REV-VAR-2324-ASM-SET 1-MATH
Suggested solutions

Multiple Choice Questions

1. C	2. D	3. D	4. B	5. A
6. A	7. D	8. A	9. D	10. A
11. D	12. B	13. B	14. C	15. C
16. D	17. A	18. B	19. A	20. A
21. D	22. D	23. D	24. B	25. D
26. B	27. D	28. A	29. B	30. C

1. C

Let $x = \dfrac{k\sqrt{y}}{z^3}$, where k is a non-zero constant.

$$\frac{3}{2} = \frac{k\sqrt{4}}{3^3}$$
$$k = \frac{81}{4}$$

Thus, $x = \dfrac{81\sqrt{y}}{4z^3}$.

2. D

Let $z = \dfrac{kx}{\sqrt{y}}$, where k is a non-zero constant. Then $k = \dfrac{z\sqrt{y}}{x}$.

$$k = \frac{4\sqrt{16}}{4} = \frac{z\sqrt{4}}{16} \quad \Rightarrow \quad z = 32$$

3. D

Let $z = kx^2\sqrt{y}$, where k is a non-zero constant.

$$7 = k(1)^2\sqrt{36}$$
$$k = \frac{7}{6}$$

$$\text{Required value} = \frac{7}{6}(3)^2\sqrt{144}$$
$$= 126$$

4. B

Let $c = ka\sqrt{n}$, where k is a non-zero constant.

$$312\,000 = k(600)\sqrt{25}$$

$$k = 104$$

$$\text{Cost} = 104(800)\sqrt{9}$$

$$= 249\,600$$

5. A

Let $x = \dfrac{k}{y^3 z}$, where k is a constant.

Then $k = xy^3 z$ is a constant.

6. A

Let $y = \dfrac{k\sqrt{x}}{z^2}$, where k is a non-zero constant.

Then $k = \dfrac{yz^2}{\sqrt{x}}$ and so $\dfrac{x}{y^2 z^4} = k^{-2}$ is a constant.

7. D

Let $z = \dfrac{k\sqrt{x}}{y^2}$, where k is a non-zero constant.

Then $k = \dfrac{y^2 z}{\sqrt{x}}$.

$\dfrac{y^4 z^2}{x} = k^2$ is a constant.

8. A

Let $a = \dfrac{kb^2}{\sqrt{c}}$, where k is a non-zero constant. Then $k = \dfrac{a\sqrt{c}}{b^2}$ is a constant.

9. D

Let $y = ax^3$ and $z = \dfrac{b}{y^2}$, where a and b are non-zero constants.

Then $z = \dfrac{b}{a^2 x^6}$

I. ✓. $zy^2 = b$ is a constant.

II. ✓. $zx^6 = \dfrac{b}{a^2}$ is a constant.

III. ✓. $\dfrac{zy^4}{x^6} = \dfrac{(zy^2)^2}{zx^6} = \dfrac{b^2}{\left(\frac{b}{a^2}\right)}$ is a constant.

10. A

Let $y = \dfrac{kx}{\sqrt{z}}$, where k is a non-zero constant.

Then $k = \dfrac{y\sqrt{z}}{x}$.

Therefore, $\dfrac{x}{y\sqrt{z}} = \dfrac{1}{k}$ must be a constant.

11. D

Let $x = \dfrac{ky^3}{\sqrt{z}}$, where k is a non-zero constant.

$$k = \frac{x\sqrt{z}}{y^3}$$

$$k^2 = \frac{x^2 z}{y^6}$$

$$\frac{y^6}{x^2 z} = \frac{1}{k^2} = \text{constant}$$

12. B

We have $x = \dfrac{k\sqrt{z}}{y}$.

So, $k = \dfrac{xy}{\sqrt{z}}$ is a constant $\Rightarrow k^2 = \dfrac{x^2 y^2}{z}$ is also a constant.

13. B

Let $w = \dfrac{k\sqrt{u}}{v}$, where k is a non-zero constant. Then $k = \dfrac{vw}{\sqrt{u}}$.

So, $\dfrac{wv}{\sqrt{u}} = k$ is a constant.

14. C

Let $z = \dfrac{a}{x^2}$ and $x = b\sqrt{y}$, where a and b are non-zero constants.

$$z = \frac{a}{(b\sqrt{y})^2}$$

$$= \frac{a}{b^2 y}$$

Thus, $z \propto \dfrac{1}{y}$.

