

REG-RF-2324-ASM-SET 2-MATH

建議題解

多項選擇題

1. B	2. C	3. B	4. D	5. A
6. B	7. A	8. B	9. B	10. A
11. A	12. D	13. D	14. B	15. D
16. C	17. A	18. C	19. B	20. D
21. D	22. A	23. C	24. A	25. C
26. C	27. C	28. A	29. B	30. C

1. B

$$\begin{aligned}\frac{a^2 + a - 2}{a^2 + 7a + 10} &= \frac{(a + 2)(a - 1)}{(a + 2)(a + 5)} \\ &= \frac{a - 1}{a + 5}\end{aligned}$$

2. C

$$\begin{aligned}\frac{\frac{2}{a} + \frac{1}{b}}{\frac{a}{b} - \frac{4b}{a}} &= \frac{\frac{2}{a} + \frac{1}{b}}{\frac{a}{b} - \frac{4b}{a}} \times \frac{ab}{ab} \\ &= \frac{2b + a}{a^2 - 4b^2} \\ &= \frac{2b + a}{(a + 2b)(a - 2b)} \\ &= \frac{1}{a - 2b}\end{aligned}$$

3. B

$$\begin{aligned}\frac{9x - \frac{4}{x}}{3 + \frac{2}{x}} &= \frac{9x - \frac{4}{x}}{3 + \frac{2}{x}} \times \frac{x}{x} \\ &= \frac{9x^2 - 4}{3x + 2} \\ &= \frac{(3x + 2)(3x - 2)}{3x + 2} \\ &= 3x - 2\end{aligned}$$

4. D

$$\begin{aligned}1 + \frac{3x}{x + \frac{2}{x}} &= 1 + \frac{3x}{x + \frac{2}{x}} \times \frac{x}{x} \\ &= 1 + \frac{3x^2}{x^2 + 2} \\ &= \frac{(x^2 + 2) + 3x^2}{x^2 + 2} \\ &= \frac{4x^2 + 2}{x^2 + 2}\end{aligned}$$

5. A

$$\begin{aligned}\frac{\frac{1}{a^2} - \frac{1}{b^2}}{\left(\frac{a}{b} + 1\right)\left(1 - \frac{b}{a}\right)} &= \frac{\frac{1}{a^2} - \frac{1}{b^2}}{\left(\frac{a}{b} + 1\right)\left(1 - \frac{b}{a}\right)} \times \frac{a^2 b^2}{a^2 b^2} \\ &= \frac{b^2 - a^2}{ab(a+b)(a-b)} \\ &= \frac{(b+a)(b-a)}{ab(a+b)(a-b)} \\ &= -\frac{1}{ab}\end{aligned}$$

6. B

$$\begin{aligned}\frac{4x^2 - 25}{2x^2 - 3x - 5} \div \frac{2x + 5}{x^2 + 3x + 2} &= \frac{(2x + 5)(2x - 5)}{(2x - 5)(x + 1)} \times \frac{(x + 1)(x + 2)}{2x + 5} \\ &= x + 2\end{aligned}$$

7. A

$$\begin{aligned}\frac{1}{1-x} - \frac{x+5}{x^2-1} - \frac{2}{x+1} &= \frac{-(x+1) - (x+5) - 2(x-1)}{(x+1)(x-1)} \\ &= \frac{-4x-4}{(x+1)(x-1)} \\ &= \frac{-4(x+1)}{(x+1)(x-1)} \\ &= \frac{4}{1-x}\end{aligned}$$

8. B

$$\begin{aligned}\frac{x}{x-y} - \frac{y}{y+x} + \frac{2xy}{y^2-x^2} &= \frac{x(x+y) - y(x-y) - 2xy}{(x-y)(x+y)} \\ &= \frac{x^2 - 2xy + y^2}{(x-y)(x+y)} \\ &= \frac{(x-y)^2}{(x-y)(x+y)} \\ &= \frac{x-y}{x+y}\end{aligned}$$

9. B

$$\begin{aligned}\frac{3}{x^2-4x-5} + \frac{1}{1-x^2} &= \frac{3}{(x-5)(x+1)} + \frac{1}{(1-x)(1+x)} \\ &= \frac{3(1-x) + (x-5)}{(x-5)(x+1)(1-x)} \\ &= \frac{-2x-2}{(x-5)(x+1)(1-x)} \\ &= \frac{2}{(x-5)(x-1)}\end{aligned}$$

