M2REG-DIFF-2324-ASM-SET 4-MATH
Suggested solutions

1. $\frac{dy}{dx} = \frac{1}{2}(5x^2+2x)^{-\frac{1}{2}}(10x+2)$ 1M

$= (5x+1)(5x^2+2x)^{-\frac{1}{2}}$ 1A

$\frac{d^2y}{dx^2} = 5(5x^2+2x)^{-\frac{1}{2}} + (5x+1)\left(-\frac{1}{2}\right)(5x^2+2x)^{-\frac{3}{2}}(10x+2)$

$= (5x^2+2x)^{-\frac{3}{2}}[5(5x^2+2x) - (5x+1)^2]$

$= -(5x^2+2x)^{-\frac{3}{2}}$ 1A

2. $\frac{dy}{dx} = 2xe^{x^2} + x^2e^{x^2}(2x)$ 1M

$= 2e^{x^2}(x+x^3)$ 1A

$\frac{d^2y}{dx^2} = 2e^{x^2}(2x)(x+x^3) + 2e^{x^2}(1+3x^2)$

$= 2e^{x^2}[(2x^2+2x^4)+(1+3x^2)]$

$= 2e^{x^2}(2x^4+5x^2+1)$ 1A

3. $\frac{dy}{dx} = e^{-\frac{x}{2}}\left(-\frac{1}{2}\right)\cos\frac{x}{2} + e^{-\frac{x}{2}}\left(-\sin\frac{x}{2}\right)\left(\frac{1}{2}\right)$ 1M

$= -\frac{1}{2}e^{-\frac{x}{2}}\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)$ 1A

$\frac{d^2y}{dx^2} = -\frac{1}{2}e^{-\frac{x}{2}}\left(-\frac{1}{2}\right)\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right) - \frac{1}{2}e^{-\frac{x}{2}}\left(-\frac{1}{2}\sin\frac{x}{2}+\frac{1}{2}\cos\frac{x}{2}\right)$

$= \frac{1}{4}e^{-\frac{x}{2}}\left[\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)+\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)\right]$

$= \frac{1}{2}e^{-\frac{x}{2}}\sin\frac{x}{2}$ 1A

4. $\frac{dy}{dx} = \cos x\ln(\cos x) + \sin x\cdot\frac{-\sin x}{\cos x}$ 1M

$= \cos x\ln(\cos x) - \sin x\tan x$ 1A

$\frac{d^2y}{dx^2} = -\sin x\ln(\cos x) + \cos x\cdot\frac{-\sin x}{\cos x} - \cos x\tan x - \sin x\sec^2 x$

$= -\sin x\left[\ln(\cos x)+1+1+\sec^2 x\right]$

$= -\sin x\left[\ln(\cos x)+\sec^2 x+2\right]$ 1A

5. (a) $\frac{\mathrm{d}y}{\mathrm{d}x} = 5\left(\frac{2x+3}{2x-1}\right)^4 \cdot \frac{2(2x-1)-(2x+3)(2)}{(2x-1)^2}$ 1M

$= 5\left(\frac{2x+3}{2x-1}\right)^4 \cdot \frac{-8}{(2x-1)^2}$

$= \frac{-40(2x+3)^4}{(2x-1)^6}$ 1

(b) $\frac{\mathrm{d}y}{\mathrm{d}x} = -40(2x+3)^4(2x-1)^{-6}$

$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -160(2x+3)^3(2)(2x-1)^{-6} - 40(2x+3)^4(-6)(2x-1)^{-7}(2)$ 1M

$= 160(2x+3)^3(2x-1)^{-7}\,[-2(2x-1)+3(2x+3)]$

$= 160(2x+3)^3(2x-1)^{-7}(2x+11)$ 1A

6. (a) $y = \ln\left(1+x^2\right) - \ln\left(1-x^2\right)$

$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x}{1+x^2} - \frac{-2x}{1-x^2}$ 1M

$= \frac{2x(1-x^2)+2x(1+x^2)}{(1-x^2)(1+x^2)}$

$= \frac{4x}{1-x^4}$ 1

(b) $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{4(1-x^4)-4x(-4x^3)}{(1-x^4)^2}$ 1M

$= \frac{4(3x^4+1)}{(1-x^4)^2}$ 1A

7. $\frac{\mathrm{d}y}{\mathrm{d}x} = e^{-4x} + xe^{-4x}(-4)$ 1M

$= e^{-4x}(1-4x)$ 1A

$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = e^{-4x}(-4)(1-4x) + e^{-4x}(-4)$

$= -4e^{-4x}[(1-4x)+1]$

$= 8e^{-4x}(2x-1)$ 1A

$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 3\frac{\mathrm{d}y}{\mathrm{d}x} - 4y$

$= \left[8e^{-4x}(2x-1)\right] + 3\left[e^{-4x}(1-4x)\right] - 4xe^{-4x}$

$= e^{-4x}\,[8(2x-1)+3(1-4x)-4x]$

$= -5e^{-4x}$ 1

8. $\dfrac{dy}{dx} = \dfrac{1}{x} \cdot \dfrac{1}{x^2} + \ln x(-2x^{-3})$ 1M

$= x^{-3}(1 - 2\ln x)$ 1A

$\dfrac{d^2y}{dx^2} = -3x^{-4}(1 - 2\ln x) + x^{-3}\left(-\dfrac{2}{x}\right)$

$= -x^{-4}[3(1 - 2\ln x) + 2]$

$= -x^{-4}(5 - 6\ln x)$ 1A

$x^2\dfrac{d^2y}{dx^2} + 5x\dfrac{dy}{dx} + 4y$

$= x^2\left[-x^{-4}(5 - 6\ln x)\right] + 5x\left[x^{-3}(1 - 2\ln x)\right] + 4x^{-2}\ln x$

