REV-LOG-2324-ASM-SET 3-MATH
Suggested solutions

Multiple Choice Questions

1. D	2. C	3. B	4. C	5. A
6. B	7. A	8. D	9. A	10. D
11. A	12. C	13. A	14. C	15. A
16. D	17. A	18. B	19. A	20. D
21. C	22. D	23. C	24. D	25. A
26. C	27. B	28. B	29. C	30. B
31. A	32. C	33. B	34. B	35. C
36. D	37. C	38. C	39. A	40. A
41. B	42. B			

1. $\boxed{D}$

$$y = a + b^x$$
$$y - a = b^x$$
$$\log(y - a) = x \log b$$

The graph of $\log(y - a)$ against x is a straight line.
Slope = $\log b$ and passes through the origin.

2. $\boxed{C}$

For the point $(0,\ -2)$.

$$\log x = 0 \quad \text{and} \quad \log y = -2$$
$$x = 1 \qquad\qquad y = 10^{-2} = \frac{1}{100}$$

$$\frac{1}{100} = k(1)^2$$
$$k = \frac{1}{100}$$

3. $\boxed{B}$

Base of log is 2, which is greater than 1.
The graph of y against x is also increasing.

$$x = 0 \quad \to \quad \log_2 y = -1 \quad \longrightarrow \quad y = 2^{-1} = 0.5$$

Only option B satisfies these.

4. C

$$y = 3x^2$$

$$\log y = 2\log x + \log 3$$

I. ✓.

II. ✗. There is no y-intercept indeed, we have only the $\log y$-intercept.

III. ✓. $y = 3x^2$

$\log_3 y = 2\log_3 x + \log_3 3$‘

Slope of the line is also 2.

5. A

$$\log_2 y - 1 = \frac{1}{3}(\log_4 x + 3)$$

$$\log_2 y = \frac{1}{3}\log_4 x + 2$$

$$\frac{\log y}{\log 2} = \frac{\log x}{3(2\log 2)} + 2$$

$$\log y = \frac{1}{6}\log x + 2\log 2$$

$$= \log 2^2 x^{\frac{1}{6}}$$

$$y = 4x^{\frac{1}{6}}$$

Thus, we have $n = \frac{1}{6}$.

6. B

I. ✓.

II. ✓.

III. ✗. The graph of $y = \log_a x$ should pass through $(1,\ 0)$.

7. A

A. ✓.

B. ✗. The graph passes through $(1,\ 0)$ but $y = 5^1 = 5 \neq 0$.

C. ✗. The graph passes through $(1,\ 0)$ but $y = 0.2^1 = 0.2 \neq 0$.

D. ✗. When $y = 1$, $x = 0.2^1 = 0.2$ but the graph shown does not pass through $(0.2,\ 1)$ obviously.

8. D

Slope of the graph $= \dfrac{0+2}{4-0} = \dfrac{1}{2}$. The equation is

$$\log_9 y = \frac{1}{2}x - 2$$
$$y = 9^{\frac{1}{2}x-2}$$
$$= 9^{-2} \times 3^x$$

So, $b = 3$.

9. A

Consider the point $(0,\ 2)$.

$$\log_3 x = 0 \quad \text{and} \quad \log_3 y = 2$$
$$x = 1 \qquad\qquad y = 3^2 = 9$$

Consider the point $(4,\ 0)$.

$$\log_3 x = 4 \quad \text{and} \quad \log_3 y = 0$$
$$x = 3^4 = 81 \qquad\qquad y = 1$$

Check the relation using the values of x and y.

	$x = 1$ & $y = 9$	$x = 81$ & $y = 1$
A.	✓	✓
B.	✗	
C.	✓	✗
D.	✗	

The answer is A.

10. D

For the point $(0,\ -3)$.

$$\log_9 x = 0 \quad \text{and} \quad \log_3 y = -3$$
$$x = 1 \qquad\qquad y = 3^{-3} = \frac{1}{27}$$

Only option D satisfies this pair of x and y.

