REG-POLY-2324-ASM-SET 1-MATH
Suggested solutions

Multiple Choice Questions

1. B	2. B	3. C	4. D	5. C
6. D	7. D	8. B	9. B	10. C
11. C	12. B	13. B	14. A	15. C
16. D	17. B	18. A		

1. $\boxed{\text{B}}$

$$
\begin{array}{rr}
 & x - 3 \\ \cline{2-2}
x+1 \big) & x^2 - 2x + 3 \\
 & \underline{x^2 + x\phantom{{}+3}} \\
 & -3x + 3 \\
 & \underline{-3x - 3} \\
 & 6
\end{array}
$$

2. $\boxed{\text{B}}$

$$
\begin{array}{rr}
 & x^2 + 0x + 1 \\ \cline{2-2}
x^2+0x-1 \big) & x^4 + 0x^3 + 0x^2 + 0x + 1 \\
 & \underline{x^4 + 0x^3 - x^2\phantom{{}+0x+1}} \\
 & x^2 + 0x + 1 \\
 & \underline{x^2 + 0x - 1} \\
 & 2
\end{array}
$$

3. $\boxed{\text{C}}$

$$\frac{2x^3 + 54}{x + 3} = 2x^2 - 6x + 18$$

4. $\boxed{\text{D}}$

$$\begin{array}{rr}
 & 2x^2 - 5x + 10 \\ \cline{2-2}
x+2 \big) & 2x^3 - x^2 + 0x - 4 \\
 & \underline{2x^3 + 4x^2} \qquad\qquad \\
 & -5x^2 + 0x \qquad \\
 & -\underline{5x^2 - 10x} \qquad \\
 & 10x - 4 \\
 & \underline{10x + 20} \\
 & -24
\end{array}$$

5. $\boxed{\text{C}}$

$$\begin{array}{rr}
 & -x^2 + 2x + 2 \\ \cline{2-2}
-x+1 \big) & x^3 - 3x^2 + 0x + 1 \\
 & \underline{x^3 - x^2} \qquad\qquad \\
 & -2x^2 + 0x \qquad \\
 & -\underline{2x^2 + 2x} \qquad \\
 & -2x + 1 \\
 & -\underline{2x + 2} \\
 & -1
\end{array}$$

6. $\boxed{\text{D}}$

$$\begin{array}{rr}
 & 2x + 1 \\ \cline{2-2}
x-2 \big) & 2x^2 - 3x + 5 \\
 & \underline{2x^2 - 4x} \qquad \\
 & x + 5 \\
 & \underline{x - 2} \\
 & 7
\end{array}$$

7. $\boxed{\text{D}}$

$$\begin{array}{rr}
 & 3x + 5 \\ \cline{2-2}
x^2 - x + 2 \big) & 3x^3 + 2x^2 - 4x + 6 \\
 & \underline{3x^3 - 3x^2 + 6x} \qquad \\
 & 5x^2 - 10x + 6 \\
 & \underline{5x^2 - 5x + 10} \\
 & -5x - 4
\end{array}$$

8. $\boxed{\text{B}}$

$$
\begin{array}{r}
x^2 + x + 3 \\
-x+3 \overline{\big)\, -x^3 + 2x^2 + 0x + 5} \\
\underline{-x^3 + 3x^2} \phantom{{}+0x+5} \\
-x^2 + 0x \phantom{{}+5} \\
\underline{-x^2 + 3x} \phantom{{}+5} \\
-3x + 5 \\
\underline{-3x + 9} \\
-4
\end{array}
$$

9. $\boxed{\text{B}}$

$$
\begin{array}{r}
x^2 + 0x + 3 \\
x^2-2x+0 \overline{\big)\, x^4 - 2x^3 + 3x^2 - 4x + 1} \\
\underline{x^4 - 2x^3 + 0x^2} \phantom{{}-4x+1} \\
3x^2 - 4x + 1 \\
\underline{3x^2 - 6x + 0} \\
2x + 1
\end{array}
$$

10. $\boxed{\text{C}}$

$$
\begin{array}{r}
2x^2 - 6x + 10 \\
x+3 \overline{\big)\, 2x^3 + 0x^2 - 8x + 5} \\
\underline{2x^3 + 6x^2} \phantom{{}-8x+5} \\
-6x^2 - 8x \phantom{{}+5} \\
\underline{-6x^2 - 18x} \phantom{{}+5} \\
10x + 5 \\
\underline{10x + 30} \\
-25
\end{array}
$$

11. $\boxed{\text{C}}$

Degree of remainder must be smaller than the degree of the divisor (degree 2).
Thus, only II and III are possible.