10. A

$$\begin{aligned}\frac{2x}{x^2-9} - \frac{x-2}{x^2-x-6} &= \frac{2x}{(x-3)(x+3)} - \frac{x-2}{(x-3)(x+2)} \\ &= \frac{2x(x+2) - (x-2)(x+3)}{(x-3)(x+3)(x+2)} \\ &= \frac{x^2+3x+6}{(x-3)(x+3)(x+2)}\end{aligned}$$

11. A

$$\begin{aligned}\frac{3x+1}{3x^2+8x-3} - \frac{1}{7x-3x^2-2} &= \frac{3x+1}{(3x-1)(x+3)} - \frac{1}{-(3x-1)(x-2)} \\ &= \frac{(3x+1)(x-2) + (x+3)}{(3x-1)(x+3)(x-2)} \\ &= \frac{3x^2-4x+1}{(3x-1)(x+3)(x-2)} \\ &= \frac{(x-1)(3x-1)}{(3x-1)(x+3)(x-2)} \\ &= \frac{x-1}{(x+3)(x-2)}\end{aligned}$$

12. D

$$\begin{aligned}\frac{x}{x^2-4} - \frac{1}{2x-4} &= \frac{2x-(x+2)}{2(x-2)(x+2)} \\ &= \frac{x-2}{2(x-2)(x+2)} \\ &= \frac{1}{2(x+2)}\end{aligned}$$

13. D

$$\begin{aligned}\frac{1}{(x-1)(x-2)} + \frac{3}{(x-1)(x+2)} &= \frac{x+2+3(x-2)}{(x-1)(x-2)(x+2)} \\ &= \frac{4(x-1)}{(x-1)(x-2)(x+2)} \\ &= \frac{4}{(x-2)(x+2)}\end{aligned}$$

14. B

$$\begin{aligned}1 - \frac{ab}{a^2-b^2} - \frac{b}{b-a} &= \frac{(a^2-b^2) - ab + b(a+b)}{(a+b)(a-b)} \\ &= \frac{a^2}{a^2-b^2}\end{aligned}$$

15. D

$$\begin{aligned}\frac{1}{6x^2-45x+81} - \frac{1}{x-3} &= \frac{1}{3(x-3)(2x-9)} - \frac{1}{x-3} \\ &= \frac{1-3(2x-9)}{3(x-3)(2x-9)} \\ &= \frac{2(3x-14)}{-3(x-3)(2x-9)}\end{aligned}$$

16. C

$$\begin{aligned}\frac{1}{x^2 - 2x + 1} - \frac{1}{x^2 + x - 2} &= \frac{1}{(x-1)^2} - \frac{1}{(x+2)(x-1)} \\ &= \frac{(x+2) - (x-1)}{(x-1)^2(x+2)} \\ &= \frac{3}{(x-1)^2(x+2)}\end{aligned}$$

17. A

$$\begin{aligned}\frac{x^2 - 3x}{x^3 + 27} - \frac{1}{x+3} &= \frac{x^2 - 3x}{(x+3)(x^2 - 3x + 9)} - \frac{x^2 - 3x + 9}{(x+3)(x^2 - 3x + 9)} \\ &= \frac{-9}{x^3 + 27}\end{aligned}$$

18. C

$$\begin{aligned}\frac{4}{x^2 - x - 12} + \frac{x}{x+3} &= \frac{4 + x(x-4)}{(x+3)(x-4)} \\ &= \frac{(x-2)^2}{(x+3)(x-4)}\end{aligned}$$

19. B

$$(a+2)^2, (a+2)(a-2), (a+2)(a^2 - 2a + 4)$$

$$\text{L.C.M.} = (a-2)(a+2)^2(a^2 - 2a + 4)$$

20. D

該三個數式為 $(x-3)^2$ 、 $(x-3)(x-1)$ 及 $(x+3)(x-3)$ 。

$$\text{L.C.M.} = (x-3)^2(x-1)(x+3)$$

21. D

$$\begin{aligned}8x^3 - 1 &= (2x-1)(4x^2 + 2x + 1) \\ 4x^2 - 1 &= (2x+1)(2x-1) \\ 6x^2 + 3x - 3 &= 3(2x-1)(x+1) \\ \text{L.C.M.} &= 3(x+1)(2x-1)(2x+1)(4x^2 + 2x + 1)\end{aligned}$$

