

M2REG-DIFF-2324-ASM-SET 2-MATH

建議題解

$$1. \frac{dy}{dx} = \frac{4(1-5x) - (2+4x)(-5)}{(1-5x)^2} \quad 1M$$

$$= \frac{14}{(1-5x)^2} \quad 1A$$

$$2. \frac{dy}{dx} = \frac{4(2x^2+x) - (4x-1)(4x+1)}{(2x^2+x)^2} \quad 1M$$

$$= \frac{-8x^2+4x+1}{(2x^2+x)^2} \quad 1A$$

$$3. \quad y = \frac{x^2+10x+25}{x-11} \quad 1M$$

$$\frac{dy}{dx} = \frac{(2x+10)(x-11) - (x^2+10x+25)(1)}{(x-11)^2} \quad 1M$$

$$= \frac{x^2-22x-135}{(x-11)^2} \quad 1A$$

$$4. \quad y = 4 \left(\frac{2x^2+1}{5-3x^2} \right)^3 \cdot \frac{4x(5-3x^2) - (2x^2+1)(-6x)}{(5-3x^2)^2} \quad 1M$$

$$= 4 \left(\frac{2x^2+1}{5-3x^2} \right)^3 \cdot \frac{26x}{(5-3x^2)^2}$$

$$= \frac{104x(2x^2+1)^3}{(5-3x^2)^5} \quad 1A$$

$$5. \quad \frac{dy}{dx} = \frac{3x^2(5x^6+6)^{\frac{1}{2}} - (x^3+1) \left(\frac{1}{2} \right) (5x^6+6)^{-\frac{1}{2}} (30x^5)}{5x^6+6} \quad 1M$$

$$= \frac{3x^2(5x^6+6) - 15x^5(x^3+1)}{(5x^6+6)^{\frac{3}{2}}}$$

$$= \frac{-15x^5+18x^2}{(5x^6+6)^{\frac{3}{2}}} \quad 1A$$

$$6. \quad \frac{d}{dx}(x^3 - y^3 + x^2y^4) = \frac{d}{dx}(50)$$

$$3x^2 - 3y^2 \frac{dy}{dx} + 2xy^4 + 4x^2y^3 \frac{dy}{dx} = 0 \quad 1M$$

$$(3y^2 - 4x^2y^3) \frac{dy}{dx} = 3x^2 + 2xy^4$$

$$\frac{dy}{dx} = \frac{3x^2 + 2xy^4}{3y^2 - 4x^2y^3} \quad 1A$$

$$\begin{aligned}
7. \quad & \frac{xy - 1}{x^2y^2 + 1} = 1 \\
& \frac{d}{dx}(xy - 1) = \frac{d}{dx}(x^2y^2 + 1) \\
& y + x \frac{dy}{dx} = 2xy^2 + 2x^2y \frac{dy}{dx} \quad 1M \\
& (x - 2x^2y) \frac{dy}{dx} = 2xy^2 - y \\
& \frac{dy}{dx} = \frac{y(2xy - 1)}{x(1 - 2xy)} \\
& = -\frac{y}{x} \quad 1A
\end{aligned}$$

$$\begin{aligned}
8. \quad & (x + y)(2y - x) = xy \\
& -x^2 + xy + 2y^2 = xy \\
& \frac{d}{dx}(-x^2 + 2y^2) = \frac{d}{dx}(0) \\
& -2x + 4y \frac{dy}{dx} = 0 \quad 1M \\
& \frac{dy}{dx} = \frac{x}{2y} \quad 1A
\end{aligned}$$

$$\begin{aligned}
9. \quad (a) \quad & \frac{4x}{y^2} + 2xy = 25 \\
& \frac{d}{dx}(4x + 2xy^3) = \frac{d}{dx}(25y^2) \\
& 4 + 2y^3 + 6xy^2 \frac{dy}{dx} = 50y \frac{dy}{dx} \quad 1M \\
& (6xy^2 - 50y) \frac{dy}{dx} = -2y^3 - 4 \\
& \frac{dy}{dx} = \frac{y^3 + 2}{25y - 3xy^2} \quad 1A \\
(b) \quad & \frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)} \\
& = \frac{25y - 3xy^2}{y^3 + 2} \quad 1A \\
& \left. \frac{dx}{dy} \right|_{(5, 2)} = \frac{25(2) - 3(5)(2)^2}{2^3 + 2} \\
& = -1 \quad 1A
\end{aligned}$$

$$10. \quad (a) \quad \frac{d}{dy}(\sqrt{x}) = \frac{d}{dx}\left(y^2 - 5\sqrt{y} - \frac{12}{y}\right)$$

$$\frac{1}{2}x^{-\frac{1}{2}}\frac{dx}{dy} = 2y - \frac{5}{2}y^{-\frac{1}{2}} + 12y^{-2}$$

1M

$$\frac{dx}{dy} = x^{\frac{1}{2}}(4y - 5y^{-\frac{1}{2}} + 24y^{-2})$$

1A

$$(b) \quad \frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$$

$$= \frac{1}{x^{\frac{1}{2}}(4y^2 - 5y^{-\frac{1}{2}} + 24y^{-2})}$$

1A

$$\left.\frac{dy}{dx}\right|_{(9, 4)} = \frac{1}{9^{\frac{1}{2}}[4(4)^2 - 5(4)^{-\frac{1}{2}} + 24(4)^{-2}]}$$

$$= \frac{1}{45}$$

1A

$$11. \quad (a) \quad x^2 + y^2 = 30$$

$$2x + 2y\frac{dy}{dx} = 0$$

1M

$$\frac{dy}{dx} = -\frac{x}{y}$$

1A

$$\left.\frac{dy}{dx}\right|_{(p, q)} = \frac{-p}{q} = -\frac{1}{2}$$

$$q = 2p$$

1

$$(b) \quad p^2 + (2p)^2 = 30$$

1M

$$p^2 = 6$$

$$p = \pm\sqrt{6}$$

因此， $(p, q) = (\sqrt{6}, 2\sqrt{6})$ 或 $(-\sqrt{6}, -2\sqrt{6})$ 。

1A+1A