REG-MEN-2324-ASM-SET 1-MATH
Suggested solutions

Multiple Choice Questions

1. C	2. C	3. D	4. A	5. A
6. B	7. C	8. C	9. B	10. C
11. C	12. A	13. B	14. D	15. A
16. D	17. B	18. D	19. B	20. D
21. C	22. D	23. C	24. B	25. B
26. B	27. D	28. A	29. B	30. D
31. B	32. D	33. A	34. B	35. D
36. D	37. C	38. C	39. B	40. C
41. B	42. D			

1. $\boxed{\text{C}}$

Let the base radius of the cone be r cm.

$$\frac{1}{3}\pi r^2(24) = 3 \times \frac{2}{3}\pi r^3$$
$$r = 4$$

Total surface area

$= 3(2\pi r^2 + \pi r^2)$

$= 144\pi \text{ cm}^2$

2. $\boxed{\text{C}}$

Let r cm and R cm be the radii of the smaller spheres and the larger sphere respectively.

$$\frac{4}{3}\pi R^3 = 64 \times \frac{4}{3}\pi r^3$$
$$R^3 = 64r^3$$
$$R = 4r$$

$$\begin{aligned} A_2 &= A_1(1 + x\%) \\ 64 \times 4\pi r^2 &= 4\pi R^2(1 + x\%) \\ 64r^2 &= (4r)^2(1 + x\%) \\ 1 + x\% &= 4 \\ x &= 300 \end{aligned}$$

3. $\boxed{\text{D}}$

Let the base radius of the cone be r cm.

$$\frac{1}{3}\pi r^2(6) = 128\pi$$
$$r = 8$$

Total surface area

$= \pi(8)^2 + \pi(8)(\sqrt{8^2 + 6^2})$

$= 144\pi \text{ cm}^2$

4. $\boxed{\text{A}}$

$CM = \sqrt{26^2 - 10^2}$

$= 24 \text{ cm}$

$BM = \sqrt{20^2 - \left(\frac{24}{2}\right)^2}$

$= 16 \text{ cm}$

Required area $= \frac{(12)(16)}{2}$

$= 96 \text{ cm}^2$

5. $\boxed{\text{A}}$

Let the slant height of the cone be ℓ.

$$4\pi r^2 = \pi r \ell$$
$$\ell = 4r$$

Volume $= \frac{1}{3}\pi r^2 \sqrt{\ell^2 - r^2}$

$= \frac{\sqrt{15}}{3}\pi r^3$

6. $\boxed{\text{B}}$

Let O be the centre. Radius $= \frac{AD}{2} = \frac{\sqrt{10^2 + 24^2}}{2} = 13 \text{ cm}$

$AC^2 = OA^2 + OC^2 - 2(OA)(OC)\cos\angle AOC$

$\angle AOC \approx 135°$

Area $= 13^2\pi \times \frac{\angle AOC}{360°} - \frac{1}{2}(13)^2 \sin\angle AOC \approx 139 \text{ cm}^2$

7. $\boxed{\text{C}}$

Area ratio $= \frac{0.54(1000 \times 100)^2}{216}$

$= 25\,000\,000$

Required scale $= 1 : \sqrt{25\,000\,000}$

$= 1 : 5000$

8. $\boxed{\text{C}}$

$$\text{Radius} = \sqrt{\left(\frac{24}{2}\right)^2 + 5^2}$$
$$= 13\text{ cm}$$

Let the angle subtended by the minor segment be 2θ.

$$\cos\theta = \frac{5}{13}$$
$$\theta \approx 67.4°$$

$$\text{Required perimeter} = 2\pi(13) \times \frac{2\theta}{360°} + 24$$
$$\approx 55\text{ cm}$$

9. $\boxed{\text{B}}$

$$OR = \frac{12}{2} = 6\text{ cm}$$

Required area

$$= \pi(12)^2 \times \frac{30°}{360°} - \frac{1}{2}(6)(12\sin 30°)$$
$$= 12\pi - 18$$
$$= 6(2\pi - 3)\text{ cm}^2$$

10. $\boxed{\text{C}}$

Let the radius be r cm.

$$3\pi = r^2\pi \times \frac{120°}{360°}$$
$$r = 3 \quad \text{or} \quad -3 \text{ (rejected)}$$

$$\overset{\frown}{AB} = 2\pi(3) \times \frac{120°}{360°}$$
$$= 2\pi\text{ cm}$$

$$\text{Area of } \triangle OAB = \frac{2(3\sin 60°)(3\cos 60°)}{2}$$
$$= \frac{9\sqrt{3}}{4}\text{ cm}^2$$

$$AB = 2(3\sin 60°)$$
$$= 3\sqrt{3}\text{ cm}$$

11. $\boxed{\text{C}}$

Base radius of the cone $= \sqrt{20^2 - 16^2} = 12\text{ cm}$

Volume of the cone $= \frac{1}{3}\pi(12)^2(16)$

$= 768\pi\text{ cm}^3$

Let the height of the frustum be h cm.

$$\frac{768\pi - 756\pi}{768\pi} = \left(\frac{16-h}{16}\right)^3$$

$$(16-h)^3 = 64$$

$$16 - h = 4$$

$$h = 12$$

12. $\boxed{\text{A}}$

Let F be the point of intersection of AE produced and CD produced such that $ABCF$ is a square.

