

REG-EOC-2324-ASM-SET 3-MATH**Suggested solutions****Multiple Choice Questions**

1. A	2. A	3. C	4. D	5. C
6. C	7. C	8. C	9. A	10. D
11. C	12. A	13. C	14. B	15. B
16. B	17. B	18. C	19. A	20. A
21. D	22. B	23. A	24. C	25. B
26. A	27. D	28. C	29. A	30. B

1. A

Required equation is

$$(x + 2)^2 + (y - 5)^2 = (2 + 2)^2 + (1 - 5)^2$$
$$x^2 + y^2 + 4x - 10y - 3 = 0$$

2. A

Required equation is

$$(x + 2)^2 + (y - 4)^2 = (3 + 2)^2 + (5 - 4)^2$$
$$x^2 + y^2 + 4x - 8y - 6 = 0$$

3. CRadius = $0 - (-4) = 4$. The equation is

$$(x + 4)^2 + (y - 3)^2 = 4^2$$
$$x^2 + y^2 + 8x - 6y + 9 = 0$$

4. D

Required equation is

$$(x + 5)^2 + (y - 12)^2 = (0 + 5)^2 + (0 - 12)^2$$
$$(x + 5)^2 + (y - 12)^2 = 169$$

5. C

Centre $(-10, 12)$. The equation is in the form $x^2 + y^2 + 20x - 24y + F = 0$, where F is a constant.

The coordinates of mid-point of AB are $(-10, 0)$.

Thus, the x -coordinates of A and B are $-10 \pm 16 = 6$ or -26 .

$$(6)^2 + (0)^2 + 20(6) - 24(0) + F = 0$$

$$F = -156$$

$$C: x^2 + y^2 + 20x - 24y - 156 = 0$$

6. C

The circle passes through points $(5 \pm 12, 0)$, i.e., $(-7, 0)$ and $(17, 0)$.

Centre $(5, -7) \Rightarrow$ The circle is in the form $x^2 + y^2 - 10x + 14y + F = 0$.

Substitute $(-7, 0)$, $F = -119$.

7. C

$$\text{Radius} = \sqrt{\left(\frac{8}{2}\right)^2 + 3^2} = 5$$

Required equation is

$$(x + 2)^2 + (y - 3)^2 = 5^2$$

$$x^2 + y^2 + 4x - 6y - 12 = 0$$

8. C

Centre is the mid-point of AB .

Coordinates of centre are $\left(-\frac{3}{2}, 2\right)$.

Required equation is

$$\left(x + \frac{3}{2}\right)^2 + (y - 2)^2 = \left(0 + \frac{3}{2}\right)^2 + (1 - 2)^2$$

$$x^2 + y^2 + 3x - 4y + 3 = 0$$

9. A

Centre is the mid-point of AB .

The coordinates of the centre are $(-1, -1)$.

Required equation is

$$(x + 1)^2 + (y + 1)^2 = (2 + 1)^2 + (3 + 1)^2$$

$$x^2 + y^2 + 2x + 2y - 23 = 0$$

10. D

$$\text{Distance between the two points} = \sqrt{(0+8)^2 + (0+6)^2} = 10 = 2(5)$$

The line segment joining the two points is therefore a diameter of the required circle.

The coordinates of the centre are $(-4, -3)$.

Required equation is

$$(x+4)^2 + (y+3)^2 = 5^2$$

$$x^2 + y^2 + 8x + 6y = 0$$

11. C

The centre is the mid-point of AC , the coordinates are $(7, 5)$.

Required equation is $x^2 + y^2 - 14x - 10y + F = 0$, where F is a constant.

$$8^2 + 8^2 - 14(8) - 10(8) + F = 0$$

$$F = 64$$

12. A

Since $\angle AOB = 90^\circ$, where O is the origin.

Centre of the circle is the mid-point of AB .

The coordinates of the centre are $\left(\frac{-3}{2}, 2\right)$.

Required equation is

$$\left(x + \frac{3}{2}\right)^2 + (y - 2)^2 = \left(0 + \frac{3}{2}\right)^2 + (0 - 2)^2$$

$$x^2 + y^2 + 3x - 4y = 0$$

13. C

Centre is the mid-point of $(4, 0)$ and $(0, -6)$.

Coordinates of centre are $(2, -3)$.

Required equation is

$$(x - 2)^2 + (y + 3)^2 = (0 - 2)^2 + (0 + 3)^2$$

$$x^2 + y^2 - 4x + 6y = 0$$

14. B

Line segment joining $(0, 8)$ and $(-6, 0)$ is a diameter.

Coordinates of centre are $(-3, 4)$.

The equation is $x^2 + y^2 + 6x - 8y = 0$ (passes through origin).

15. B

$$x\text{-coordinate of centre} = \frac{1+5}{2} = 3$$

$$y\text{-coordinate of centre} = \frac{1+(-3)}{2} = -1$$

$$\text{Radius} = 5 - 3 = 2$$

$$\text{Required equation is } (x - 3)^2 + (y + 1)^2 = 4.$$

16. B

Let the coordinates of C be $(h, 0)$, where $h < 0$.

