

REV-TRIG-2324-ASM-SET 2-MATH**Suggested solutions****Multiple Choice Questions**

1. D	2. D	3. B	4. D	5. D
6. A	7. A	8. B	9. C	10. A
11. C	12. C	13. C	14. A	15. D
16. B	17. C	18. B	19. C	20. B
21. D	22. D	23. A	24. B	25. A
26. C	27. C	28. C	29. B	30. C

1. D

$$7 \sin^2 x = \sin x$$

$$\sin x = 0 \quad \text{or} \quad \frac{1}{7}$$

When $\sin x = 0$, $x = 0^\circ$ or 180° or 360° ; when $\sin x = \frac{1}{7}$, there are two roots.

There are total 5 roots.

2. D

$$3 \tan^2 x = \tan x$$

$$\tan x(3 \tan x - 1) = 0$$

$$\tan x = 0 \quad \text{or} \quad \frac{1}{3}$$

When $\tan x = 0$, $x = 0^\circ$ or 180° or 360°

When $\tan x = \frac{1}{3}$, there are 2 roots (in quadrants I and III)

Altogether 5 roots.

3. B

$$2 \tan \theta = \sin \theta$$

$$2 \sin \theta = \sin \theta \cos \theta$$

$$\sin \theta(2 - \cos \theta) = 0$$

$$\sin \theta = 0 \quad \text{or} \quad \cos \theta = 2 \text{ (rejected)}$$

There are 3 roots.

4. D

We have $\sin \theta = 0$ or $\tan \theta = -3$.

When $\sin \theta = 0$, $\theta = 0^\circ$ or 180° or 360° .

When $\tan \theta = -3$, there are two roots.

Thus, there are 5 roots.

5. D

$$\sin \theta = \sin^3 \theta$$

$$0 = \sin \theta (\sin^2 \theta - 1)$$

$$\sin \theta = 0 \quad \text{or} \quad \pm 1$$

With reference to the graph of sine, there are 5 roots.

6. A

$$\frac{3}{\tan^2(90^\circ - \alpha)} - \frac{5}{\sin(270^\circ + \alpha)} = \frac{1}{\cos^2 \alpha}$$

$$3 \tan^2 \alpha + \frac{5}{\cos \alpha} = \frac{1}{\cos^2 \alpha}$$

$$3 \sin^2 \alpha + 5 \cos \alpha = 1$$

$$-3 \cos^2 \alpha + 5 \cos \alpha + 2 = 0$$

$$\cos \alpha = -\frac{1}{3} \quad \text{or} \quad 2 \text{ (rejected)}$$

There are two roots for $\cos \alpha = -\frac{1}{3}$. Required number of roots is 2.

7. A

$$3 \cos^2 \theta + \cos \theta - 4 = 0$$

$$\cos \theta = 1 \quad \text{or} \quad -\frac{4}{3} \text{ (rejected)}$$

$$\theta = 0^\circ$$

There is 1 root.

8. B

$$5 \sin^2 \theta + \sin \theta - 4 = 0$$

$$\sin \theta = -1 \quad \text{or} \quad \frac{4}{5}$$

$$\sin \theta = -1 \Rightarrow \theta = 270^\circ; \quad \sin \theta = \frac{4}{5} \Rightarrow \theta \approx 53.1^\circ \text{ or } 127^\circ.$$

$\Rightarrow$ 3 roots

9. C

A. 3 roots

B. 3 roots

C. 4 roots

D. 2 roots

10. A

$$9 \cos^4 \theta - 13 \cos^2 \theta + 4 = 0$$

$$\cos^2 \theta = 1 \quad \text{or} \quad \frac{4}{9}$$

$$\cos \theta = \pm 1 \quad \text{or} \quad \pm \frac{2}{3}$$

When $\cos \theta = \pm 1$, $\theta = 180^\circ$

When $\cos \theta = \pm \frac{2}{3}$, there are 3 roots (one root per quadrant).

There are total 4 roots.