We have $\angle FDE = 45°$ and $FD = 4\sqrt{2}\cos 45° = 4\text{ cm}$.

Area of pentagon $= 12^2 - \frac{(4)(4)}{2} = 136\text{ cm}^2$

13. $\boxed{\text{B}}$

$TS = \frac{144 + 24}{2}$

$= 84\text{ cm}$

Radius $= \sqrt{12^2 + 84^2}$

$= 60\sqrt{2}\text{ cm}$

Let V be a point on OQ such that $RV \perp OQ$.

Consider $\triangle ORV$.

$$\sin\angle ROV = \frac{RV}{OR}$$

$$\sin\angle ROV = \frac{84 - 24}{60\sqrt{2}}$$

$$\angle ROV = 45°$$

Required area $= \pi(60\sqrt{2})^2 \times \frac{45°}{360°}$

$= 900\pi\text{ cm}^2$

14. $\boxed{\text{D}}$

Horizontal separation of $AG = 1 + 2 - 1 = 2\text{ cm}$

Vertical separation of $AG = 2 + 2(2) - x = (6 - x)\text{ cm}$

$$2^2 + (6-x)^2 = \left(\frac{2\sqrt{194}}{5}\right)^2$$

$$x = 0.8$$

15. $\boxed{\text{A}}$

Let r cm and h cm be the base radius and the height of the right circular cylinder respectively.
Let V cm^3 be the volume of the pyramid.

$$\frac{\pi r^2 h}{\frac{1}{3}r^2(3h)} = \frac{320\pi}{V}$$

$$V = 320$$

The volume of the pyramid is $320\,\text{cm}^3$.

16. $\boxed{\text{D}}$

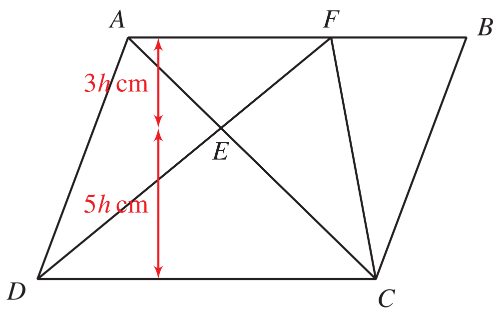
$\triangle AFE \sim \triangle CDE$ (ratio 3 : 5)

Let $AF = 3\,\text{cm}$, then $CD = 5\,\text{cm}$ and $BF = 2\,\text{cm}$.

$$\frac{(2)(3h+5h)}{2} = 16$$

$$h = 2$$

Area of $\triangle CDE = \dfrac{(5)(5\times 2)}{2} = 25\,\text{cm}^2$



17. $\boxed{\text{B}}$

Let $BE = 1\,\text{cm}$. Then $CE = 2\,\text{cm}$ and $AD = 6\,\text{cm}$.

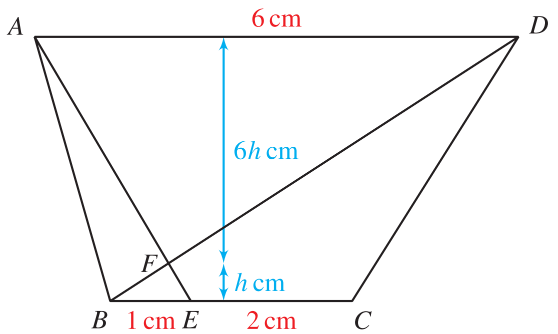
$\triangle ADF \sim \triangle EBF$ (ratio 6 : 1)

$$6 = \frac{(1)(7h)}{2} - \frac{(1)(h)}{2}$$

$$h = 2$$

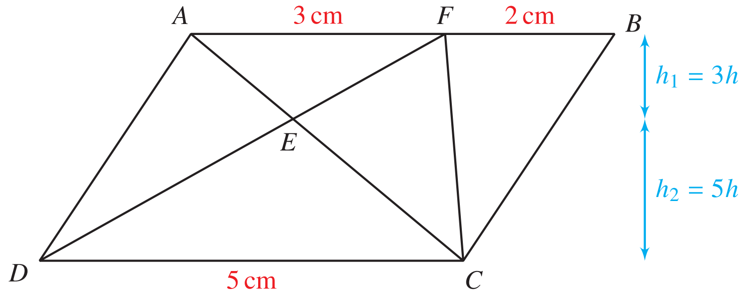
Required area $= \dfrac{(3)(7h)}{2} - \dfrac{(1)(h)}{2}$

$= 20\,\text{cm}^2$



18. $\boxed{\text{D}}$

Let $AF = 3\text{ cm}$. Then we have the lengths as shown in the figure.



