

M2REG-DIFF-2324-ASM-SET 1-MATH

建議題解

$$\begin{aligned} 1. \quad \frac{dy}{dx} &= 12(2x) - 17 + 0 \\ &= 24x - 17 \end{aligned} \quad 1A$$

$$\begin{aligned} 2. \quad \frac{dy}{dx} &= -x^{-2} - 4(-2x^{-3}) + 9(-3x^{-4}) \\ &= -x^{-2} + 8x^{-3} - 27x^{-4} \end{aligned} \quad 1A$$

$$\begin{aligned} 3. \quad y &= \frac{1}{x} - \sqrt{9x^3} + \sqrt[4]{x^5} + 4x^{\frac{3}{7}} \\ &= x^{-1} - 3x^{\frac{3}{2}} + x^{\frac{5}{4}} + 4x^{\frac{3}{7}} \\ \frac{dy}{dx} &= -x^{-2} - \frac{9}{2}x^{\frac{1}{2}} + \frac{5}{4}x^{\frac{1}{4}} + \frac{12}{7}x^{-\frac{4}{7}} \end{aligned} \quad \begin{array}{l} 1M \\ 1A \end{array}$$

$$\begin{aligned} 4. \quad y &= 4x^3 \cdot \sqrt{x} + 9x^2 \cdot \sqrt[3]{x^4} - 5x \cdot \sqrt[5]{x^2} \\ &= 4x^{\frac{7}{2}} + 9x^{\frac{10}{3}} - 5x^{\frac{7}{5}} \\ \frac{dy}{dx} &= 14x^{\frac{5}{2}} + 30x^{\frac{7}{3}} - 7x^{\frac{2}{5}} \end{aligned} \quad \begin{array}{l} 1M \\ 1A \end{array}$$

$$\begin{aligned} 5. \quad y &= x^{-4}(2x^7 - 3x^{-1} + 9) \\ &= 2x^3 - 3x^{-5} + 9x^{-4} \\ \frac{dy}{dx} &= 6x^2 + 15x^{-6} - 36x^{-5} \end{aligned} \quad \begin{array}{l} 1M \\ 1A \end{array}$$

$$\begin{aligned} 6. \quad y &= \frac{3x^{-1} + 4x^5 - 7x^6}{2x^2} \\ &= \frac{3}{2}x^{-3} + 2x^3 - \frac{7}{2}x^4 \\ \frac{dy}{dx} &= -\frac{9}{2}x^{-4} + 6x^2 - 14x^3 \end{aligned} \quad \begin{array}{l} 1M \\ 1A \end{array}$$

$$\begin{aligned} 7. \quad y &= \frac{x^{\frac{3}{2}} - 6x^2 + 3x^{\frac{8}{3}}}{\sqrt[3]{x}} \\ &= x^{\frac{7}{6}} - 6x^{\frac{5}{3}} + 3x^{\frac{7}{3}} \\ \frac{dy}{dx} &= \frac{7}{6}x^{\frac{1}{6}} - 10x^{\frac{2}{3}} + 7x^{\frac{4}{3}} \end{aligned} \quad \begin{array}{l} 1M \\ 1A \end{array}$$

$$\begin{aligned} 8. \quad y &= \frac{x^2 \cdot \sqrt{x^3} - 3x\sqrt{x} + x^{\frac{9}{8}}}{\sqrt[4]{x^{11}}} \\ &= x^{\frac{3}{4}} - 3x^{-\frac{5}{4}} + x^{-\frac{13}{8}} \\ \frac{dy}{dx} &= \frac{3}{4}x^{-\frac{1}{4}} + \frac{15}{4}x^{-\frac{9}{4}} - \frac{13}{8}x^{-\frac{21}{8}} \end{aligned} \quad \begin{array}{l} 1M \\ 1A \end{array}$$