$= x^{-2}[-(5 - 6\ln x) + 5(1 - 2\ln x) + 4\ln x]$

$= 0$ 1

9. $\dfrac{dy}{dx} = \dfrac{3}{2}x^{\frac{1}{2}}\ln x + x^{\frac{3}{2}} \cdot \dfrac{1}{x}$ 1M

$= \dfrac{1}{2}x^{\frac{1}{2}}(3\ln x + 2)$ 1A

$\dfrac{d^2y}{dx^2} = \dfrac{1}{4}x^{-\frac{1}{2}}(3\ln x + 2) + \dfrac{1}{2}x^{\frac{1}{2}}\left(\dfrac{3}{x}\right)$

$= \dfrac{1}{4}x^{-\frac{1}{2}}[(3\ln x + 2) + 6]$

$= \dfrac{1}{4}x^{-\frac{1}{2}}(3\ln x + 8)$ 1A

$4x^2\dfrac{d^2y}{dx^2} - 8x\dfrac{dy}{dx} + 9y$

$= 4x^2\left[\dfrac{1}{4}x^{-\frac{1}{2}}(3\ln x + 8)\right] - 8x\left[\dfrac{1}{2}x^{\frac{1}{2}}(3\ln x + 2)\right] + 9x^{\frac{3}{2}}\ln x$

$= x^{\frac{3}{2}}[(3\ln x + 8) - 4(3\ln x + 2) + 9\ln x]$

$= 0$ 1

10. $\dfrac{dy}{dx} = -2x^{-3}\cos x + x^{-2}(-\sin x)$ 1M

$= -x^{-3}(2\cos x + x\sin x)$ 1A

$\dfrac{d^2y}{dx^2} = 3x^{-4}(2\cos x + x\sin x) - x^{-3}[-2\sin x + \sin x + x\cos x]$

$= x^{-4}[3(2\cos x + x\sin x) - x(-2\sin x + \sin x + x\cos x)]$

$= x^{-4}(4x\sin x - x^2\cos x + 6\cos x)$ 1A

$x^2\dfrac{d^2y}{dx^2} + 4x\dfrac{dy}{dx} + (x^2 + 2)y$

$= x^2\left[x^{-4}(4x\sin x - x^2\cos x + 6\cos x)\right] + 4x\left[-x^{-3}(2\cos x + x\sin x)\right] + (x^2 + 2)x^{-2}\cos x$

$= x^{-2}\left[(4x\sin x - x^2\cos x + 6\cos x) - 4(2\cos x + x\sin x) + (x^2 + 2)\cos x\right]$

$= 0$ 1

11. $\dfrac{dy}{dx} = 9x^2 - 8x$ 1A

$\dfrac{d^2y}{dx^2} = 18x - 8$ 1A

$0 = x^2\dfrac{d^2y}{dx^2} + kx\dfrac{dy}{dx} + my$

$= x^2(18x - 8) + kx(9x^2 - 8x) + m(3x^3 - 4x^2)$

$= (18 + 9k + 3m)x^3 + (-8 - 8k - 4m)x^2$

We have $18 + 9k + 3m = 0$ and $-8 - 8k - 4m = 0$. 1M

Solving, we have $k = -4$ and $m = 6$. 1A

12. (a) $\dfrac{dy}{dx} = e^x \sin x + e^x \cos x$ 1M

$= e^x(\sin x + \cos x)$ 1A

$\dfrac{d^2y}{dx^2} = e^x(\sin x + \cos x) + e^x(\cos x - \sin x)$

$= 2e^x \cos x$ 1A

(b) $0 = \dfrac{d^2y}{dx^2} - k\dfrac{dy}{dx} + 2y$

$0 = 2e^x \cos x - ke^x(\sin x + \cos x) + 2e^x \sin x$

$= e^x[(2 - k)\cos x + (-k + 2)\sin x]$

We have $2 - k = 0$. 1M

Thus, $k = 2$. 1A

13. (a) (i) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = k\left(\dfrac{x+1}{x-1}\right)^{k-1} \cdot \dfrac{1(x-1)-(x+1)(1)}{(x-1)^2}$ 1M

$$= k\left(\frac{x+1}{x-1}\right)^{k}\left(\frac{x+1}{x-1}\right)^{-1} \cdot \frac{-2}{(x-1)^2}$$

$$= ky\frac{-2}{(x+1)(x-1)}$$

$$= \frac{2ky}{1-x^2}$$ 1

(ii) $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = \dfrac{2k\frac{\mathrm{d}y}{\mathrm{d}x} - 2ky(-2x)}{(1-x^2)^2}$

$$= \frac{2k}{1-x^2}\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{4kxy}{(1-x^2)^2}$$ 1A

$$(x^2-1)\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2(x+k)\frac{\mathrm{d}y}{\mathrm{d}x}$$

$$= (x^2-1)\left[\frac{2k}{1-x^2}\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{4kxy}{(1-x^2)^2}\right] + 2(x+k)\left[\frac{2ky}{1-x^2}\right]$$

$$= -2k\left(\frac{2ky}{1-x^2}\right) - \frac{4kxy}{1-x^2} + \frac{4kxy}{1-x^2} + \frac{4k^2y}{1-x^2}$$

$$= 0$$ 1

(b) $y = \left(\dfrac{0+1}{0-1}\right)^k$

$$= (-1)^k$$

$$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=0} = \frac{2k(-1)^k}{1-0}$$

$$= 2k(-1)^k$$

$$\left.\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right|_{x=0} = \frac{2k}{1}[2k(-1)^k] + 0$$

$$16 = 4k^2(-1)^k$$ 1A

We have k being an positive even number and $4k^2 = 16$. 1M

Thus, $k = 2$. 1A

 END OF PAPER