22. A

該三個數式為 $2^2m^2n^5$ 、 $2 \cdot 3m^3n^3$ 及 $2^3m^5n^4$ 。

$$\text{H.C.F. 為 } 2m^2n^3。$$

23. C

$$2x^2 + 5x - 3 = (x+3)(2x-1) \text{ 及 } x^4 + 27x = x(x+3)(x^2 - 3x + 9)$$

$$\text{L.C.M.} = x(x+3)(2x-1)(x^2 - 3x + 9)$$

24. A

$$\text{H.C.F.} = xy^2$$

25. C

$$3x^4y^2z \quad 2^2xy^5z \quad (2)(3)x^2y^3$$

$$\text{L.C.M.} = 2^2(3)x^4y^5z = 12x^4y^5z$$

26. C

$$3^2a^2b, (2)^2(3)a^4b^3, (3)(5)a^6$$

$$\text{L.C.M.} = (2)^2(3)^2(5)a^6b^3 = 180a^6b^3$$

27. C

L.C.M. 是由選取每一因式的最大次方組成。
L.C.M. 為 x^2y^3z 。

28. A

考慮三個數式中的變量 x 、 y 及 z 的次方。從 H.C.F. 及 L.C.M. 的數式中，可得出以下結果：

變量	x	y	z
最小次方	2	2	1
最大次方	3	4	5

因此，第三數式是 x^2y^4z 。

29. B

- A. ✗。L.C.M. 會變成 $4a^4b^4c^6$ 。
- B. ✓。
- C. ✗。H.C.F. 會包含因數 2。
- D. ✗。H.C.F. 會包含因數 2。

30. C

考慮 a 的次方， $\max \{1, 4, r\} = 6 \Rightarrow r = 6$
考慮 b 的次方， $\min \{3, 3, s\} = 1 \Rightarrow s = 1$

結構式試題

$$\begin{aligned}
 31. \quad & \frac{x^2 + 7x + 6}{x^2 - x - 2} \times \frac{x - 2}{x - 3} \times \frac{x^2 - x - 12}{x - 4} \\
 &= \frac{(x + 1)(x + 6)}{(x - 2)(x + 1)} \times \frac{x - 2}{x - 3} \times \frac{(x - 4)(x + 3)}{x - 4} \\
 &= \frac{(x + 6)(x + 3)}{x - 3}
 \end{aligned}$$

1M+1A
1A

$$\begin{aligned}
 32. \quad & \frac{a}{1+a} + \frac{b}{1-b} = \frac{\left(\frac{2x+1}{2x-1}\right)}{1 + \frac{2x+1}{2x-1}} + \frac{\left(\frac{2x-1}{2x+1}\right)}{1 - \frac{2x-1}{2x+1}} \\
 &= \frac{2x+1}{2x-1} \div \frac{(2x-1) + (2x+1)}{2x-1} + \frac{2x-1}{2x+1} \div \frac{(2x+1) - (2x-1)}{2x+1} \\
 &= \frac{2x+1}{4x} + \frac{2x-1}{2} \\
 &= \frac{(2x+1) + 2x(2x-1)}{4x} \\
 &= \frac{4x^2 + 1}{4x}
 \end{aligned}$$

1M
1M
1M
1A

$$\begin{aligned}
 33. \quad & \frac{A}{2x-1} + \frac{B}{3x-2} = \frac{A(3x-2) + B(2x-1)}{(2x-1)(3x-2)} \\
 &= \frac{(3A+2B)x + (-2A-B)}{(2x-1)(3x-2)}
 \end{aligned}$$

