 85°$

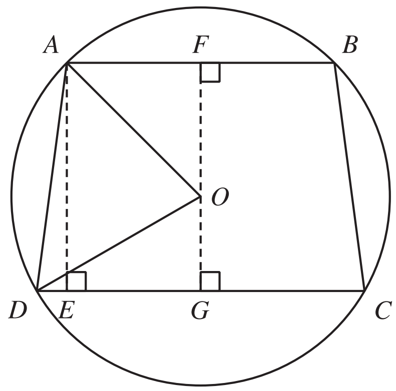
48. $\boxed{\text{D}}$

Note that $\angle ADC = \angle BCD = 80°$ and $AB // CD$.

Let E be a point on CD such that $AE \perp CD$.

Let F and G be the mid-points of AB and CD respectively.

Denote the centre of the circle by O.



Consider $\triangle ADE$.

$$\tan \angle ADE = \frac{AE}{DE}$$

$$\tan 80° = \frac{AE}{\left(\frac{8-6}{2}\right)}$$

$$AE = \tan 80° \text{ cm}$$

Let $OG = x$ cm. Consider $\triangle ODG$ and $\triangle OAF$.

$$r^2 = 3^2 + (AE - x)^2 = 4^2 + x^2$$

$$-2(AE)x = 7 - AE^2$$

$$x \approx 2.22$$

Required area $= r^2\pi$

$= (4^2 + x^2)\pi$

$\approx 65.7 \text{ cm}^2$

49. $\boxed{\text{C}}$

C: $x^2 + y^2 - 2x + 8y - \dfrac{534}{5} = 0$

I. ✓. Centre $(1,\ -4)$. Since $3(1) + 7(-4) + 25 = 0$, the line passes through centre of circle.

II. ✓. $2^2 + 16^2 - 2(2) + 8(-16) - \dfrac{534}{5} = \dfrac{106}{5} > 0$. $(2,\ -16)$ lies outside the circle.

III. ✗.

50. $\boxed{\text{B}}$

Let the radius of C be r.

The coordinates of the centre of C are $(6,\ r)$.

$$r = \sqrt{(6+2)^2 + (r-4)^2}$$
$$r^2 = r^2 - 8r + 80$$
$$r = 10$$

Radius of $C' = \dfrac{10}{\sqrt{4}} = 5$

The equation of C' is

$$(x-6)^2 + (y-10)^2 = 5^2$$
$$x^2 + y^2 - 12x - 20y + 111 = 0$$

51. $\boxed{\text{C}}$

$x^2 + y^2 = 4$ is a circle with centre $(0,\ 0)$ and radius 2.

The locus of P is a pair of concentric circles with centre $(0,\ 0)$ and radii 1 and 3 respectively.

The answer is C.

52. $\boxed{\text{A}}$

Let $P\ (x,\ y)$.

Then the coordinates of A and B are $(2x,\ 0)$ and $(0,\ 2y)$ respectively.

$$AB = \sqrt{(2x)^2 + (2y)^2}$$
$$x^2 + y^2 = \frac{(AB)^2}{4}$$

Locus of P is a arc (part of circle) with centre $(0,\ 0)$ and radius $\dfrac{AB}{2}$.

53. $\boxed{\text{D}}$

I. ✗. Coordinates of centres are $\left(-\dfrac{a}{2},\ \dfrac{b}{2}\right)$ and $\left(\dfrac{a}{2},\ -\dfrac{b}{2}\right)$ respectively.

II. ✓. Both radius $= \sqrt{\left(\dfrac{a}{2}\right)^2 + \left(\dfrac{b}{2}\right)^2 - 0} = \sqrt{\dfrac{a^2}{4} + \dfrac{b^2}{4}}$

III. ✓. Origin is indeed the mid-point of two centres.

54. $\boxed{\text{A}}$

It is the perpendicular bisector of AB.
Slope of $AB = \dfrac{5+1}{1+5} = 1$. Slope of locus $= -1$. Only option A has a line with slope -1.

55. $\boxed{\text{A}}$

The locus of P is a circle with centre $A\ (2,\ -5)$ and radius AB.

A. ✓. Centre $(2,\ -5)$ and $8^2 + 3^2 - 4(8) + 10(3) - 71 = 0$.

B. ✗. Centre $(-2,\ 5)$

C. ✗. Centre $(-2,\ 5)$

D. ✗. Centre $(2,\ -5)$ but $8^2 + 3^2 - 4(8) + 10(3) - 75 = -4 \neq 0$.

56. $\boxed{\text{C}}$

Centre $(-10,\ 12)$. The equation is in the form $x^2 + y^2 + 20x - 24y + F = 0$, where F is a constant.
The coordinates of mid-point of AB are $(-10,\ 0)$.
Thus, the x-coordinates of A and B are $-10 \pm 16 = 6$ or -26.

$$\begin{aligned}(6)^2 + (0)^2 + 20(6) - 24(0) + F &= 0\\ F &= -156\end{aligned}$$

C: $x^2 + y^2 + 20x - 24y - 156 = 0$

57. $\boxed{\text{B}}$

Distance from P to the line AB is the height of $\triangle PAB$, which is a constant.
The locus of P is a pair of straight lines, with a fixed distance to the line AB.