$\triangle AFE \sim \triangle CDE$

$$\frac{h_1}{h_2} = \frac{AF}{CD} = \frac{3}{5}$$

Consider the area of $\triangle BFC$,

$$\frac{(2)(8h)}{2} = 16$$

$$h = 2$$

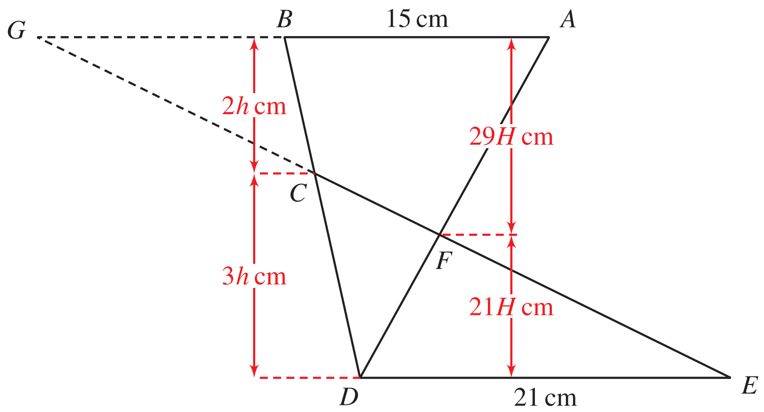
Required area $= \dfrac{(5)(10)}{2} = 25\text{ cm}^2$

19. $\boxed{\text{B}}$

Let $AB = 15$ cm. Then $DE = 21$ cm.

Let G be the intersection of AB produced and EC produced.

Rotate the figure such that the parallel lines appear horizontal as shown.



$\boxed{\text{Point } C}$

We have $\triangle GBC \sim \triangle EDC$ (ratio 2 : 3).

Let $2h$ cm and $3h$ cm be the heights of two triangles respectively.

$GB = \frac{2}{3} \times DE = 14$ cm

$\boxed{\text{Point } F}$

We have $\triangle AFG \sim \triangle DFE$ (ratio 29 : 21).

Let $29H$ cm and $21H$ cm be the heights of two triangles respectively.

Consider the area of $\triangle CDF$.

$$\frac{21(3h)}{2} - \frac{21(21H)}{2} = 63$$
$$3h - 21H = 6$$

Consider the total height of the figure.

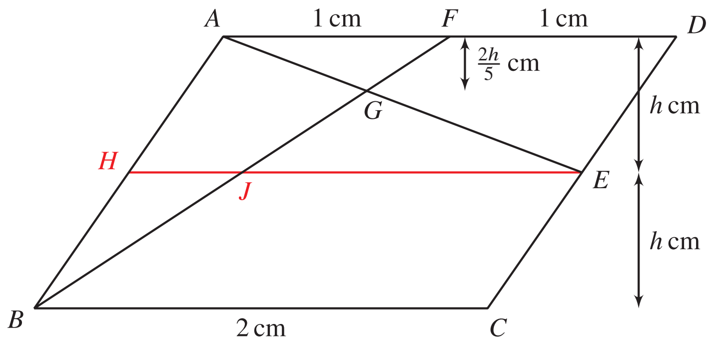
$$\begin{cases} 3h - 21H = 6 \\ 2h + 3h = 29H + 21H \end{cases}$$

Solving, we have $h = \frac{20}{3}$ and $H = \frac{2}{3}$.

$$\text{Required area} = \frac{29(29H)}{2} - \frac{14(2h)}{2}$$
$$= 187 \text{ cm}^2$$

20. $\boxed{\text{D}}$

Let $AF = 1$ cm and height of parallelogram $ABCD$ be $2h$ cm. Let H be mid-point of AB such that $HE // AD$. BF and EH intersect at J.



$FD = 1$ cm and $BC = 2$ cm

E is mid-point of CD $\Rightarrow$ height of $\triangle ADE = h$ cm

$\triangle ABF \sim \triangle HBJ$ $\Rightarrow$ $HJ = 0.5$ cm and $EJ = 1.5$ cm

$\triangle AFG \sim \triangle EJG$ $\Rightarrow$ height of $\triangle AFG = h \times \dfrac{1}{1+1.5} = \dfrac{2h}{5}$ cm

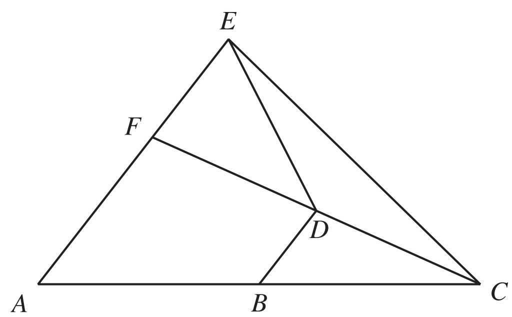
$$32 = \frac{(1+1.5)h}{2} - \frac{(1.5)\left(h - \frac{2h}{5}\right)}{2}$$

$h = 40$

Required area $= \dfrac{(1+2)(2h)}{2} - 32 = 88\text{ cm}^2$

21. $\boxed{\text{C}}$

Let F be the intersection of AE and CD produced.



Let $AB : BC = 1 : r$.

Since $\triangle CBD \sim \triangle CAF$, $FD : DC = 1 : r$.

Area of $\triangle DEF = \dfrac{8}{r}\text{ cm}^2$

Area of $\triangle CAF = \dfrac{4(1+r)^2}{r^2}\text{ cm}^2$

$$8 + \frac{8}{r} + \frac{4(1+r)^2}{r^2} = 45$$

$$r = \frac{2}{3} \quad \text{or} \quad -\frac{2}{11} \text{ (rejected)}$$

So, $AB : BC = 3 : 2$.