11. A

For the point $(0,\ 5)$,

$$\begin{aligned} \log_2 x &= 0 \quad \text{and} \quad & \log_8 y &= 5 \\ x &= 1 & y &= 8^5 \\ & & &= 2^{15} \end{aligned}$$

Only option A satisfies this.

12. C

Bases of $\log_4 x$ and $\log_4 y$ are both 4, which is greater than 1.
The resulting graph is also decreasing.
When $\log_4 x = 0$, we have $x = 1$ and $\log_4 y = \log_4(4 \cdot 1^n) = 1 > 0$.
Value of vertical intercept is positive. Only option C satisfies all of these.

13. A

For the point $(0,\ 3)$.

$$\begin{aligned} \log_2 t &= 0 \quad \text{and} \quad & \log_2 x &= 3 \\ t &= 1 & x &= 2^3 = 8 \end{aligned}$$

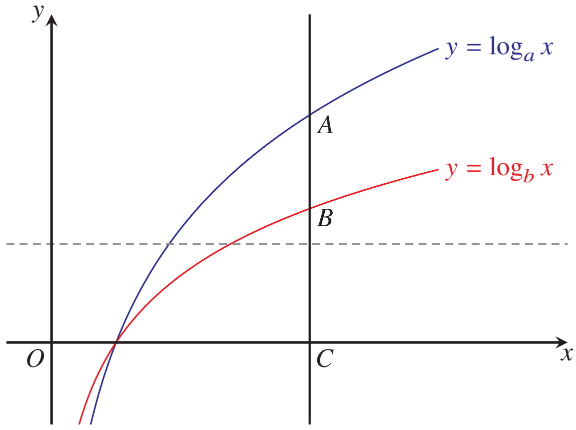
For the point $(4,\ 0)$.

$$\begin{aligned} \log_2 t &= 4 \quad \text{and} \quad & \log_2 x &= 0 \\ t &= 2^4 = 16 & x &= 1 \end{aligned}$$

The answer is A.

14. C

The x-intercept of the graphs are 1. Draw the line $y = 1$.



I. ✓. $y = \log_a x$ intersects the line $y = 1$ at $(a,\ 1)$.
Compare with the x-intercept, we have $a > 1$.

II. ✗. $y = \log_b x$ intersects the line $y = 1$ at $(b,\ 1)$.
Therefore, $b > a$

III. ✓. Let $x = k$. Then $A\left(k,\ \log_a k\right)$ and $B\left(k,\ \log_b k\right)$.

$$\begin{aligned}\frac{AB}{BC} &= \frac{\log_a k - \log_b k}{\log_b k} \\ &= \frac{\frac{\log k}{\log a} - \frac{\log k}{\log b}}{\frac{\log k}{\log b}} \times \frac{\log a \log b}{\log a \log b} \\ &= \frac{\log b - \log a}{\log a} \\ &= \log_a \frac{b}{a}\end{aligned}$$

15. A

For the point $\left(\frac{1}{3},\ 0\right)$,

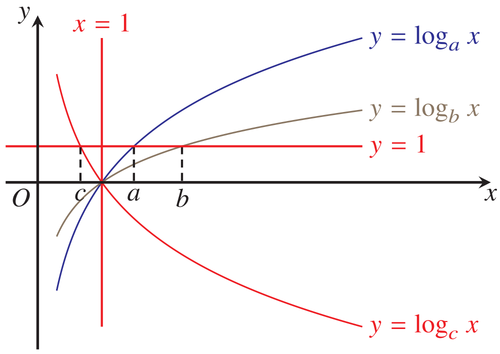
$$\log_8 x = \frac{1}{3} \qquad \text{and} \log_4 y = 0$$
$$x = 8^{\frac{1}{3}} = 2 \qquad y = 1$$

Only option A satisfies this.

16. D

Draw the line $y = 1$.

The line intersects the graphs at $(a,\ 1)$, $(b,\ 1)$ and $(c,\ 1)$.
From the graph, we have $c < a < b$.



