REG-EOSL-2324-ASM-SET 3-MATH
Suggested solutions

Multiple Choice Questions

1. A	2. B	3. C	4. A	5. B
6. A	7. D	8. C	9. C	10. A
11. C	12. B	13. C	14. A	15. A
16. A	17. D	18. A	19. A	20. A
21. B	22. D	23. D	24. A	25. A
26. B	27. B	28. D	29. D	30. C

1. A

Let the x-coordinates of A and B be $-3p$ and $5q$ respectively.
Then $A\,(-3p,\ 4p)$ and $B\,(5q,\ 12q)$.

$$OA = \sqrt{(3p)^2 + (4p)^2} \quad \text{and} \quad OB = \sqrt{(5q)^2 + (12q)^2}$$
$$5 = 5p \qquad\qquad 13 = 13q$$
$$p = 1 \qquad\qquad q = 1$$

Slope of $AB = 1$. Equation of AB is

$$y - 4 = 1(x + 3)$$
$$x - y + 7 = 0$$

2. B

Slope of $L_2 = -2$ and so $m = \frac{1}{2}$

I. ✓. y-intercept $= b > 0$

II. ✓. $m = \frac{1}{2} > 0$

III. ✗. It is possible that y-intercept = 10, then $b = \frac{1}{10} < \frac{1}{2} = m$.

3. C

Let the x-intercept be a. Then the y-intercept is also a.
Slope of the line $= \frac{a - 0}{0 - a} = -1$
Required equation is

$$y - 5 = -1(x - 3)$$
$$x + y - 8 = 0$$

4. A

Let the coordinates of P be $(p,\ 0)$.

Consider the slopes.

$$\frac{-3-3}{1-5} = \frac{0-3}{p-5}$$
$$p = 3$$

5. B

Coordinates of A and B are $(3,\ 0)$ and $(0,\ 6)$. mid-point of AB is at $\left(\frac{3}{2},\ 3\right)$.

Slope of $L_2 = \frac{3}{\left(\frac{3}{2}\right)} = 2$

Equation of L_2 is $y = 2x$, i.e., $2x - y = 0$.

6. A

y-intercept of $L_2 = 2 \times \tan(180° - 90° - 30°) = 2\sqrt{3}$

7. D

$$\frac{-k}{4} \times \frac{6}{9} = -1$$
$$k = 6$$

L: $6x + 4y - 12 = 0$. The y-intercept is 3.

8. C

$$\frac{x}{3} - \frac{y}{4} = 1$$
$$y = \frac{4x}{3} - 4$$

Slope $= \frac{4}{3}$

Only option C is a straight line with slope $\frac{4}{3}$.

9. C

$$5h - 2(4) - 2 = 0$$
$$h = 2$$

Equation of L is in the form $2x + 5y + k = 0$, where k is a constant.

$$2(2) + 5(4) + k = 0$$
$$k = -24$$

Equation of L is $2x + 5y - 24 = 0$.

10. A

$$\begin{cases} 2x + y - 13 = 0 \\ 3x + 2y - 3 = 0 \end{cases}$$

We have $x = 23$ and $y = -33$.

The coordinates of S are $(23,\ -33)$.

11. C

Let the coordinates of P be $(p,\ 0)$.

$$\frac{-3-0}{3-p} = \frac{1+3}{7-3}$$

$$p = 6$$

Slope of required line $= \frac{3-0}{0-6} = -\frac{1}{2}$

Required equation is

$$y - 3 = -\frac{1}{2}(x - 0)$$

$$x + 2y - 6 = 0$$

12. B

The coordinates of B' are $(3,\ 4)$.

Let the coordinates of D be $(0,\ d)$.

A, D and B are collinear.

$$\frac{4-0}{-3-1} = \frac{d-0}{0-1}$$

$$d = 1$$

Slope of $B'D = \frac{4-1}{3-0} = 1$

Equation of $B'D$ is $y = x + 1$.

Slope of $BC = \frac{4-2}{-3-3} = -\frac{1}{3}$

Equation of BC is

$$y - 2 = -\frac{1}{3}(x - 3)$$

$$x + 3y - 9 = 0$$

Solve $\begin{cases} x + 3y - 9 = 0 \\ y = x + 1 \end{cases}$, we have $x = \frac{3}{2}$ and $y = \frac{5}{2}$.