22. $\boxed{\text{D}}$

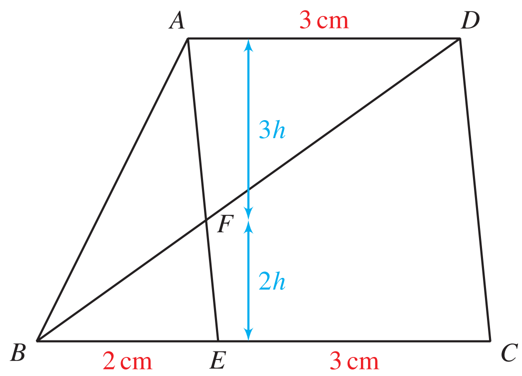
Let $BE = 2$ cm. Then $CE = AD = 3$ cm.

$\triangle ADF \sim \triangle EBF$ (ratio $= 3 : 2$)

Required ratio

$$= \frac{(3)(5h)}{2} : \left[(3)(5h) - \frac{(3)(3h)}{2}\right]$$

$$= 5 : 7$$



23. $\boxed{\text{C}}$

Let $PQ = 3$ cm. Then $SK = KR = 2$ cm.

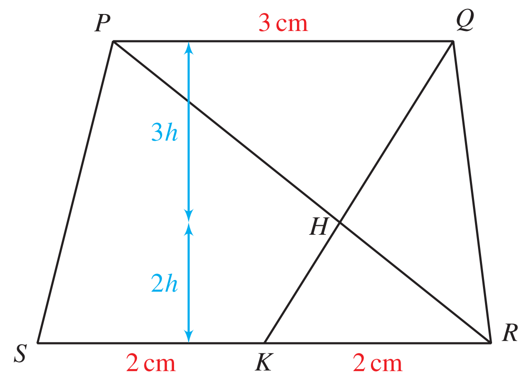
$\triangle PQH \sim \triangle RKH$ (ratio 3 : 2)

$$\frac{(3)(3h)}{2} = 90$$

$$h = 20$$

$$\text{Required area} = \frac{(3+4)(100)}{2}$$

$$= 350\text{ cm}^2$$



24. $\boxed{\text{B}}$

Let $BF = 1$ cm. Then $FC = 2$ cm and $AD = 3$ cm.

Consider the area of $\triangle BEF$,

$$\frac{(1)(BE)}{2} = 2$$

$$BE = 4\text{ cm}$$

$AE = BE = 4$ cm

$$\text{Required area} = (3)(8) - \frac{1}{2}(3)(4) - 2 - \frac{1}{2}(8)(2)$$

$$= 8\text{ cm}^2$$

25. $\boxed{\text{B}}$

Rotate the figure such that BC and AD are horizontal.

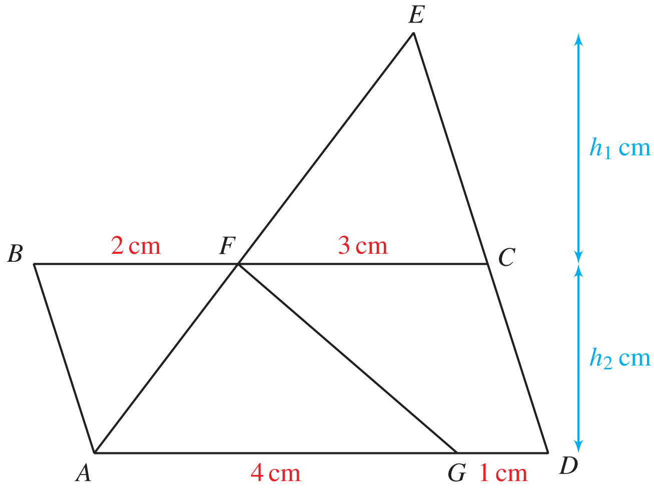
$\triangle ABF \sim \triangle ECF$ and $BF : FC = AB : CE = 2 : 3$.

Let $BF = 2$ cm and we have the lengths in the figure.

Consider the area of $\triangle ECF$,

$$\frac{(3)(h_1)}{2} = 18$$

$$h_1 = 12$$



$\triangle ECF \sim \triangle EDA$

$$\frac{h_1}{h_1 + h_2} = \frac{FC}{AD}$$

$$h_2 = 8$$

$$\text{Required area} = \frac{(3+1)(8)}{2}$$

$$= 16\,\text{cm}^2$$

26. $\boxed{\text{B}}$

$AE : EC = 24 : 36 = 2 : 3$

$\boxed{\text{Point } E}$

Note that $\triangle CDE \sim \triangle AFE$ (ratio $3 : 2$).

Let $CD = 3$ cm and the heights of two triangles be $3h$ cm and $2h$ cm respectively.

We have $AF = BF = 2$ cm.

