REG-POLY-2324-ASM-SET 3-MATH
Suggested solutions

Multiple Choice Questions

1. C	2. C	3. B	4. B	5. D
6. C	7. C	8. A	9. B	10. D
11. B	12. C	13. A	14. B	15. C
16. C	17. B	18. D		

1. $\boxed{\text{C}}$

   No steps required.

2. $\boxed{\text{C}}$

   Put $x = \frac{1}{3}$.

   A. $\left(\frac{1}{3}\right)^2 - 2\left(\frac{1}{3}\right) - 3 = -\frac{32}{9} \neq 0$

   B. $3\left(\frac{1}{3}\right)^2 + 4\left(\frac{1}{3}\right) + 1 = \frac{8}{3} \neq 0$

   C. $3\left(\frac{1}{3}\right)^2 + 8\left(\frac{1}{3}\right) - 3 = 0$

   D. $3\left(\frac{1}{3}\right)^2 + 10\left(\frac{1}{3}\right) + 3 = \frac{20}{3}$

   The polynomial $x^2 + 8x - 3$ has a factor $3x - 1$.

3. $\boxed{\text{B}}$

   Put $x = 1$.

   I. $1^2 + 1 - y - y = 2 - 2y$

   II. $1(1 - y) - 1 + y = 0$

   III. $1^2 + y + 1(1 + y) = 2 + 2y$

   Only $x(x - y) - x + y$ has a factor of $x - 1$.

4. $\boxed{\text{B}}$

   Let $f(x) = x^3 - 3x^2 - x + 6$.

   A. ✗. $f(1) = 3 \neq 0$

   B. ✓. $f(2) = 0$

   C. ✗. $f(3) = 3 \neq 0$

   D. ✗. $f(6) = 108 \neq 0$

5. $\boxed{\text{D}}$

Let $f(x) = (6x^2 + 7x - 3)Q(x)$, where $Q(x)$ is a polynomial.

$$\begin{aligned} f(x) &= (6x^2 + 7x - 3)Q(x) \\ &= (2x + 3)(3x - 1)Q(x) \end{aligned}$$

We have $f\left(-\dfrac{3}{2}\right) = f\left(\dfrac{1}{3}\right) = 0$.

6. $\boxed{\text{C}}$

Let $q(x) = (x + 3)Q(x)$, where $Q(x)$ is a polynomial.

$$\begin{aligned} q(3x + 2) &= [(3x + 2) + 3]Q(3x + 2) \\ &= (3x + 5)Q(3x + 2) \end{aligned}$$

Thus, $3x + 5$ must be a factor of $q(3x + 2)$.

7. $\boxed{\text{C}}$

$$\begin{aligned} 2(-k)^3 - 8(-k)^2 + 3(-k - k) &= 0 \\ -2k^3 - 8k^2 - 6k &= 0 \\ 2k(k + 1)(k + 3) &= 0 \\ k &= 0 \quad \text{or} \quad -1 \quad \text{or} \quad -3 \end{aligned}$$

8. $\boxed{\text{A}}$

$$\begin{aligned} P(2) = 0 &= (2 - 2)Q(2) + 2a + 6 \\ 0 &= 2a + 6 \\ a &= -3 \end{aligned}$$

9. $\boxed{\text{B}}$

$$\begin{aligned} f(-2) = 0 &= 3(-2)^2 + 5(-2) - k \\ k &= 2 \end{aligned}$$

Remainder $= f(2) = 12 + 10 - 2 = 20$

10. $\boxed{\text{D}}$

$$\begin{aligned} (-k)^3 + (k - 2)(-k)^2 - 7(-k) + (k - 8) &= 0 \\ -2k^2 + 8k - 8 &= 0 \\ k &= 2 \end{aligned}$$

11. $\boxed{\text{B}}$

$3^2 + 3h + k = 0 \qquad$ and $\qquad 3^3 + (h - 9)(3^2) + 19(3) + k = 0$

$3h + k = -9 \qquad\qquad\qquad 9h + k = -3$

Solving, we have $h = 1$ and $k = -12$.

12. $\boxed{\text{C}}$

$$\begin{aligned} 2\left(\frac{3}{2}\right)^3 - 3\left(\frac{3}{2}\right)^2 + k\left(\frac{3}{2}\right) + 12 &= 0 \\ 12 + \frac{3k}{2} &= 0 \\ k &= -8 \end{aligned}$$

13. $\boxed{\text{A}}$

$$\begin{aligned} 2k^3 - 3k + k &= 0 \\ 2k^3 - 2k &= 0 \\ k^3 &= k \end{aligned}$$

$$\begin{aligned} \text{Remainder} &= 2(-k)^3 - 3(-k) + k \\ &= -2k^3 + 4k \\ &= -2(k) + 4k \\ &= 2k \end{aligned}$$

14. $\boxed{\text{B}}$

$$\begin{aligned} 6(-p)^3 + (6p+1)(-p)^2 - 12(-p) + 36 &= 0 \\ p^2 + 12p + 36 &= 0 \\ p &= -6 \end{aligned}$$

