REG-MEN-2324-ASM-SET 2-MATH
Suggested solutions

Conventional Questions

1. (a) Volume of cone

$= \frac{1}{3}(24)^2\pi(36)$ 1M

$= 6912\pi \text{ cm}^3$

Volume of water

$= 6912 \times \left(\frac{1}{3}\right)^3$ 1M

$= 256\pi \text{ cm}^3$ 1A

(b) Let h cm be the new height of the water layer.

$$\frac{6912\pi - 256\pi}{6912\pi} = \left(\frac{36-h}{36}\right)^3$$ 1M

$$(36-h)^3 = 44\,928$$

$$36 - h = \sqrt[3]{44\,928}$$

$$h \approx 0.450$$ 1A

Required height is 0.450 cm.

(c) $\frac{\text{lower base radius of oil layer}}{\text{lower base radius of water layer}} = \frac{24}{24} = 1$ 1M

$\frac{\text{volume of oil layer}}{\text{volume of water layer}} = \frac{6912\pi - 256\pi}{256\pi} = 26 \neq 1^3$

They are not similar.

The claim is disagreed. 1A

2. (a) Denote reflex $\angle COD$ by θ.

$$\frac{2\pi(OA)\times\frac{\theta}{360^\circ}}{2\pi(OC)\times\frac{\theta}{360^\circ}}=\frac{180\pi}{60\pi} \qquad \text{1M}$$

$$\frac{OA}{OA-100}=3$$

$$OA=150\,\text{cm} \qquad \text{1A}$$

(b) Upper base radius $=\dfrac{180\pi}{2\pi}=90\,\text{cm}$

Lower base radius $=\dfrac{60\pi}{2\pi}=30\,\text{cm}$

Height of frustum $=\sqrt{100^2-(90-30)^2}$ 1M

$=80\,\text{cm}$

Let h cm be the height of the large cone that form the frustum.

$$\frac{h-80}{h}=\frac{30}{90}$$

$$h=120$$

Capacity of the bowl

$=\frac{1}{3}(90)^2(120)-\frac{1}{3}(30)^2(40)-\frac{2}{3}\pi(30)^3$ 1M+1M

$\approx 924\,000\,\text{cm}^3$

$=0.924\,\text{m}^3$

$<0.95\,\text{m}^3$

The water will overflow. 1A

3. (a) Volume of the cylinder $= \pi(4)^2\left(\frac{70}{9}\right)$

$= \frac{1120\pi}{9}\text{ cm}^3$

Length ratio of the cones $= \sqrt{4} : \sqrt{9} = 2 : 3$

Volume ratio of the cones $= 2^3 : 3^3 = 8 : 27$ 1M

Required volume $= \frac{1120\pi}{9} \times \frac{27}{8+27}$ 1M

$= 96\pi\text{ cm}^3$ 1A

(b) Height of the larger cone $= \frac{96\pi}{\frac{1}{3}\pi(6)^2}$

$= 8\text{ cm}$ 1A

$\frac{\text{curved surface area of the larger cone}}{\text{curved surface area of } A}$

$= \frac{\pi(6)(\sqrt{6^2+8^2})}{240\pi}$ 1M

$= \frac{1}{4}$

$\left(\frac{\text{base radius of the larger cone}}{\text{base radius of } A}\right)^2$

$= \left(\frac{6}{15}\right)^2$

$= \frac{4}{25}$

$\neq \frac{1}{4}$ 1M

They are not similar.

The claim is disagreed. 1A

4. (a) $(10 + 6\pi) = 2(5) + 2\pi(5) \times \frac{m°}{360°}$ 1M

$m = 216$ 1A

(b) Let r cm be the base radius of the cone.

$2\pi r = (10 + 6\pi) - 2(5)$ 1M

$r = 3$

Required height $= \sqrt{5^2 - 3^2}$ 1M

$= 4\text{ cm}$ 1A

5. (a) Volume $= \pi(12)^2(27)$ 1M

$= 3888\pi \text{ cm}^3$ 1A

(b) Capacity of the conical vessel

$= \frac{1}{3}\pi(21)^2(42)$ 1M

$= 6174\pi \text{ cm}^3$

Let r cm be the radius of the water surface in the vessel.

$$\left(\frac{r}{21}\right)^3 = \frac{3888\pi}{6174\pi}$$ 1M+1A

$$r^3 = 5832$$

$$r = 18$$ 1A

Required radius is 18 cm.