Consider $\triangle CDE$.

$$\frac{3(3h)}{2} = 36$$

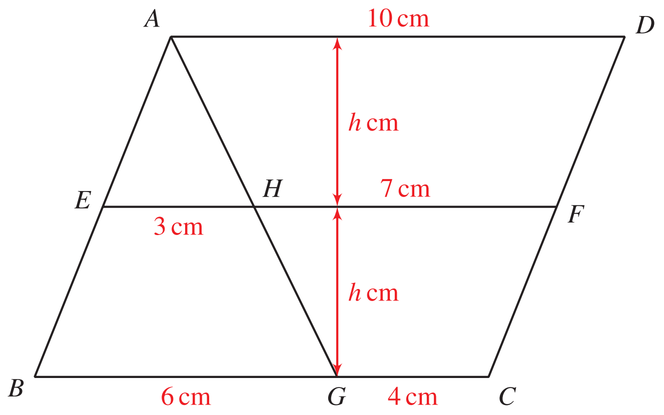
$$h = 8$$

$$\text{Required area} = \frac{(3+4)(5h)}{2}$$

$$= 140\,\text{cm}^2$$

27. $\boxed{\text{D}}$

Let $BG = 6$ cm. Then $GC = 4$ cm and $AD = 10$ cm.



Note that $\triangle AEH \sim \triangle ABG$ (ratio 1 : 2).

We have $EH = 3$ cm and $HF = 7$ cm.

Consider the area of $\triangle AEH$.

$$\frac{3(h)}{2} = 36$$

$$h = 24$$

$$\text{Required area} = \frac{(7+4)h}{2}$$

$$= 132\,\text{cm}^2$$

28. $\boxed{\text{A}}$

$\triangle ABC \sim \triangle BEC$ Area of $\triangle ABD = 18 \times \dfrac{1}{3} = 6\,\text{cm}^2$

Let the area of $\triangle BEC$ be x cm^2.

$$\frac{x}{x+18+6} = \left(\frac{BE}{AB}\right)^2$$

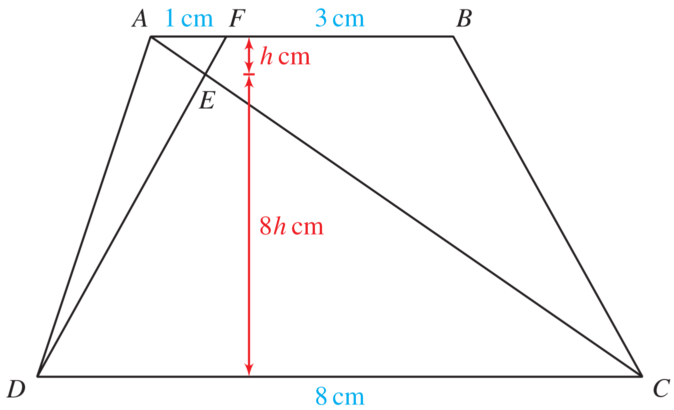
$$x = 8$$

Area of $\triangle ABC = 18 + 6 + 8 = 32\,\text{cm}^2$

29. $\boxed{\text{B}}$

Let $AF = 1$ cm.

Then $BF = 3$ cm and $CD = 8$ cm.



$\boxed{\text{Point } E}$ We have $\triangle AEF \sim \triangle CED$ (ratio 1 : 8).

Let the heights of $\triangle AEF$ and $\triangle AED$ be h cm and $8h$ cm respectively.

Consider the area of $CBFE$.

$$105 = \frac{(4)(9h)}{2} - \frac{(1)(h)}{2}$$

$$h = 6$$

$$\text{Required area} = \frac{(4+8)(9h)}{2}$$

$$= 324\text{ cm}^2$$

30. $\boxed{\text{D}}$

$$\frac{\text{area of } \triangle AQC}{\text{area of } \triangle ABC} = \frac{QC}{BC}$$

$$= \frac{4}{5}$$

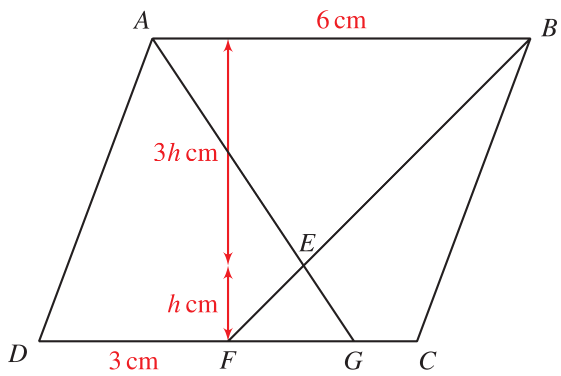
$$\frac{\text{area of } \triangle BPC}{\text{area of } \triangle ABC} = \frac{1}{4}$$

$$\frac{\text{area of } \triangle BPC}{\text{area of } \triangle AQC} = \frac{1}{4} \div \frac{4}{5}$$

$$= \frac{5}{16}$$

31. $\boxed{\text{B}}$

Let $CG = 1$ cm. Then $FG = 2$ cm and $DF = 3$ cm and $AB = 6$ cm.