17. A

Slope = 2 and vertical intercept = −2. We have $\log_3 y = 2\log_3 x - 2$.

$$\log_3 y = 2\log_3 x - 2$$
$$\log_3 y = \log_3 x^2 - \log_3 9$$
$$y = \frac{x^2}{9}$$

It has a shape of parabola opening upwards and passing through origin.

18. B

$y = m\log_{\frac{1}{3}} x + c = -m\log_3 x + c$

$$\text{Slope} = -m = \frac{2}{4}$$
$$m = -\frac{1}{2}$$

19. A

Let $\log y = mx + c$, where m is a constant.
From the linear graph, we have $m > 0$ and $c < 0$.

$$\log y = mx + c$$
$$y = 10^{mx+c}$$
$$y = 10^c \cdot (10^m)^x$$

Consider the graph of y against x.
y-intercept $= 10^c < 10^0 = 1$
Since $10^m > 10^0 = 1$, the curve is increasing.
The answer is A.

20. D

The base of log is 6 (greater than 1), the new graph is also decreasing.
When $x = 0$, $y = 0.5$, then $\log_6 y = \log_6 0.5 < 0$.
The new graph has a negative intercept on vertical axis.

21. C

$$y = ab^x$$

$\log y = \log a + x \log b$
Slope $= \log b < 0 \Rightarrow 0 < b < 1$
Vertical intercept $= \log a > 0 \Rightarrow a > 1$

22. D

Substitute $(\log_4 x,\ \log_4 y) = (1,\ 2)$ and $(9,\ 6)$ into $\log_4 y = \log_4 k + a \log_4 x$,

$$\begin{cases} 2 = \log_4 k + a(1) \\ 6 = \log_4 k + a(9) \end{cases}$$

Solving, we have $\log_4 k = \dfrac{3}{2}$ and $a = \dfrac{1}{2}$.
So, $k = 4^{\frac{3}{2}} = 8$.

23. C

Slope of the line $= \dfrac{6-0}{0-3} = -2$
Equation of the graph is

$$\begin{aligned} \log_a y - 6 &= -2(x-0) \\ \log_a y &= -2x + 6 \\ y &= a^{-2x+6} \\ &= a^6 (a^{-2})^x \end{aligned}$$

We have $m = a^6$ and $n = a^{-2}$.

I. ✓.

II. ✗. Take $a = 0.5$, then $n = 4$.

III. ✓. $mn^3 = (a^6)(a^{-2})^3 = 1$.

24. D

Suppose the equation of L is $x = k$.

I. ✗. $y = \log_b x$ and the line $y = 1$ intersect at $(b,\ 1)$.
The x-intercept of the graph of $y = \log_b x$ is 1.
Thus, we have $0 < b < 1$ from the graph.

II. ✓. The coordinates of A and B are $\left(k,\ \log_a k\right)$ and $\left(k,\ \log_b k\right)$. Since $AC = BC$,

$$\begin{aligned}\log_a k &= -\log_b k\\ \frac{\log k}{\log a} &= -\frac{\log k}{\log b}\\ \log b &= -\log a\\ \log ab &= 0\\ ab &= 1\end{aligned}$$

III. ✓. The x-intercept of the curves is 1.
Thus, $OC > 1$.

25. A

For the point $(0,\ 2)$.

$$\begin{aligned}\log_3 x &= 0 &\quad\text{and}\quad \log_3 y &= 2\\ x &= 1 & y &= 3^2 = 9\end{aligned}$$

Only option A satisfies this.

26. C

Slope of the graph $= \dfrac{4-0}{0+2} = 2.$ The equation is

$$\begin{aligned}\log_3 y &= 2x + 4\\ y &= 3^{2x+4}\\ &= 3^{2x} \times 3^4\\ &= 81 \times 9^x\end{aligned}$$

So, $n = 9$.

27. B

(4, 0)

$\log_2 x = 4$ and $\log_4 y = 0$

$x = 2^4$ $\quad$ $y = 1$

(0, 5)

$\log_2 x = 0$ and $\log_4 y = 5$

$x = 1$ $\quad$ $y = 4^5$

$y = 2^{10}$

	x^5y^2	x^2y^5	x^4y^2	x^5y^4
$x = 2^4$ and $y = 1$	2^{20}	2^8	2^{16}	2^{20}
$x = 1$ and $y = 2^{10}$	2^{20}	2^{50}	2^{20}	2^{40}

The answer is B.