(c) Total volume of the sphere and water

$= \frac{4}{3}\pi(12)^3 + 3888\pi$ 1M

$= 6192\pi \text{ cm}^3$ 1M

$> 6174\pi \text{ cm}^3$

The water will overflow. 1A

6. (a) Height of the original cone = 24 cm

Curved surface area of the original cone

$= \pi(10)(\sqrt{10^2 + 24^2})$

$= 260\pi \text{ cm}^2$

Total surface area of X

$= 260\pi \times \left(1 - \frac{5^2}{10^2}\right) + 2\pi(10)^2 + 5^2$ 1M+1M

$= 420\pi \text{ cm}^2$ 1A

(b) Volume of X

$= \frac{1}{3}\pi(10)^2(24) \times \left(1 - \frac{5^3}{10^3}\right) + \frac{2}{3}\pi(10)^3$ 1M

$= \frac{4100\pi}{3} \text{ cm}^3$

Volume of Y

$= \frac{4100\pi}{3} \times \left(\sqrt{\frac{9}{16}}\right)^3$ 1M

$\approx 1810 \text{ cm}^3$

$> 1800 \text{ cm}^3$

The claim is disagreed. 1A

7. (a) Let the base radius be r cm.

$$80\pi = 10(2r\pi) \qquad \text{1M}$$

$$r = 4 \qquad \text{1A}$$

The base radius is 4 cm.

(b) Capacity $= (4)^2\pi(10)$ 1M

$= 160\pi \text{ cm}^3$ 1A

(c) Base radius of bucket $B = \dfrac{8\pi}{2\pi} = 4$ cm.

$\dfrac{\text{base radius of } A}{\text{base radius of } B} = 1 \neq \dfrac{\text{height of } A}{\text{height of } B} = \dfrac{10}{12}$ 1M

Thus, they are not similar. 1A

8. (a) Radius ratio of the smaller sphere to the larger sphere is 3 : 4. 1M

Required surface area $= 4\pi\left[18 \times \dfrac{4}{3}\right]^2$ 1M

$= 2304\pi \text{ cm}^2$ 1A

(b) Let r cm be the base radius of X.

$$\pi r^2 = 2304\pi$$

$$r = 48 \quad \text{or} \quad -48 \text{ (rejected)}$$

Volume of $X = \dfrac{1}{3}\pi(48)^2(30)$ 1M

$= 23\,040\pi \text{ cm}^3$

Volume of $Y = \dfrac{4}{3}\pi(18)^3 + \dfrac{4}{3}\pi\left(18 \times \dfrac{4}{3}\right)^3 - 23\,040\pi$ 1M

$= 3168\pi \text{ cm}^3$

Let R cm be the base radius of Y.

$$\pi R^2(22) = 3168\pi$$

$$R = 12 \quad \text{or} \quad \text{(rejected)}$$

Required area $= 2\pi(12)^2 + 2\pi(12)(22)$

$= 816\pi \text{ cm}^2$ 1A

9. (a) Volume of milk $= \frac{1}{3}\pi(78)^2\sqrt{130^2 - 78^2}$ 1M

$= 210\,912\pi \text{ cm}^3$ 1A

(b) Let the height of the frustum inside the hemispherical vessel be h cm.

$$h^2 + \left(\frac{AC}{2}\right)^2 = 75^2$$ 1M

$$h^2 + 60^2 = 75^2$$

$$h = 45$$

Volume of the frustum

$= \frac{1}{3}(96)(72)(150)\left[1 - \left(\frac{150 - 45}{150}\right)^3\right]$ 1M

$= 227\,059.2 \text{ cm}^3$

Capacity of the vessel

$= \frac{2}{3}\pi(75)^3 - 227\,059.2$ 1M

$\approx 656\,514 \text{ cm}^3$ 1A

$< 210\,912\pi \text{ cm}^3$

The milk will overflow.

The claim is agreed. 1A

END OF PAPER