15. C

Let $x = \dfrac{k\sqrt{y}}{z}$, where k is a non-zero constant.

$$x = \frac{k\sqrt{\frac{4z}{3}}}{z}$$
$$= k\sqrt{\frac{4}{3}} \cdot \frac{1}{\sqrt{z}}$$

Thus, $x \propto \dfrac{1}{\sqrt{z}}$.

16. D

Let $x = \dfrac{ky^2}{\sqrt{z}}$, where k is a non-zero constant.

$$x = \frac{ky^2}{\sqrt{z}}$$
$$\sqrt{z} = \frac{ky^2}{x}$$
$$z = \frac{k^2y^4}{x^2}$$

Thus, $z \propto \dfrac{y^4}{x^2}$.

17. A

Let $c = \dfrac{ka^2}{b}$, where k is a non-zero constant.

$$\text{Percentage change} = \frac{\frac{k(0.4a)^2}{1.6b} - \frac{ka^2}{b}}{\frac{ka^2}{b}} \times 100\%$$
$$= -90\%$$

18. B

Let $z = \dfrac{ky}{x^2}$, where k is a non-zero constant.

$$\text{Percentage change} = \frac{\frac{k(1.35y)}{(1.25x)^2} - \frac{ky}{x^2}}{\frac{ky}{x^2}} \times 100\% = -13.6\%$$

19. A

Let $z = \dfrac{kx^2}{\sqrt{y}}$, where k is a non-zero constant.

$$\text{Percentage change of } z = \frac{\frac{k(0.6x)^2}{\sqrt{1.44y}} - \frac{kx^2}{\sqrt{y}}}{\frac{kx^2}{\sqrt{y}}} \times 100\% = -70\%$$

20. A

Let $p = \dfrac{kr}{q^2}$, where k is a non-zero constant.

$$\begin{aligned}\frac{p_2}{p_1} &= \frac{1-10\%}{(1+20\%)^2}\\ &= 0.625\end{aligned}$$

p is decreased by 37.5%.

21. D

Let $z = \dfrac{k\sqrt{x}}{y^3}$, where k is a non-zero constant.

$$\begin{aligned}\text{Percentage change} &= \frac{\frac{k\sqrt{1.1x}}{(0.9y)^3} - \frac{k\sqrt{x}}{y^3}}{\frac{k\sqrt{x}}{y^3}} \times 100\%\\ &\approx 43.9\%\end{aligned}$$

22. D

Let $z = \dfrac{kx^2}{\sqrt{y}}$, where k is a non-zero constant.

$$\begin{aligned}z' &= \frac{(1.1)^2}{\sqrt{1-0.36}}\\ &= 1.5125\end{aligned}$$

z is increased by 51.25%.

23. D

Let $x = \dfrac{ky^2}{z}$, where k is a non-zero constant.

$$\begin{aligned}\frac{x_2}{x_1} &= \frac{\left(\frac{y_2}{y_1}\right)^2}{\left(\frac{z_2}{z_1}\right)}\\ &= \frac{1.2^2}{0.8}\\ &= 1.8\end{aligned}$$

Percentage increase = 80%

24. B

Let $x = \dfrac{kz}{\sqrt{y}}$, where k is a non-zero constant.

$$\begin{aligned}\text{Percentage change} &= \frac{\frac{k(0.88z)}{\sqrt{1.21y}} - \frac{kz}{\sqrt{y}}}{\frac{kz}{\sqrt{y}}} \times 100\%\\ &= -20\%\end{aligned}$$

25. D

Let $z = \dfrac{kx}{y^2}$, where k is a non-zero constant.

$$\text{Percentage change in } z = \frac{\frac{k(0.9x)}{(0.75y)^2} - \frac{kx}{y^2}}{\frac{kx}{y^2}}$$

$$= +60\%$$

26. B

Let $x = \dfrac{k\sqrt{y}}{z^2}$, where k is a non-zero constant.

$$\text{Percentage change} = \frac{\frac{k\sqrt{1.44y}}{(0.8z)^2} - \frac{k\sqrt{y}}{z^2}}{\frac{k\sqrt{y}}{z^2}} \times 100\%$$

$$= 87.5\%$$

27. D

Let $z = \dfrac{k\sqrt{x}}{y}$, where k is a non-zero constant.

$$\frac{z_2}{z_1} = \frac{\sqrt{\frac{x_2}{x_1}}}{\left(\frac{y_2}{y_1}\right)}$$

$$= \frac{\sqrt{1 - 0.36}}{1 + 0.6}$$

$$= 0.5$$

z is decreased by 50%.

28. A

Let $w = \dfrac{ku^3}{\sqrt{v}}$, where k is a non-zero constant.