$\triangle EFG \sim \triangle EBA$ (ratio 1 : 3)

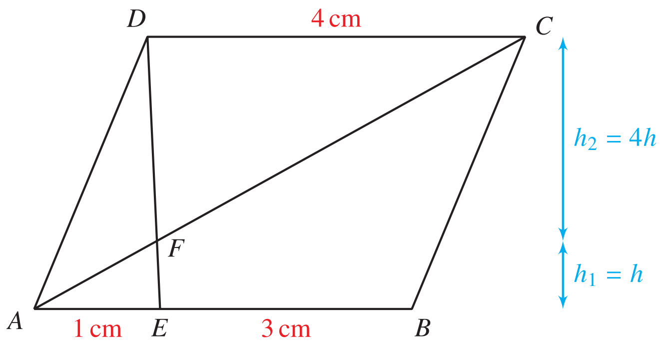
$$1265 = \frac{(3)(4h)}{2} - \frac{(2)(h)}{2}$$

$$h = 253$$

$$\text{Area of } \triangle EBA = \frac{(3 \times 253)(6)}{2} = 2277\text{ cm}^2$$

32. $\boxed{\text{D}}$

Let $AE = 1$ cm. Then we have the lengths as shown in the figure.



$\triangle CDF \sim \triangle AEF$

$$\frac{h_1}{h_2} = \frac{AE}{CD} = \frac{1}{4}$$

Consider the area of the parallelogram,

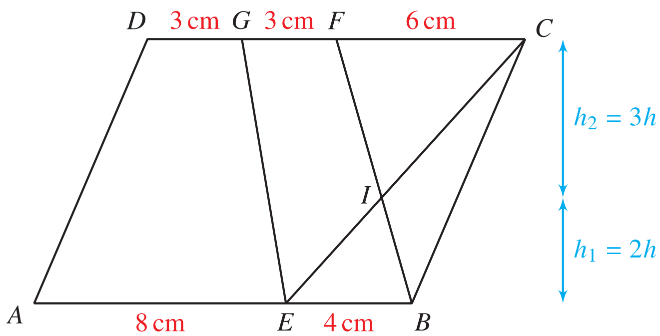
$$(4)(5h) = 40$$

$$h = 2$$

$$\text{Required area} = \frac{(4)(8)}{2} = 16\text{ cm}^2$$

33. $\boxed{\text{A}}$

Let $DG = 3$ cm. Then we have the lengths as shown in the figure.



$\triangle FIC \sim \triangle BIE$

$$\frac{h_2}{h_1} = \frac{FC}{EB} = \frac{3}{2}$$

Consider the area of $\triangle BIC$,

$$\frac{(4)(5h)}{2} - \frac{(4)(2h)}{2} = 6$$

$$h = 1$$

Required area

$$= \frac{(9)(5)}{2} - \frac{(6)(3)}{2}$$

$$= 13.5\text{ cm}^2$$

34. $\boxed{\text{B}}$

Let $BC = 3$ cm.

Consider the area of $\triangle DBC$,

$$\frac{(3)(h_1)}{2} = 12$$

$$h_1 = 8 \text{ cm}$$

Consider the area of $\triangle DEC$,

$$\frac{(DE)(8)}{2} = 8$$

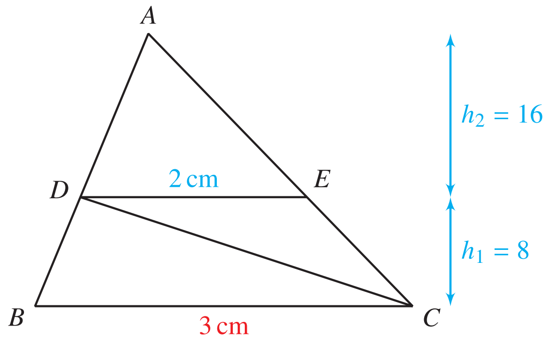
$$DE = 2 \text{ cm}$$

$\triangle ADE \sim \triangle ABC$

$$\frac{h_2}{8 + h_2} = \frac{DE}{BC}$$

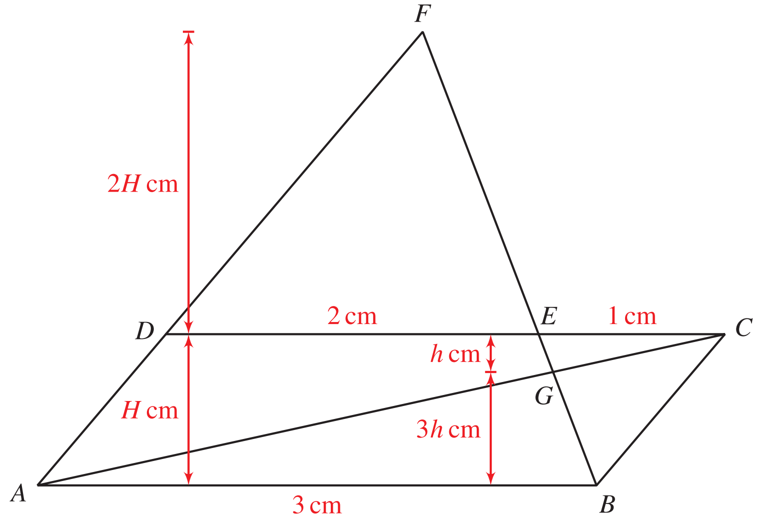
$$h_2 = 16 \text{ cm}$$

$$\text{Required area} = \frac{(2)(16)}{2} = 16 \text{ cm}^2$$



35. $\boxed{\text{D}}$

Let $DE = 2$ cm. Then $EC = 1$ cm and $AB = 3$ cm.