11. C

$$\sin \theta \tan \theta = 1$$

$$\frac{\sin^2 \theta}{\cos \theta} = 1$$

$$1 - \cos^2 \theta = \cos \theta$$

$$-\cos^2 \theta - \cos \theta + 1 = 0$$

$$\begin{aligned} \cos \theta &= \frac{1 \pm \sqrt{1^2 - 4(-1)(1)}}{-2} \\ &= \frac{-1 \pm \sqrt{5}}{2} \end{aligned}$$

Since $\sin \theta \tan \theta = 1$, $\sin \theta$ and $\tan \theta$ are both positive or both negative.

In any of the case, $\cos \theta > 0$.

Thus, $\cos \theta = \frac{\sqrt{5} - 1}{2}$.

12. C

$$4 \cos^2 \theta - 7 \sin \theta - 7 = 0$$

$$4(1 - \sin^2 \theta) - 7 \sin \theta - 7 = 0$$

$$-4 \sin^2 \theta - 7 \sin \theta - 3 = 0$$

$$\sin \theta = -1 \quad \text{or} \quad -\frac{3}{4}$$

$\sin \theta = -1$ has one root and $\sin \theta = -\frac{3}{4}$ has two roots.

13. C

$$5(1 - \sin^2 x) = \sin x + 1$$

$$0 = 5 \sin^2 x + \sin x - 4$$

$$\sin x = \pm \frac{2}{\sqrt{5}}$$

When $\sin x = \frac{2}{\sqrt{5}}$, there are two roots; when $\sin x = -\frac{2}{\sqrt{5}}$, there are also two roots.

There are total 4 roots.

14. A

$$\cos x = \tan x$$

$$\cos^2 x = \sin x$$

$$-\sin^2 x - \sin x + 1 = 0$$

$$\sin x \approx -1.62 \text{ (rejected) or } 0.618$$

There are 2 roots and hence 2 intersections.

15. D

$$2 \cos^2 \theta = 2 - \sin \theta$$

$$2(1 - \sin^2 \theta) = 2 - \sin \theta$$

$$-2 \sin^2 \theta + \sin \theta = 0$$

$$\sin \theta = 0 \quad \text{or} \quad \frac{1}{2}$$

When $\sin \theta = 0$, $\theta = 0^\circ$ or 180° or 360° .

When $\sin \theta = \frac{1}{2}$, $\theta = 30^\circ$ or 150° .

There are 5 roots.

16. B

$$\frac{1 + \cos \theta}{\sin \theta} = 3 \tan \theta$$

$$\cos \theta(1 + \cos \theta) = 3 \sin^2 \theta$$

$$\cos \theta + \cos^2 \theta = 3(1 - \cos^2 \theta)$$

$$4 \cos^2 \theta + \cos \theta - 3 = 0$$

$$\cos \theta = \frac{3}{4} \quad \text{or} \quad -1$$

Note that when $\cos \theta = -1$, we have $\theta = 180^\circ$ and $\sin \theta = 0$, which should be rejected.

There are two roots for $\cos \theta = \frac{3}{4}$.

Thus, there are 2 roots.

17. C

$$\cos^2 x - \sin x = 1$$

$$1 - \sin^2 x - \sin x = 1$$

$$\sin x(\sin x + 1) = 0$$

$$\sin x = 0 \quad \text{or} \quad -1$$

There are three roots for $\sin x = 0$ and one root for $\sin x = -1$.

Thus, there are 4 roots.

18. B

The graph passes through (0, 2).

- A. ✓. When $x = 0$, $y = 2 - 0 = 2$.
- B. ✓. When $x = 0$, $y = 2 - 0 = 2$.
- C. ✗. When $x = 0$, $y = 1 - 1 = 0 \neq 2$.
- D. ✓. When $x = 0$, $y = 1 + 1 = 2$.

The graph passes through (45, 1).

- A. ✗. When $x = 45$, $y = 2 - \sin 22.5^\circ \neq 1$.
- B. ✓. When $x = 45$, $y = 2 - \sin 90^\circ = 1$.
- C.
- D. ✗. When $x = 45$, $y = 1 + \cos 22.5^\circ \neq 1$.

19. C

The graph passes through (60, 0).