Since $y = 3$ is a tangent to the circle, radius of the circle is 3.

Since $x = 1$ is a tangent to the circle, we have

$$1 - h = 3$$

$$h = -2$$

$$\text{Required equation is } (x + 2)^2 + y^2 = 9.$$

17. B

$$\text{Radius} = 3$$

Required equation is

$$(x + 5)^2 + (y + 3)^2 = 3^2$$

$$x^2 + y^2 + 10x + 6y + 25 = 0$$

18. C

$$y\text{-coordinate of centre} = 2$$

$$x\text{-coordinate of centre} = \frac{1+4}{2} = \frac{5}{2}$$

$$\text{Radius} = \frac{5}{2}$$

Required equation is

$$\left(x - \frac{5}{2}\right)^2 + (y - 2)^2 = \left(\frac{5}{2}\right)^2$$

$$x^2 + y^2 - 5x - 4y + 4 = 0$$

19. A

$$y\text{-coordinate of centre} = \frac{(-1) + (-9)}{2} = -5$$

$$\text{Radius of circle} = 0 - (-5) = 5$$

Let the coordinates of centre be $(h, -5)$.

$$\sqrt{(h - 0)^2 + (-5 + 1)^2} = 5$$

$$h = -3 \quad \text{or} \quad (\text{rejected})$$

20. A

$$x\text{-coordinate of centre} = \frac{2+8}{2} = 5$$

Radius = 5 and y-coordinate of centre = 4

Required equation is

$$(x - 5)^2 + (y - 4)^2 = 16$$

$$x^2 + y^2 - 10x - 8y + 16 = 0$$

21. D

Let the radius be r .

The coordinates of the centre are $(r, -r)$.

Centre lies on L .

$$(r) + 2(-r) + 4 = 0$$

$$r = 4$$

Required equation is

$$(x - 4)^2 + (y + 4)^2 = 4^2$$

$$x^2 + y^2 - 8x + 8y + 16 = 0$$

22. B

Let the coordinates of centre be $(h, 0)$.

The equation of C is $(x - h)^2 + y^2 = (-h)^2$.

$$(-2 - h)^2 + (-4)^2 = h^2$$

$$h^2 + 4h + 20 = h^2$$

$$h = -5$$

Required equation is

$$(x + 5)^2 + y^2 = 5^2$$

$$x^2 + y^2 + 10x = 0$$

23. A

Let the coordinates of the centre be $(1, k)$.

The equation of C is $(x - 1)^2 + (y - k)^2 = k^2$.

$$(5 - 1)^2 + (8 - k)^2 = k^2$$

$$k^2 - 16k + 80 = k^2$$

$$k = 5$$

Required equation is

$$(x - 1)^2 + (y - 5)^2 = 5^2$$

$$x^2 + y^2 - 2x - 10y + 1 = 0$$

24. C

Let the radius of the circle be r .

The coordinates of the centre are $(-r, r)$.

Equation of C is $(x + r)^2 + (y - r)^2 = r^2$.

$$\frac{r - 12}{-r - 0} = \frac{12 - 0}{0 + 24}$$

$$2r - 24 = -r$$

$$r = 8$$

Required equation is

$$(x + 8)^2 + (y - 8)^2 = 8^2$$

$$x^2 + y^2 + 16x - 16y + 64 = 0$$

25. B

Let the coordinates of centre be $(h, 0)$.

$$\sqrt{(h - 0)^2 + (5 - 0)^2} = \sqrt{(h - 6)^2 + (1 - 0)^2}$$

$$h^2 + 25 = h^2 - 12h + 37$$

$$h = 1$$

Required equation is

$$(x - 1)^2 + y^2 = (0 - 1)^2 + 5^2$$

$$x^2 + y^2 - 2x - 25 = 0$$

26. A

A. ✓.

B. ✗. Centre $(1, -7)$ does not lie on L .

C. ✗. Centre $\left(\frac{5}{2}, \frac{7}{2}\right)$ does not lie on L .

D. ✗. $(3, 4)$ does not lie on the circle.

27. D

- A. ✗. Centre $\left(-\frac{5}{2}, 5\right)$ does not lie on $x + y - 1 = 0$.
- B. ✗. Centre $\left(\frac{19}{2}, -4\right)$ does not lie on $x + y - 1 = 0$.
- C. ✗. Centre $\left(\frac{17}{2}, -\frac{19}{2}\right)$ does not lie on $x + y - 1 = 0$.
- D. ✓. Centre $\left(-\frac{17}{2}, \frac{19}{2}\right)$ lie on $x + y - 1 = 0$,
and both $(0, 0)$ and $(3, 4)$ satisfy the equation.