Note that $\triangle CEG \sim \triangle ABG$ (ratio 1 : 3).

The heights of two triangles are h cm and $3h$ cm respectively.

Consider the area of $\triangle ABF$.

$$\frac{(1)(4h)}{2} - \frac{(1)(h)}{2} = 6$$
$$h = 4$$

Note that $\triangle DEF \sim \triangle ABF$ (ratio 2 : 3).

The heights of two triangles are therefore 32 cm and 48 cm respectively $(H = 16)$.

$$\text{Required area} = \frac{(3)(48)}{2}$$
$$= 72\text{ cm}^2$$

36. $\boxed{\text{D}}$

Rotate the graph such that BC and AD are horizontal.

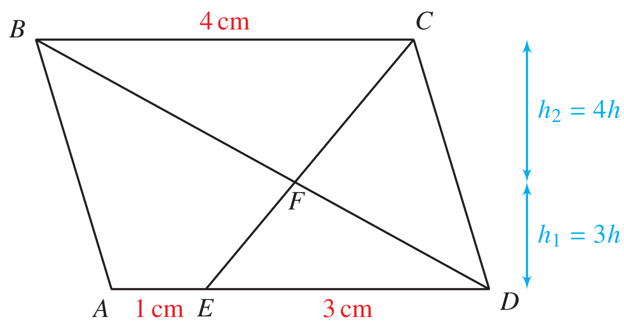
Let $AE = 1$ cm. Then we have the lengths as shown in the figure.

$\triangle BCF \sim \triangle DEF$

$$\frac{h_1}{h_2} = \frac{DE}{BC}$$
$$= \frac{3}{4}$$

Required ratio

$$= \left(\frac{(4)(7h)}{2} - \frac{(3)(3h)}{2}\right) : \frac{(4)(4h)}{2}$$
$$= 19 : 16$$



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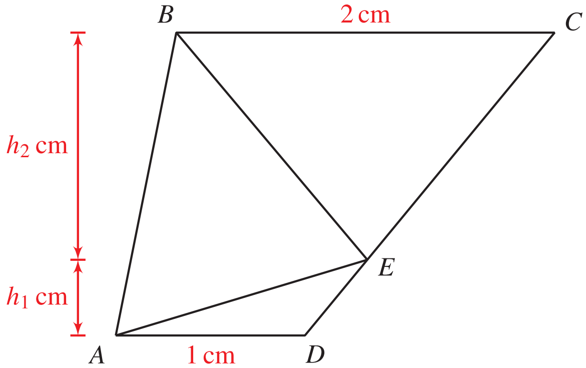
37. $\boxed{C}$

Assume $AD = 1$ cm. Then $BC = 2$ cm.



Note that $h_2 : h_1 = CE : DE = 3 : 1$.

Consider the area of $\triangle ABE$.

$$\frac{(1+2)(h_1+h_2)}{2} - \frac{2h_2}{2} - \frac{1h_1}{2} = 10$$
$$\frac{3}{2}(h_1+3h_1) - 3h_1 - \frac{h_1}{2} = 10$$
$$h_1 = 4$$

Required area $= \dfrac{(1+2)(4+12)}{2}$

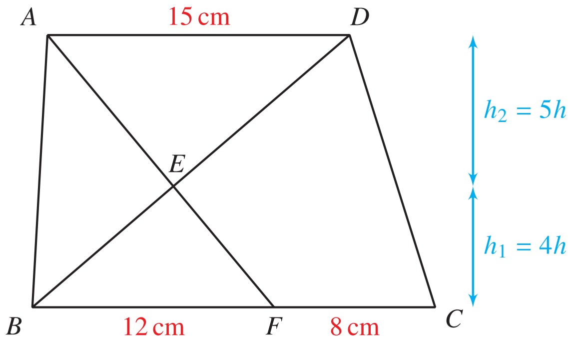
$= 24\text{ cm}^2$

38. $\boxed{C}$

Let $AD = 15$ cm. Then we have the lengths as shown in the figure.

$\triangle ADE \sim \triangle FBE$

$$\frac{h_1}{h_2} = \frac{BF}{AD}$$
$$= \frac{4}{5}$$



Consider the area of $ABCD$,

$$\frac{(15+20)(9h)}{2} = 210$$
$$h = \frac{4}{3}$$

Required area

$$= \frac{(20)(12)}{2} - \frac{(12)\left(\frac{16}{3}\right)}{2}$$
$$= 88\text{ cm}^2$$

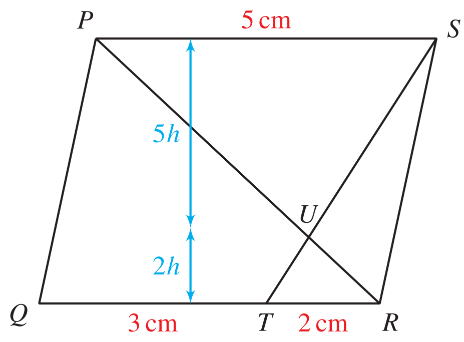
39. $\boxed{\text{B}}$

Let $QT = 3$ cm. Then $TR = 2$ cm and $PS = 5$ cm.