Required coordinates are $\left(\frac{3}{2},\ \frac{5}{2}\right)$.

13. C

Solve $\begin{cases} -2x + 3y - 10 = 0 \\ 3x - 2y - 5 = 0 \end{cases}$, we have $x = 7$ and $y = 8$.

Slope of $AB = \dfrac{8 - 1}{7 + 1} = \dfrac{7}{8}$

Required equation is

$$y - 8 = \frac{7}{8}(x - 7)$$

$$7x - 8y + 15 = 0$$

14. A

Let the coordinates of P be $(a,\ b)$.

Since P lies on L, $3a + 4b + 30 = 0$.

When P is nearest to R, $PR \perp L$.

$$\frac{b - 3}{a - 1} \times \frac{-3}{4} = -1$$

$$4a - 3b + 5 = 0$$

Solving, we have $(a,\ b) = \left(-\dfrac{22}{5},\ -\dfrac{21}{5}\right)$.

Required distance $= \sqrt{\left(1 + \dfrac{22}{5}\right)^2 + \left(3 + \dfrac{21}{5}\right)^2} = 9$

15. A

Solve $\begin{cases} y = 2x + 5 \\ x - 2y = 5 \end{cases}$, we have $x = -5$ and $y = -5$.

Required coordinates are $(-5,\ -5)$.

16. A

Solve $\begin{cases} 4x + y + 16 = 0 \\ 3x + 2y + 17 = 0 \end{cases}$, we have $x = -3$ and $y = -4$.

The coordinates of the intersection are $(-3,\ -4)$.

Since $(-3) + 2(-4) + 11 = 0$, $x + 2y + 11 = 0$ passes through the point of intersection.

17. D

Solve $\begin{cases} 2x - 3y + 4 = 0 \\ x + 6y + 5 = 0 \end{cases}$, we have $x = -\dfrac{13}{5}$ and $y = -\dfrac{2}{5}$.

Required equation is

$$y + \frac{2}{5} = -\frac{1}{2}\left(x + \frac{13}{5}\right)$$

$$5x + 10y + 17 = 0$$

18. A

$8k - 5(-5) + 7 = 0$

$k = -4$

$ak - (-5) + 3 = 0$

$-4a + 8 = 0$

$a = 2$

19. A

$2(k) + (-1) + 3 = 0$

$k = -1$

$3(-1) + h(-1) + 1 = 0$

$h = -2$

20. A

Slope of $AC = \dfrac{-6+2}{8+8} = -\dfrac{1}{4}$

Equation of AC is

$$y + 6 = -\frac{1}{4}(x - 8)$$

$$x + 4y + 16 = 0$$

Solve $\begin{cases} x + 4y + 16 = 0 \\ 3x - 4y - 12 = 0 \end{cases}$, the coordinates of C are $\left(-1,\ -\dfrac{15}{4}\right)$.

Consider the x-intercepts of two lines.

The coordinates of A and B are $(-16,\ 0)$ and $(4,\ 0)$ respectively.

Required area $= \dfrac{(4+16)\left(\frac{15}{4}\right)}{2}$

$= 37.5$

21. B

$2x + 3(2) + 6 = 0$ and $2(0) + 3y + 6 = 0$

$x = -6$ $y = -2$

The coordinates of A and C are $(-6,\ 2)$ and $(0,\ -2)$ respectively.

Required area $= \dfrac{(0+6)(2+2)}{2}$

$= 12$

22. D

x-intercept = 18 and y-intercept = 6.

Required area $= \dfrac{(18)(6)}{2}$

$= 54$

23. D

Solve $\begin{cases} 2x + y = 0 \\ x - y + 3 = 0 \end{cases}$, we have $x = -1$ and $y = 2$.

The coordinates of the intersection are $(-1,\ 2)$.

$$3(-1) - 2 + k = 0$$
$$k = 5$$

24. A

Solve $\begin{cases} 2x + 3y - 7 = 0 \\ x - 3y - 5 = 0 \end{cases}$, we have $x = 4$ and $y = -\dfrac{1}{3}$.

L_2 and L_3 intersect at $\left(4,\ -\dfrac{1}{3}\right)$.