28. C

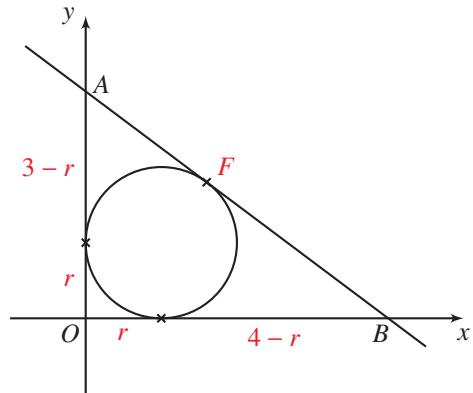
Let F be the point of contact of AB and the circle. Let r be the radius.

$$AF = 3 - r \text{ and } BF = 4 - r$$

$$(3 - r) + (4 - r) = \sqrt{3^2 + 4^2}$$

$$r = 1$$

$$\text{The equation of circle is } (x - 1)^2 + (y - 1)^2 = 1.$$

29. A

Radius of circle is 2. Let the centre of C_3 be (h, k) . Note that the polygon formed by joining the centres is an equilateral triangle.

By considering the line segment joining centres of C_1 and C_3 ,

$$h - 2 = (2 + 2) \cos 60^\circ \quad \text{and} \quad k - 2 = (2 + 2) \sin 60^\circ$$

$$h = 4 \quad k = 2 + 2\sqrt{3}$$

Only option A has its centre at $(4, 2 + 2\sqrt{3})$.

30. B

$$\begin{aligned} \text{y-coordinate of centre} &= \frac{2\sqrt{3} + 8\sqrt{3}}{2} \\ &= 5\sqrt{3} \end{aligned}$$

Centre of circle is the centroid/orthocentre/circumcentre/incentre of $\triangle CAB$.

(Note: four centres coincide when it is equilateral triangle.)

- A. ✗. Centre $(-5, 5\sqrt{3})$ should not lie on the line AB .
- B. ✓.
- C. ✗. y-coordinate of centre $= -5\sqrt{3} \neq 5\sqrt{3}$
- D. ✗. y-coordinate of centre $= -5\sqrt{3} \neq 5\sqrt{3}$

Conventional Questions

31. (a) $(x - 7)^2 + (y + 3)^2 = (-8 - 7)^2 + (5 + 3)^2$ 1M
 $(x - 7)^2 + (y + 3)^2 = 289$ 1A

(b) $JK = \sqrt{(7 + 1)^2 + (-3 - 25)^2} = \sqrt{848} >$ radius of C . 1M
So, J lies outside the circle. 1A

(c) (i) J, K and P are collinear. 1A
(ii) Slope of the line $= \frac{25 + 3}{-1 - 7} = -\frac{7}{2}$.
Required equation is
 $y - 25 = -\frac{7}{2}(x + 1)$ 1M
 $7x + 2y - 43 = 0$ 1A

32. (a) (i) Consider $\triangle ABC$ and $\triangle ODA$,

$$\begin{aligned}
 AB &= AD && \text{(given)} \\
 \angle ACB &= \angle ACD && \text{(equal chords, equal } \angle\text{s)} \\
 \angle OAD &= \angle ACD && \text{(\angle in alt. segment)} \\
 &= \angle ACB \\
 \angle ADO &= \angle ABC && \text{(ext. } \angle, \text{ cyclic quad.)} \\
 \triangle ABC &\sim \triangle ODA && \text{(AA)} \\
 \frac{AB}{BC} &= \frac{OD}{AD} && \text{(corr. sides, } \sim \text{ of } \triangle\text{s)} \\
 AB^2 &= BC \cdot OD
 \end{aligned}$$

Marking Scheme

Case 1	Any correct proof with correct reasons.	3
Case 2	Any correct proof without reasons.	2
Case 3	Incomplete proof with any one correct step with reason.	1

(ii) Since $\angle ACD = \angle ACB$,

$$\begin{aligned}
 \cos \angle ACD &= \cos \angle ACB \\
 \frac{CD^2 + AC^2 - AD^2}{2(CD)(AC)} &= \frac{AC^2 + BC^2 - AB^2}{2(AC)(BC)} && 1M \\
 2(CD^2 + AC^2 - AD^2) &= AC^2 + BC^2 - AB^2 \\
 2CD^2 + 2AC^2 - 2AD^2 &= AC^2 + (2CD)^2 - AD^2 && 1M \\
 AC^2 &= 2CD^2 + AD^2 && 1
 \end{aligned}$$

(b) (i) Since $\angle BCD = 90^\circ$, BD is a diameter.

$$\text{Centre} = \left(\frac{15+27}{2}, \frac{0+24}{2} \right) = (21, 12).$$

The equation of the circle is

$$\begin{aligned}
 (x - 21)^2 + (y - 12)^2 &= (15 - 21)^2 + (0 - 12)^2 && 1M \\
 (x - 21)^2 + (y - 12)^2 &= 180 && 1A
 \end{aligned}$$

(ii) $AB^2 = BC \cdot OD$

$$\begin{aligned}
 AB &= \sqrt{24 \cdot 15} \\
 &= 6\sqrt{10} && 1A \\
 AC^2 &= 2CD^2 + AD^2 \\
 AC^2 &= 2(12)^2 + (6\sqrt{10})^2 && 1M \\
 AC &= 18\sqrt{2} && 1A
 \end{aligned}$$