$\triangle PSU \sim \triangle RTU$ (ratio 5 : 2)

$$\frac{(2)(2h)}{2} = 12$$

$$h = 6$$

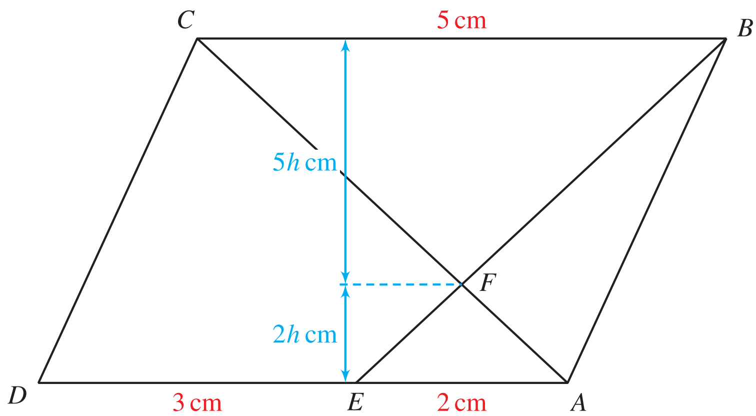


$$\text{Required area} = \frac{(5)(7h)}{2} - \frac{(2)(2h)}{2}$$

$$= 93\text{ cm}^2$$

40. $\boxed{\text{C}}$

Rotate the figure such that AD and BC appear horizontal as shown below.



Let $AE = 2$ cm. Then $DE = 3$ cm and $BC = 5$ cm.

$\boxed{\text{Point } F}$

We have $\triangle AEF \sim \triangle BCF$ (ratio 2 : 5).

Let the heights of the triangles be $2h$ cm and $5h$ cm respectively.

Consider the area of $\triangle DEF$.

$$\frac{(2)(2h)}{2} = 4$$

$$h = 2$$

$$\text{Required area} = 5(5h + 2h)$$

$$= 70\text{ cm}^2$$

41. $\boxed{\text{B}}$

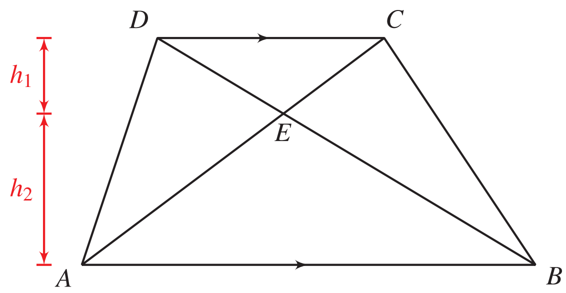
Construct the heights of the upper and lower triangles as follows:

$\dfrac{1}{2} = \dfrac{\text{Area of } \triangle CDE}{\text{Area of } \triangle BCE} = \dfrac{DE}{BE}$

Since $\triangle CDE \sim \triangle AEB$, $\dfrac{1}{2} = \dfrac{h_1}{h_2} = \dfrac{CD}{AB}$.

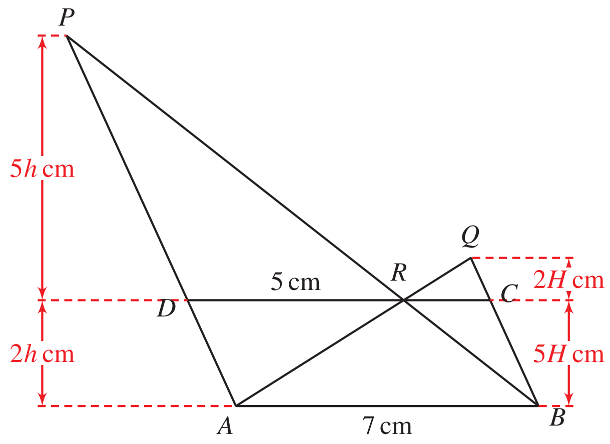
Let $CD = 1$.

$$\frac{\text{Area of } \triangle CDE}{\text{Area of trapezium } ABCD}$$
$$= \frac{\frac{1}{2}(1)(h_1)}{\frac{1}{2}(1+2)(h_1+h_2)}$$
$$= \frac{h_1}{3(h_1+2h_1)} = \frac{1}{9}$$



42. $\boxed{\text{D}}$

Let $AB = 7$ cm.



$\boxed{\text{Point } P}$

We have $\triangle PDR \sim \triangle PAB$ (ratio 5 : 7).

Let $5h$ cm and $7h$ cm be the heights of two triangles respectively.

We have $DR = AB \times \frac{5}{7} = 5$ cm and $RC = 7 - 5 = 2$ cm.

$\boxed{\text{Point } Q}$

We have $\triangle QRC \sim \triangle QAB$ (ratio 2 : 7).

Let $2H$ cm and $7H$ cm be the heights of two triangles respectively.

Consider the area of $\triangle DPR$.

$$\frac{5(5h)}{2} = 50$$
$$h = 4$$

Consider the height of $ABCD$.

$$2h = 5H$$
$$H = 1.6$$

$$\text{Required area} = \frac{7(7H)}{2}$$
$$= 39.2 \text{ cm}^2$$