12. $\boxed{\text{B}}$

$$
\begin{aligned}
\text{Required polynomial} &= (x-2)(x^2-x-2)+3 \\
&= x^3 - 3x^2 + 7
\end{aligned}
$$

13. $\boxed{\text{B}}$

Required polynomial $= (x^2 - 3x - 10)(3x - 1) + (3x - 2)$

$= 3x^3 - 10x^2 - 24x + 8$

14. $\boxed{\text{A}}$

Let the polynomial be $p(x)$.

$$\begin{aligned} x^3 + 7x^2 - 5 &= p(x)(x+3) - (11x+2) \\ p(x) &= \frac{(x^3 + 7x^2 - 5) + (11x + 2)}{x+3} \\ &= \frac{x^3 + 7x^2 + 11x - 3}{x+3} \\ &= x^2 + 4x - 1 \end{aligned}$$

15. $\boxed{\text{C}}$

Let the required polynomial be $p(x)$.

$$\begin{aligned} 2x^3 - 3x + 5 &= p(x)(2x^2 + 4x + 5) + 15 \\ 2x^3 - 3x - 10 &= p(x)(2x^2 + 4x + 5) \\ p(x) &= \frac{2x^3 - 3x - 10}{2x^2 + 4x + 5} \\ &= x - 2 \end{aligned}$$

16. $\boxed{\text{D}}$

$-5 = (1)(-a) + (2)(1)$

$a = 7$

17. $\boxed{\text{B}}$

$2x^3 + px^2 - 4x - 1 = (x^2 - 2x + q)(2x + 3) - 4$

$2x^3 + px^2 - 4x + 3 = (x^2 - 2x + q)(2x + 3)$

Compare the coefficients of x^2 and the constant terms.

$$\begin{cases} 3q = 3 \\ (1)(3) + (-2)(2) = p \end{cases}$$

Solving, we have $p = -1$ and $q = 1$.

18. $\boxed{\text{A}}$

Let $4x^3 + 8x^2 - 11x + 3 = (2x^2 + 5x + k)(Ax + B)$, where A and B are constants.

R.H.S. $= 2Ax^3 + (2B + 5A)x^2 + (5B + Ak)x + Bk$

We have $2A = 4$, $2B + 5A = 8$, $5B + Ak = -11$ and $Bk = 3$.

Solving, we have $A = 2$, $B = -1$ and $k = -3$.

Conventional Questions

19. (a) $f(x) = 2(x-2)(x^2-4x+1)+ax+b$ 1M

$= 2x^3 - 12x^2 + (18+a)x + (b-4)$

We have $2b = -12$, $18+a = 7a$ and $c = b-4$. 1M

Solving, we have $a = 3$, $b = -6$ and $c = -10$. 1A

(b) $0 = 2(x-2)(x^2-4x+1)+3x-6$

$= (x-2)[2(x^2-4x+1)+3]$ 1M

$= (x-2)(2x^2-8x+5)$

$x = 2 \quad \text{or} \quad \dfrac{8 \pm \sqrt{8^2-4(2)(5)}}{2(2)}$

$= 2 \quad \text{or} \quad \dfrac{4 \pm \sqrt{6}}{2}$ 1A

There is 1 rational root. 1A

20. (a) $f(x) = (8x^2+ax+8)(3x^2+7x+r)+bx+c$ 1M

$= 24x^4 + (56+3a)x^3 + \ldots$

$56 + 3a = 47$ 1M

$a = -3$ 1A

(b) (i) Let $g(x) = A(8x^2+ax+8)+bx+c$, where A is a constant. 1M

$f(x) - g(x) = [(8x^2+ax+8)(3x^2+7x+r)+bx+c] - [A(8x^2+ax+8)+bx+c]$

$= (8x^2+ax+8)(3x^2+7x+r-A)$

Thus, $f(x)-g(x)$ is divisible by $8x^2+ax+8$. 1

(ii) $f(x) - g(x) = 0$

$(8x^2-3x+8)(3x^2+7x+r-A) = 0$

$8x^2-3x+8 = 0 \quad \text{or} \quad 3x^2+7x+r-A = 0$ 1M

For $8x^2-3x+8=0$,

$\Delta = (-3)^2 - 4(8)(8) = -247 < 0$. The equation has no real roots. 1M

For $3x^2+7x+r-A=0$, the equation has at most 2 real roots.

Thus, $f(x)-g(x)=0$ has at most 2 real roots.

The claim is disagreed. 1A