58. $\boxed{\text{D}}$

$$\begin{aligned}PA &= AB\\ \sqrt{(x-3)^2 + (y+5)^2} &= \sqrt{(3+2)^2 + (-5-7)^2}\\ x^2 + y^2 - 6x + 10y + 34 &= 169\\ x^2 + y^2 - 6x + 10y - 135 &= 0\end{aligned}$$

59. $\boxed{\text{C}}$

Let the coordinates of P be $(x,\ y)$.

$$\begin{aligned}AB &= 2AP\\ \sqrt{(3+5)^2 + (1+5)^2} &= 2\sqrt{(x-3)^2 + (y-1)^2}\\ 100 &= 4(x^2 + y^2 - 6x - 2y + 10)\\ x^2 + y^2 - 6x - 2y - 15 &= 0\end{aligned}$$

60. $\boxed{\text{B}}$

$x^2 + y^2 - 2x + 4y - 3 = 0$

I. ✓.

II. ✗. Radius $= \sqrt{1^2 + 2^2 + 3} = \sqrt{8} \neq 3$

III. ✓. $2^2 + 1^2 - 2(2) + 4(1) - 3 = 2$

> 0

(2, 1) lies outside the circle.

61. $\boxed{\text{C}}$

Let $P = (x,\ y)$.

$$\begin{aligned} PX &= 2PY \\ \sqrt{x^2 + (y-5)^2} &= 2\sqrt{(x-1)^2 + y^2} \\ x^2 + (y-5)^2 &= 4[(x-1)^2 + y^2] \\ 0 &= 3x^2 + 3y^2 - 8x + 10y - 21 \end{aligned}$$

62. $\boxed{\text{C}}$

C_2: $x^2 + y^2 + 6x - 8y + \dfrac{33}{2} = 0$

G_1 (4, 3) and G_2 (−3, 4).

I. ✓. Slope of $G_1O\times$ slope of $G_2O = \dfrac{3}{4} \times \dfrac{4}{-3} = -1$

II. ✗. Radius of $C_1 = \sqrt{4^2 + 3^2 - 20} = \sqrt{5}$ and radius of $C_2 = \sqrt{3^2 + 4^2 \dfrac{33}{2}} = \sqrt{8.5} > \sqrt{5}$

So, area of C_1 is smaller than the area of C_2.

III. ✓. $OG_1 = OG_2 = \sqrt{3^2 + 4^2} = 5$

63. $\boxed{\text{D}}$

C_2: $x^2 + y^2 + 10x - 14y + \dfrac{75}{2} = 0$

The coordinates of G_1 and G_2 are (0, 10) and (−5, 7) respectively.

Radius of $C_1 = \sqrt{0^2 + 10^2 - 0} = 10$

Radius of $C_2 = \sqrt{5^2 + 7^2 - \dfrac{75}{2}} = \sqrt{\dfrac{73}{2}}$

I. ✗.

II. ✓. $G_1G_2 = \sqrt{(0+5)^2 + (10-7)^2} = \sqrt{34} < 10$

Thus, G_2 lies inside C_1.

III. ✗. $G_1G_2 = \sqrt{34} < \sqrt{\dfrac{37}{2}}$

Thus, G_1 lies inside C_2.

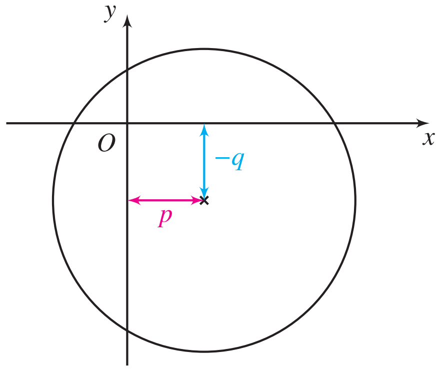
64. $\boxed{\text{D}}$

Centre $(p,\ q)$ lies in quadrant IV. So, $p > 0$ and $q < 0$.

I. ✓.

II. ✓. $p - r < 0$
(length p is shorter than the radius)

III. ✓. $\sqrt{p^2 + q^2} < r$
(distance between origin and centre is smaller than the radius)



65. $\boxed{\text{C}}$

The circle passes through points $(5 \pm 12,\ 0)$, i.e., $(-7,\ 0)$ and $(17,\ 0)$.
Centre $(5,\ -7) \Rightarrow$ The circle is in the form $x^2 + y^2 - 10x + 14y + F = 0$.
Substitute $(-7,\ 0)$, $F = -119$.