- A. ✗. When $x = 60$, $y = \sin 90^\circ - 2 = -1 \neq 0$.
- B. ✗. When $x = 60$, $y = \sin 30^\circ - 2 = -\frac{3}{2} \neq 0$.
- C. ✓. When $x = 60$, $y = \sin 90^\circ - 1 = 0$.
- D. ✗. When $x = 60$, $y = \sin 30^\circ - 1 = -\frac{1}{2} \neq 0$.

20. B

The graph passes through (120, 3).

- A. ✗. When $x = 120$, $y = -3 \sin(120^\circ - 30^\circ) = -3 \neq 3$.
- B. ✓. When $x = 120$, $y = 3 \sin(120^\circ - 30^\circ) = 3$.
- C. ✗. When $x = 120$, $y = -3 \sin(120^\circ + 30^\circ) = -\frac{3}{2} \neq 3$.
- D. ✗. When $x = 120$, $y = 3 \sin(120^\circ + 30^\circ) = \frac{3}{2} \neq 3$.

21. D

Check the equations by the points on the graph:

	<u>(0, 7)</u>	<u>(90, 1)</u>
A.	✗	
B.	✗	
C.	✓	✗
D.	✓	✓

22. D

The graph passes through the following points:

$$\underline{(75, 0)}$$

- A. $\times$
 B. $\times$
 C. $\times$
 D. $\checkmark$

23. A

The graph passes through the following points:

$$\underline{(b, 2)} \quad \underline{(0, \ominus)}$$

- A. $\checkmark$ $\checkmark$
 B. $\times$
 C. $\times$
 D. $\checkmark$ $\times$

$\ominus$ represents a negative number (the y-intercept).

24. B

The graph passes through $(210, -4)$.

$$\underline{(210, -4)}$$

- A. $y = -2 \cos 240^\circ + 2 \neq -4$
 B. $y = 2 \cos 180^\circ - 2 = -4$
 C. $y = 2 \cos 240^\circ + 2 \neq 4$
 D. $y = -2 \cos 180^\circ - 2 = 0 \neq -4$

25. A

If p is positive, then the maximum value and the minimum value of $y = p \sin 3x^\circ + q$ are $p + q$ and $-p + q$ respectively.

$$\text{Solve } \begin{cases} p + q = 1 \\ -p + q = -3 \end{cases}, \text{ we have } p = 2 \text{ and } q = -1.$$

26. C

The curve passes through $(0, 1)$. Only option C satisfies this.

27. C

The graph passes through $(0, 3)$.

- A. ✗. When $x = 0$, $y = \sin 30^\circ + 3 = 3.5 \neq 3$
- B. ✗. When $x = 0$, $y = 2 \sin 30^\circ + 1 = 2 \neq 3$
- C. ✓. When $x = 0$, $y = 2 \sin 30^\circ + 2 = 3$
- D. ✗. When $x = 0$, $y = 3 \sin 30^\circ + 4 = 5.5 \neq 3$

28. C

The graph passes through $(0^\circ, 1)$ and $(90^\circ, 3)$.

	<u>$(0^\circ, 1)$</u>	<u>$(90^\circ, 3)$</u>
A.	✓	✗
B.	✗	
C.	✓	✓
D.	✗	

29. B

The graph passes through $(146, 0)$.

- A. ✗. When $x = 146$, $y = -3 \sin 112^\circ \neq 0$.
- B. ✓. When $x = 146$, $y = -3 \sin 180^\circ = 0$.
- C. ✗. When $x = 146$, $y = -3 \sin 100^\circ \neq 0$.
- D. ✗. When $x = 146$, $y = -3 \sin 192^\circ \neq 0$.

30. C

The graph passes through $(0, k)$, where $-1 < k < 0$.

- A. ✗. $f(0) = -1$
- B. ✓. $f(0) = -\frac{1}{2}$, which lies between -1 and 0 .
- C. ✓. $f(0) = -\frac{1}{2}$, which lies between -1 and 0 .
- D. ✗. $f(0) = -1$

The graph passes through $\left(90, -\frac{1}{2}\right)$.

- B. ✗. $f(90) = \frac{\sin 45^\circ}{2} - \frac{1}{2} \neq -\frac{1}{2}$
- C. ✓. $f(90) = 0 - \frac{1}{2} = -\frac{1}{2}$