9. $-15 = p(2)^5 + q(2)^4 + 1$ 1M
- $2p + q = -1$
- $f'(x) = 5px^4 + 4qx^3$ 1A
- $5p(2)^4 + 4q(2)^3 = 0$ 1M
- $5p + 2q = 0$
- 求解後，可得 $p = 2$ 及 $q = -5$ 。 1A+1A
10. $f(x) = \frac{1}{3}x^3 - \frac{1}{x}$
- $= \frac{1}{3}x^3 - x^{-1}$ 1M
- $f'(x) = x^2 + x^{-2}$ 1A
- $2 = a^2 + a^{-2}$ 1M
- $2a^2 = a^4 + 1$
- $0 = a^4 - 2a^2 + 1$
- $a^2 = 1$
- $a = 1$ 或 -1 1A
11. $\frac{dy}{dx} = -5(x - x^3)^{-6}(1 - 3x^2)$ 1M
- $= \frac{5(3x^2 - 1)}{(x - x^3)^6}$ 1A
12. $y = (\sqrt[3]{x} + 2)^8$
- $= (x^{\frac{1}{3}} + 2)^8$
- $\frac{dy}{dx} = 8(x^{\frac{1}{3}} + 2)^7 \left(\frac{1}{3}x^{-\frac{2}{3}} \right)$ 1M
- $= \frac{8}{3}x^{-\frac{2}{3}}(x^{\frac{1}{3}} + 2)^7$ 1A
13. $y = \left(\frac{7}{x} - 6x\sqrt{x} \right)^{-12}$
- $= (7x^{-1} - 6x^{\frac{3}{2}})^{-12}$
- $\frac{dy}{dx} = -12(7x^{-1} - 6x^{\frac{3}{2}})^{-13}(-7x^{-2} - 9x^{\frac{1}{2}})$ 1M
- $= \frac{12(7x^{-2} + 9x^{\frac{1}{2}})}{(7x^{-1} - 6x^{\frac{3}{2}})^{13}}$ 1A
14. $y = \frac{1}{(6\sqrt[3]{x} + 5x^{\frac{3}{5}} - 2x^2)^{-4}}$
- $= (6x^{\frac{1}{3}} + 5x^{\frac{3}{5}} - 2x^2)^4$
- $\frac{dy}{dx} = 4(6x^{\frac{1}{3}} + 5x^{\frac{3}{5}} - 2x^2)^3(2x^{-\frac{2}{3}} + 3x^{-\frac{2}{5}} - 4x)$ 1M+1A

$$15. \quad \frac{dy}{dx} = 3(2x+1)^2(2)(x-4)^{\frac{1}{5}} + (2x+1)^3 \left(\frac{1}{5}\right)(x-4)^{-\frac{4}{5}} \quad 1M$$

$$= \frac{1}{5}(2x+1)^2(x-4)^{-\frac{4}{5}}[30(x-4) + (2x+1)]$$

$$= \frac{(2x+1)^2(32x-119)}{5(x-4)^{\frac{4}{5}}} \quad 1A$$

$$16. \quad y = (2x^3 - 3x)^{\frac{2}{3}}(x^2 - 1)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{3}{2}(2x^3 - 3x)^{-\frac{1}{3}}(6x^2 - 3)(x^2 - 1)^{\frac{1}{2}} + (2x^3 - 3x)^{\frac{2}{3}}\left(\frac{1}{2}\right)(x^2 - 1)^{-\frac{1}{2}}(2x) \quad 1M$$

$$= (2x^3 - 3x)^{-\frac{1}{3}}(x^2 - 1)^{-\frac{1}{2}}[2(x^2 - 1)(2x^2 - 1) + x(2x^3 - 3x)]$$

$$= \frac{6x^4 - 9x^2 + 2}{(x^2 - 1)^{\frac{1}{2}}(2x^3 - 3x)^{\frac{1}{3}}} \quad 1A$$

$$17. \quad y = (x^2 - 4x + 4)^3(\sqrt{8} - \sqrt{2}x)^8$$

$$= [(x-2)^2]^3[\sqrt{2}(2-x)]^8$$

$$= 16(x-2)^{14} \quad 1M$$

$$\frac{dy}{dx} = 224(x-2)^{13} \quad 1A$$

$$\left.\frac{dy}{dx}\right|_{x=3} = 224(3-2)^{13}$$

$$= 224 \quad 1A$$

$$18. \quad f(x) = \sqrt{(x-4)^3(x^2+4x+16)^3}$$

$$= \sqrt{[(x-4)(x^2+4x+16)]^3}$$

$$= (x^3 - 64)^{\frac{3}{2}}$$

$$f'(x) = \frac{3}{2}(x^3 - 64)^{\frac{1}{2}}(3x^2) \quad 1M$$

$$= \frac{9}{2}x^2(x^3 - 64)^{\frac{1}{2}} \quad 1A$$

$$18a^2 = \frac{9}{2}a^2(a^3 - 64)^{\frac{1}{2}} \quad 1M$$

$$\frac{9}{2}a^2[(a^3 - 64)^{\frac{1}{2}} - 4] = 0$$

$$(a^3 - 64)^{\frac{1}{2}} = 4 \quad \text{或} \quad a^2 = 0 \quad (\text{捨去})$$

$$a^3 = 80$$

$$a = 2\sqrt[3]{10} \quad 1A$$