Then $v = \dfrac{k^2u^6}{w^2}$.

$$\text{Percentage change} = \frac{\frac{k^2(1.1u)^6}{(1.21w)^2} - \frac{k^2u^6}{w^2}}{\frac{k^2u^6}{2^2}} \times 100\%$$

$$= \frac{\frac{1.1^6}{1.21^2} - 1}{1} \times 100\%$$

$$= 21\%$$

29. B

Let $a = mb^2$ and $\sqrt{b} = \dfrac{n}{c}$, where m and n are non-zero constants.

Then $c = \dfrac{n}{\sqrt{b}} = \dfrac{nm^{\frac{1}{4}}}{a^{\frac{1}{4}}} = \dfrac{p}{a^{\frac{1}{4}}}$, where $p = nm^{\frac{1}{4}}$ is a constant.

$$\text{Percentage change} = \frac{\frac{p}{(1.3a)^{\frac{1}{4}}} - \frac{p}{a^{\frac{1}{4}}}}{\frac{p}{a^{\frac{1}{4}}}} \approx -6.35\%$$

30. C

Let $z = \dfrac{kx}{\sqrt{y}}$, where k is a non-zero constant.

I. ✗. The statement is false when $k \neq 1$.

II. ✓. $k = \dfrac{z\sqrt{y}}{x} \Rightarrow \dfrac{yz^2}{x^2} = k^2 = \text{constant}$

III. ✓. $\dfrac{0.5z\sqrt{4y}}{x} = \dfrac{z\sqrt{y}}{x} = k$. The percentage changes are correct.

Conventional Questions

31. (a) Let $f(x) = ax^2 + b$, where a and b are non-zero constants. 1A

Then we have $9a + b = 27$ and $64a + b = -83$ 1M

Solving, we have $a = -2$ and $b = 45$. 1A

$f(7) = -2(7)^2 + 45 = -53$ 1A

(b) $a = f(7) = -53$ and $b = f(-7) = -53$ 1M

Area of $ABCD = \dfrac{[(7+7)+6](53)}{2}$ 1M+1M

$= 530$ 1A

32. (a) Let $f(x) = ax^2 + bx^3$, where a and b are non-zero constants. 1A

$$\begin{cases} -3 = a + b \\ -4 = 4a + 8b \end{cases}$$ 1M

Solving, we have $a = -5$ and $b = 2$. 1A

Therefore, $f(x) = -5x^2 + 2x^3$ and $f(3) = -5(3)^2 + 2(3)^3 = 9$ 1A

(b) $f(m) = 9$

$-5m^2 + 2m^3 - 9 = 0$

$(m-3)(2m^2 + m + 3) = 0$ 1M

$$m = 3 \quad \text{or} \quad \frac{-1 \pm \sqrt{1^2 - 4(2)(3)}}{2(2)}$$

$$= 3 \quad \text{or} \quad \frac{-1 \pm \sqrt{23}i}{4}$$ 1A

There is only one real root. The claim is agreed. 1A

33. (a) Let $C = ar^2 + br^3$, where a and b are non-zero constants. 1A

$$\begin{cases} 64a + 512b = 80 \\ 100a + 1000b = 150 \end{cases}$$ 1M

Solving, we have $a = \dfrac{1}{4}$ and $b = \dfrac{1}{8}$. 1A

Required cost $= \dfrac{12^2}{4} + \dfrac{12^3}{8} = \$252$ 1A

(b) Let the radius of the smaller balls be R cm.

$\dfrac{4}{3}\pi(12)^3 = 10 \times \dfrac{4}{3}\pi R^3$ 1A

$R \approx 5.57$

Percentage change $= \dfrac{10\left(\frac{R^2}{4} + \frac{R^3}{8}\right) - 252}{252} \times 100\%$ 1M

$\approx 16.5\%$ 1A

34. (a) Let $V = at + bt^2$, where a and b are non-zero constants. 1A

$$\begin{cases} 5a + 25b = 190 \\ 15a + 225b = 510 \end{cases}$$ 1M

Solving, we have $a = 40$ and $b = -\frac{2}{5}$. So, $V = 40t - \frac{2t^2}{5}$. 1A

(b) $V = 40(25) - \frac{2(25)^2}{5} = 750$ 1M

Consider the cross section in the figure. When θ is maximum, the water level touches point B. Let E be a point on AD such that BE is horizontal.

$$\frac{(AE)(10)}{2}(10) = 1000 - 750$$ 1M

$$AE = 5\,\text{cm}$$

$\theta = \tan^{-1}\frac{AE}{10} \approx 26.6°$ 1A

END OF PAPER