Slope of the linear graph $= \dfrac{5-0}{0-4} = -\dfrac{5}{4}$

$$\log_4 y - 5 = -\frac{5}{4}(\log_2 x - 0)$$

$$5\log_2 x + 4\log_4 y = 20$$

$$\frac{5\log x}{\log 2} + \frac{4\log y}{2\log 2} = 20$$

$$5\log x + 2\log y = 20\log 2$$

$$\log x^5y^2 = \log 2^{20}$$

$$x^5y^2 = 2^{20}$$

28. B

At the point (2, 1),

$\log_2 x = 2$ and $\log_4 y = 1$

$x = 4$ $\quad$ $y = 4$

Put $(x,\ y) = (4,\ 4)$ into $y = kx^a$,

$$4 = k(4^a)$$

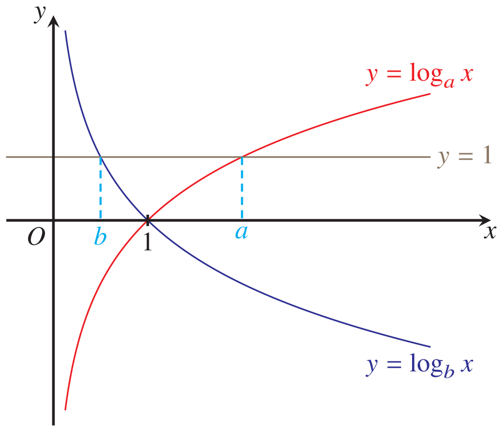
$$k = \frac{4}{4^a}$$

$$= 4^{1-a}$$

29. C

Draw the line $y = 1$.

The line intersects the graph at $(b,\ 1)$ and $(a,\ 1)$.
From the graph, we have $0 < b < 1 < a$.
The result follows.



30. B

Base of $\log_7 y$ is 7, and is greater than 1. So, the graph of $\log_7 y$ against x is also decreasing as in the original graph of y against x.

$x = 0 \quad \rightarrow \quad y = 3 \quad \rightarrow \quad \log_7 y = \log_7 3 > 0$

The result is a straight line with negative slope and positive intercept on vertical axis, which is B.

31. A

When $x = 1$, $y = \log_a 1 + b = b$.
The curve passes through $(1,\ b)$. We have $b < 0$.
Consider the x-intercept of the graph.

$$\begin{aligned} 0 &= \log_a x + b \\ \log_a x &= -b \\ x &= a^{-b} \end{aligned}$$

We have $a^{-b} < 1$ from the graph.
Thus, we have $0 < a < 1$.

32. C

From the point $(0,\ 2)$.

$$\log_5 x = 0 \quad \text{and} \quad \log_5 y = 2$$
$$x = 5^0 \qquad\qquad y = 5^2$$
$$x = 1 \qquad\qquad y = 25$$

From the point $(-4,\ 0)$.

$$\log_5 x = -4 \quad \text{and} \quad \log_5 y = 0$$
$$x = 5^{-4} \qquad\qquad y = 1$$

	$x = 1$ and $y = 25$	$x = 5^{-4}$ and $y = 1$
A.	✓	✗
B.	✗	
C.	✓	✓
D.	✗	

33. B

Base of $\log_3 y$ is 3, which is greater than 1.
The required graph is also increasing.
When $x = 0$, $\log_3 y = \log_3 0.5 < 0$.
Required graph is increasing and has a negative intercept on the vertical axis.