66. $\boxed{\text{A}}$

$x^2 + y^2 - x - \frac{y}{2} + \frac{1}{4} = 0$

I. ✗. Centre $\left(\frac{1}{2},\ \frac{1}{4}\right)$

II. ✗. Radius $= \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{4}\right)^2 - \frac{1}{4}} = \frac{1}{4} \neq 2$

III. ✓. Radius $= \frac{1}{4} = y$-coordinate of centre.
So, the circle touches the x-axis.

67. $\boxed{\text{C}}$

C_2: $x^2 + y^2 + 6x - 8y + \frac{25}{2} = 0$

I. ✓. $G_1\ (8,\ -6)$, $G_2\ (-3,\ 4)$. $OG_1 = 10 = 2OG_2$

II. ✗. $m_{OG_1} \times m_{OG_2} = \frac{-6}{8} \times \frac{4}{-3} \neq -1$

III. ✓. Radius of $C_1 = \sqrt{8^2 + 6^2 - 75} = 5$, radius of $C_2 = \sqrt{3^2 + 4^2 - \frac{25}{2}} = \frac{5}{\sqrt{2}}$

Ratio of area of C_1 to area of C_2 is $5 : \left(\frac{5}{\sqrt{2}}\right)^2 = 2 : 1$.

68. $\boxed{\text{B}}$

The locus of P is the angle bisectors of the angles formed by L_1 and L_2.
Slope of $L_1 = -\dfrac{5}{12}$ and slope of $L_2 = \dfrac{5}{12}$.
The locus of P consists of a horizontal line and a vertical line passing through the intersection of L_1 and L_2.
Required equations are $x = 12$ and $y = -3$.

69. $\boxed{\text{A}}$

C: $x^2 + y^2 - 4x + 8y - 5 = 0$

I. ✗. Centre $(2,\ -4)$ lies in the fourth quadrant.

II. ✓. Radius $= \sqrt{2^2 + 4^2 + 5} = 5$ and area $= 5^2\pi = 25\pi$.

III. ✗. $3^2 + (-2)^2 - 4(3) + 8(-2) - 5 = -20 < 0$
The point $(3,\ -2)$ lies inside C.

70. $\boxed{\text{D}}$

Locus of Q should be a pair of straight lines with infinite length, 2 units from L, one above L and one below L.

71. $\boxed{\text{C}}$

L_1 and L_2 are two intersecting straight lines.
The locus of P consists of the two angle bisectors of the two angles formed by L_1 and L_2.

72. $\boxed{\text{D}}$

Centre $(2,\ -1)$
Distance from P to centre $= \sqrt{(2+2)^2 + (1+1)^2} = \sqrt{20}$
Radius $= \sqrt{2^2 + 1^2 + 31} = 6$
Required length $= 2\sqrt{6^2 - (\sqrt{20})^2}$
$= 8$

73. $\boxed{\text{A}}$

I. ✓. Radius $= \sqrt{9} = 3$

II. ✓. The coordinates of centre are $(4,\ 5)$.
Since $3(4) + 4(5) - 32 = 0$, the centre lies on the straight line $3x + 4y - 32 = 0$.
The straight line cuts C into two equal halves.

III. ✗. Distance between the origin and the centre $= \sqrt{4^2 + 5^2} = \sqrt{41} > 3$
The origin O lies outside the circle C.

74. $\boxed{\text{C}}$

The equation of C is $x^2 + y^2 + \frac{12}{5}x - \frac{6}{5}y - \frac{11}{5} = 0$.

I. ✓. Put $(x,\ y) = (0,\ 0)$,
we have $0^2 + 0^2 + \frac{12}{5}(0) - \frac{6}{5}(0) - \frac{11}{5} = -\frac{11}{5} < 0$.
The origin lies inside C.

II. ✓. Radius $= \sqrt{\left(\frac{12}{10}\right)^2 + \left(\frac{3}{5}\right)^2 + \frac{11}{5}}$
$= 2$
Area $= 2^2\pi = 4\pi$

III. ✗. The coordinates of the centre of C are $\left(-\frac{6}{5},\ \frac{3}{5}\right)$.

75. $\boxed{\text{D}}$

C: $x^2 + y^2 - 3x + y - \frac{13}{2} = 0$

I. ✗. Centre $\left(\frac{3}{2},\ -\frac{1}{2}\right)$

II. ✓. $AB = \sqrt{(2-1)^2 + (1+2)^2} = \sqrt{10}$
Radius $= \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \frac{13}{2}} = 3 < AB$

III. ✓. Slope of $AB = \frac{1+2}{2-1} = 3$
Slope of $AG = \frac{-2+\frac{1}{2}}{1-\frac{3}{2}} = 3$, where G is the centre.
Thus, three points are collinear.

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