1M

比較係數，

$$\begin{cases} 3A + 2B = -1 \\ -2A - B = -1 \end{cases}$$

1M
1M

求解後，可得 $A = 3$ 及 $B = -5$ 。

$$\begin{aligned}
 34. \quad & \text{(a) 代 } x = 1, 3(1)^3 - 6(1) + 4(1) - 1 = 0. \\
 & \text{故此, } x - 1 \text{ 為 } 3x^3 - 6x^2 + 4x - 1 \text{ 的因式。} \\
 & \text{代 } x = -1, -6(-1)^3 + 4(-1) - 2 = 0. \\
 & \text{故此, } x + 1 \text{ 為 } -6x^3 + 4x - 2 \text{ 的因式。}
 \end{aligned}$$

1M
1
1

$$\begin{aligned}
 \text{(b) } & \frac{3x^3 - 6x^2 + 4x - 1}{-6x^3 + 4x - 2} = \frac{(x-1)(3x^2 - 3x + 1)}{(x+1)(-6x^2 + 6x - 2)} \\
 &= \frac{(x-1)(3x^2 - 3x + 1)}{-2(x+1)(3x^2 - 3x + 1)} \\
 &= -\frac{x-1}{2(x+1)}
 \end{aligned}$$

1M
1M
1A

$$35. \quad (a) \quad (i) \quad f\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right)^3 + h\left(-\frac{1}{2}\right)^2 - k\left(-\frac{1}{2}\right) - 2 = 0 \quad 1M$$

$$\frac{h}{4} + \frac{k}{2} = \frac{9}{4}$$

$$h = 9 - 2k \quad 1A$$

$$(ii) \quad f(x) = 2x^3 + (9 - 2k)x^2 - kx - 2$$

$$= (2x + 1)(x^2 + (4 - k)x - 2) \quad 1M$$

$$\text{因此, } Q(x) = x^2 + (4 - k)x - 2. \quad 1A$$

$$(b) \quad x^2 + 2x - 3 = (x + 3)(x - 1)$$

線性因式只可能為 $x + 3$ 及 $x - 1$ 。

$$f(1) = 2 + h - k - 2 \quad 1M$$

$$= h - k$$

若 $x - 1$ 為 $f(x)$ 和 $x^2 + 2x - 3$ 的公因式, 則 $h - k = 0$ 。

求解後, 可得 $h = k = 3$ 。 1A

$$\begin{aligned} f(-3) &= 2(-3)^3 + h(-3)^2 - k(-3) - 2 \\ &= 9h + 3k - 56 \end{aligned}$$

若 $x + 3$ 為公因式, 則 $9h + 3k - 56 = 0$ 。

求解後, 可得 $h = \frac{17}{3}$ 及 $k = \frac{5}{3}$ 。 1A

故此, 只有一對整數 h 及 k 使得 $f(x)$ 和 $x^2 + 2x - 3$ 有一線性公因式。

不同意該宣稱。 1A

$$36. \quad (a) \quad \text{設 } f(x) = 3x^3 - x^2 - 12x + 4.$$

$$\begin{aligned} f(2) &= 3(2)^3 - 2^2 - 12(2) + 4 \\ &= 0 \end{aligned}$$

1M

因此, $x - 2$ 為 $3x^3 - x^2 - 12x + 4$ 的因式。 1

$$(b) \quad f(x) = (x - 2)(3x^2 + 5x - 2) \quad 1M$$

$$= (x - 2)(3x - 1)(x + 2) \quad 1A$$

(c) 設商式為 $q(x)$, 其中 $q(x)$ 為一多項式。

$$3x^3 - x^2 - 7x + 5 = (ax^2 + bx + 2)q(x) + 5x + 1 \quad 1M$$

$$3x^3 - x^2 - 12x + 4 = (ax^2 + bx + 2)q(x)$$

$$(x - 2)(3x - 1)(x + 2) = (ax^2 + bx + 2)q(x)$$

由於 $ax^2 + bx + 2$ 的常數項為 2,

$$ax^2 + bx + 2 = (x - 2)(3x - 1) = 3x^2 - 7x + 2; \text{ 或} \quad 1M$$

$$ax^2 + bx + 2 = -(x + 2)(3x - 1) = -3x^2 - 5x + 2; \text{ 或}$$

$$ax^2 + bx + 2 = -\frac{1}{2}(x + 2)(x - 2) = -\frac{1}{2}x^2 + 2.$$

因此, $(a, b) = (3, -7)$ 或 $(-3, -5)$ 或 $\left(-\frac{1}{2}, 0\right)$ 。 1A+1A