34. B

$$\frac{AC}{AB} = \frac{\log_c k}{\log_b k}$$
$$= \frac{\log k}{\log c} \div \frac{\log k}{\log b}$$
$$= \frac{\log b}{\log c}$$
$$= \log_c b$$

35. C

For the point $(0,\ -1)$,

$$\log_7 x = 0 \quad \text{and} \quad \log_7 y = -1$$
$$x = 1 \qquad\qquad y = 7^{-1}$$

We have $7^{-1} = a(1)^b$, and $a = \frac{1}{7}$.
For the point $(2,\ 0)$,

$$\log_7 x = 2 \quad \text{and} \quad \log_7 y = 0$$
$$x = 49 \qquad\qquad y = 1$$

We have $1 = \frac{1}{7}(49)^b$, and $b = \frac{1}{2}$.

36. D

$\log_{27} y = \log_{27} a + x\log_{27} b$

$$\text{Slope} = \log_{27} b = \frac{0+1}{3-0}$$
$$b = 27^{\frac{1}{3}}$$
$$= 3$$

37. C

For the point $(0,\ 8)$,

$$\log_4 x = 0 \quad \text{and} \quad \log_8 y = 8$$
$$x = 1 \qquad\qquad y = 8^8$$
$$= 2^{24}$$

Only option C satisfies this.

38. C

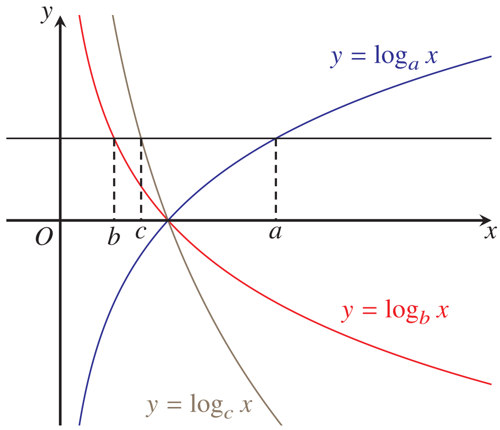
$$0 = a + \log_b x \quad \text{and} \quad 0 = \log_c x$$
$$\log_b x = -a \qquad\qquad x = 1$$
$$x = b^{-a}$$

Thus, $OT : OS = 1 : b^{-a} = b^a : 1$.

39. A

Draw the line $y = 1$.

Note that the x-intercepts of the graphs are 1.
We have $0 < b < c < 1 < a$.



40. A

Slope of the graph $= -4$ and $y^3 = -4\log_5 x + 12$.

$$2^3 = -4\log_5 x + 12$$
$$\log_5 x = 1$$
$$x = 5$$

41. B

For the point $(0,\ 7)$.

$$\log_3 x = 0 \quad \text{and} \quad \log_9 y = 7$$
$$x = 1 \qquad\qquad y = 9^7$$

A. ✗. $x^4y^7 = 9^{56} \neq 3^{56}$

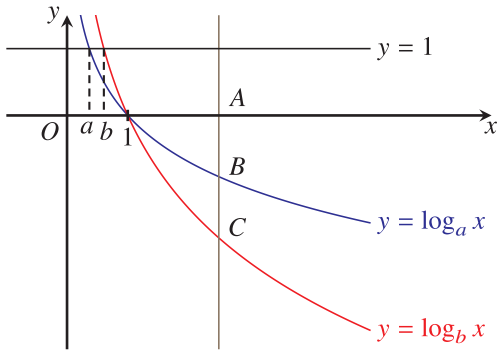
B. ✓. $x^7y^4 = 9^{28} = 3^{56}$

C. ✗. $x^7y^8 = 9^{56} \neq 3^{56}$

D. ✗. $x^8y^7 = 9^{49} \neq 3^{56}$

42. B

Draw the line $y = 1$. We have $0 < a < b < 1$.



I. ✗.

II. ✗.

III. ✓. Let the equation of ABC be $x = k$.
The coordinates of B and C are $(k,\ \log_a k)$ and $(k,\ \log_b k)$ respectively.

$$\frac{BC}{AB} = \frac{\log_a k - \log_b k}{-\log_a k}$$

$$= \frac{\frac{\log k}{\log a} - \frac{\log k}{\log b}}{-\frac{\log k}{\log a}}$$

$$= \frac{\log a}{\log b} - 1$$

$$= \log_b a - \log_b b$$

$$= \log_b \frac{a}{b}$$

END OF PAPER