If three lines intersect at a point, then L_1 passes through $\left(4,\ -\dfrac{1}{3}\right)$.

$$7(4) + k\left(-\frac{1}{3}\right) - 31 = 0$$
$$k = -9$$

25. A

Solve $\begin{cases} \dfrac{3x}{2} + y = 2 \\ 2x - 3y = 20 \end{cases}$, we have $(x,\ y) = (4,\ -4)$.

$$4(4) - 3(-4) + k = 0$$
$$k = -28$$

26. B

Solve $\begin{cases} 2x + 5y + 10 = 0 \\ 3x + 2y - 7 = 0 \end{cases}$, we have $x = 5$ and $y = -4$.

The coordinates of the intersection are $(5,\ -4)$.

$$5 + k(-4) - 8 = 0$$
$$k = -\frac{3}{4}$$

27. B

Slope of $L = \dfrac{2}{5}$

Slope of $L_1 = \dfrac{4}{10} = \dfrac{2}{5}$ and slope of $L_2 = -\dfrac{1}{1} = -1$

Thus, only L_2 has exactly one point of intersection with L.

28. $\boxed{\text{D}}$

Two straight lines have equal slopes.

$$-\frac{6}{b} = -\frac{1}{2}$$
$$b = 12$$

Two straight lines have equal x-intercepts.

$$\frac{3}{6} = \frac{-c}{1}$$
$$c = -\frac{1}{2}$$

29. $\boxed{\text{D}}$

Two straight lines are parallel. They have equal slopes.

$$\frac{4}{a} = \frac{a}{1}$$
$$a^2 = 4$$
$$a = \pm 2$$

30. $\boxed{\text{C}}$

I. ✓. Note that $L // L_1$ and they are not coincident.

II. ✓. L and L_2 are not parallel. They intersect at one point only.

III. ✗. L_3: $x + 2y - 2 = 0$ and L coincident.

Conventional Questions

31. (a) $2(a) - 3(0) + 12 = 0$ 1M

$$a = -6$$ 1A

(b) Slope of $L_1 = \frac{2}{3}$

Slope of $L_2 = -\frac{3}{2}$ 1A

The equation of L_2 is

$$y - 0 = -\frac{3}{2}(x + 6)$$ 1M

$$3x + 2y + 18 = 0$$ 1A

(c) Slope of $L_3 = -\frac{3}{2}$

$$\frac{k}{3} = -\frac{3}{2}$$ 1M

$$k = -\frac{9}{2}$$ 1A

32. (a) $\frac{-2}{-1} = \frac{-4}{k}$

$$k = -2$$ 1A

(b) The equation of L_2 is

$$4x - 2y + 6 = 0$$

$$2x - y + 3 = 0,$$ 1M

which is identical to L_1.

Thus, there are infinitely many points of intersection between L_1 and L_2. 1A

33. (a) (i) Coordinates of $B = \left(\frac{0-8}{2}, \frac{-10+14}{2}\right) = (-4, 2)$. 1A

Slope of $AC = \frac{14+10}{-8-0} = -3$.

Slope of $BE = \frac{1}{3}$ 1A

Equations of BE is

$$\frac{y-2}{x+4} = \frac{1}{3}$$ 1M

$$3y - 6 = x + 4$$

$$0 = x - 3y + 10$$ 1A

(ii) When $x = 0$, $y = \frac{10}{3}$. 1M

Thus, the coordinates of E are $\left(0, \frac{10}{3}\right)$. 1A

(b) (i) Slope of $AD = \frac{14+2}{-8-0} = -2$.

So, equation of AD is $y = -2x - 2$. 1A

Substitute $y = -2x - 2$ into $0 = x - 3y + 10$,

$$0 = x - 3(-2x - 2) + 10$$ 1M

$$x = -\frac{16}{7}$$

When $x = \frac{-16}{7}$, $y = -2\left(\frac{-16}{7}\right) - 2 = \frac{18}{7}$.

The coordinates of F are $\left(-\frac{16}{7}, \frac{18}{7}\right)$. 1A

(ii) Note that area of $\triangle ABE$ = area of $\triangle CBE$ 1M

Area of $\triangle EFC$: area of $\triangle CBE = EF : BE$

$$= \left(0 + \frac{16}{7}\right) : (0 + 4)$$ 1M

$$= 4 : 7 \neq 1 : 2$$ 1A

So, area of $\triangle CBE$ is not twice the area of $\triangle EFC$.

Thus, area of $\triangle EBA$ is not twice the area of $\triangle EFC$.

The claim is disagreed. 1A

 END OF PAPER