15. $\boxed{\text{C}}$

$$\begin{aligned} (-2)^3 + 3(-2)^2 - 10(-2) + k &= 0 \\ k &= -24 \end{aligned}$$

I. ✓. $P(3) = 3^3 + 3(3)^2 - 10(3) - 24 = 0$

II. ✗. $P(-3) = (-3)^3 + 3(-2)^2 - 10(-3) - 24 = 6 \neq 0$

III. ✓. $P(-4) = (-4)^3 + 3(-4)^2 - 10(-4) - 24 = 0$

16. $\boxed{\text{C}}$

$h(1) = 1 + c + d = 0$

$$\begin{aligned} \text{Remainder} &= (-1)^{2020} + c(-1)^{2019} + d \\ &= 1 - c + d \\ &= 1 - c + (-1 - c) \\ &= -2c \end{aligned}$$

17. $\boxed{\text{B}}$

$4^3 + k(4)^2 + 2k(4) + 8 = 0$

$k = -3$

$P(x) = x^3 - 3x^2 - 6x + 8$

By calculator, $P(1) = P(-2) = 0$. So, $(x - 1)(x + 2)$ is a factor of $P(x)$.

Only option B satisfies this.

18. $\boxed{\text{D}}$

$4(-1)^3 - 7(-1) - 3 = -4 + 7 - 3 = 0$

$x + 1$ is a factor of $4x^3 - 7x - 3$.

$$\begin{aligned} 4x^3 - 7x - 3 &= (x + 1)(4x^2 - 4x - 3) \\ &= (x + 1)(2x - 3)(2x + 1) \end{aligned}$$

Conventional Questions

19. (a) $3(-3)(-3+1)(-3+2)+k=0$ 1M

$$k=18$$ 1A

(b) $3x(x+1)(x+2)+18=0$

$$3x^3+9x^2+6x+18=0$$

$$3(x+3)(x^2+2)=0$$ 1M+1A

$$x=-3 \quad \text{or} \quad x^2=-2$$

Since the equation $x^2=-2$ has no real roots, the claim is disagreed. 1A

20. (a) $f(-2)=0$

$$4(-2)^3+b(-2)^2+c(-2)+18=0$$ 1M

$$2b-c=7$$

$$f(1)=f\left(-\frac{3}{2}\right)$$

$$4+b+c+18=4\left(-\frac{3}{2}\right)^3+b\left(-\frac{3}{2}\right)^2+c\left(-\frac{3}{2}\right)+18$$ 1M

$$b-2c=14$$

Solving the simultaneous equations $\begin{cases} 2b-c=7 \\ b-2c=14 \end{cases}$, 1M

we have $b=0$ and $c=-7$. 1A

(b) $f(x)=4x^3-7x+18$

$$=(x+2)(4x^2-8x+9)$$ 1M+1A

(c) Remainder $=f(-1-1)$

$=f(-2)$

$=4(-2)^3-7(-2)+18$ 1M

$=0$ 1A

21. (a) $f(-1) = h = -h - 1 + 9 + k$ 1M

$2h - k = 8$

$0 = f\left(\frac{1}{2}\right) = \frac{h}{8} - \frac{1}{4} - \frac{9}{2} + k$ 1M

$h + 8k = 38$

Solving, we have $h = 6$ and $k = 4$. 1A+1A

(b) $f(x) = 6$

$6x^3 - x^2 - 9x - 2 = 0$

$(x+1)(6x^2 - 7x - 2) = 0$ 1M

$x = -1$ or $\frac{7 \pm \sqrt{7^2 - 4(6)(-2)}}{2(6)}$ 1M

$= -1$ or $\frac{7 \pm \sqrt{97}}{12}$

There are some irrational roots in $f(x) = 6$.

The claim is disagreed. 1A

22. (a) $f(-1) = 0 = -4(-1)^3 + (a+2)(-1)^2 + 2(-1) - 3b$ 1M

$a - 3b = -4$

$f(2) = 9 = -4(2)^3 + (a+2)(2)^2 + 2(2) - 3b$ 1M

$4a - 3b = 29$

Solving, we have $a = 11$ and $b = 5$. 1A

(b) $f(x) = -4x^3 + 13x^2 + 2x - 15$

$= (-x^2 + 2x + 3)(4x - 5)$ 1M

So, $g(x) = 4x - 5$.

$kx(4x - 5) = (-x^2 + 2x + 3)(4x - 5)$ 1M

$(4x - 5)(x^2 + (k-2)x - 3) = 0$ 1M

$x = \frac{5}{4}$ or $x^2 + (k-2)x - 3 = 0$

$\Delta = (k-2)^2 - 4(1)(-3) = (k-2)^2 + 12 > 0$ for all real values of k.

$x^2 + (k-2)x - 3 = 0$ has two distinct real roots.

The claim is agreed. 1A

END OF PAPER