REG-CP1A-2324-ASM-SET 2-MATH
Suggested solutions

Conventional Questions

1. (a) Let $f(x) = a + b(x-3)^2$, where a and b are non-zero constants. 1A

$$\begin{cases} -20 = a + b(-1-3)^2 \\ 10 = a + b(2-3)^2 \end{cases}$$

Solving, we have $a = 12$ and $b = -2$. 1A
Thus, $f(x) = 12 - 2(x-3)^2$.

(b) -11 1A

(c) $f(x) - 5 = -2(x-3)^2 + 7$
The maximum value of $f(x) - 5$ is 7. 1M
Thus, $m \le 7$. 1A

2. (a) Let $p(x) = a + b(4-x)^2$, where a and b are non-zero constants. 1A

$$\begin{cases} 15 = a + b(4-1)^2 \\ 96 = a + b(4+2)^2 \end{cases}$$ 1M

Solving, we have $a = -12$ and $b = 3$. 1A
Thus, $p(x) = -12 + 3(4-x)^2$.

(b) $(4,\ -12)$ 1A

(c) Note that the minimum value of $p(kx)$ is -12. 1M
The equation $p(kx) + k = 0$ has no real roots.

$$\begin{aligned} -12 + k &> 0 \\ k &> 12 \end{aligned}$$ 1A

$0 = p(kx) + k$ $0 = 3k^2x^2 - 24kx + 36 + k$ The equation has no real roots. $\Delta = (-24k)^2 - 4(3k^2)(36+k) < 0$ 1M $k^2(144 - 12k) < 0$ $144 - 12k < 0$ $k > 12$ 1A

3. (a) Let $S = an + bn^2$, where a and b are non-zero constants. 1A

$$\begin{cases} 7920 = 12a + 144b \\ 12\,160 = 16a + 256b \end{cases}$$ 1M

Solving, we have $a = 360$ and $b = 25$. 1A

Required income $= 360(24) + 25(24)^2$

$= \$23\,040$ 1A

(b) $25n^2 + 360n = 17\,200$ 1M

$25n^2 + 360n - 17\,200 = 0$

$n = 20 \quad \text{or} \quad -34.4$ (rejected) 1A

The number of handbags she sells in that month is 20.

4. (a) Let $P = ax + bx^2$, where a and b are non-zero constants. 1A

$$\begin{cases} 30\,000 = 100a + 100^2 b \\ 37\,500 = 150a + 150^2 b \end{cases}$$ 1M

Solving, we have $a = 400$ and $b = -1$. 1A

Required profit $= 400(300) - (300)^2$

$= \$30\,000$ 1A

(b) Let $\$x$ be the selling price of a check shirt.

$$45\,000 = 400x - x^2$$

$$x^2 - 400x + 45\,000 = 0$$

$\Delta = 400^2 - 4(1)(45\,000) = -20\,000 < 0$ 1M

The equation has no real roots.

The shop owner cannot obtain a profit of \$45 000 by selling all these check shirts. 1A

5. (a) Let the radius of the circular cone be r cm.

$$2\pi r = 2\pi(60) \times \frac{216^\circ}{360^\circ} \quad \text{1M}$$

$$r = 36 \quad \text{1A}$$

Required radius is 36 cm.

(b) Height of circular cone $= \sqrt{60^2 - 36^2} = 48$ cm. 1A

Let the radius of the hemisphere be R cm.

$$\frac{48 - (R+6)}{48} = \frac{R}{36} \quad \text{1M}$$

$$6(42 - R) = 8R$$

$$R = 18$$

Volume of the decoration

$$= \frac{\pi}{3}(36)^2(48)\left[1 - \left(\frac{48-24}{48}\right)^3\right] - \frac{2\pi}{3}(18)^3 \quad \text{1M+1A}$$

$$= 14256\pi \text{ cm}^3$$

$$\approx 0.0448 \text{ m}^3 > 0.04 \text{ m}^3$$

The claim is agreed. 1A

6. (a) Suppose the frustum is formed by cutting off a small cone of height h cm from a larger cone of height $(h+3)$ cm.

$$\frac{h+3}{h} = \frac{4}{2} \quad \text{1M}$$

$$h = 3$$

Volume of the cream $= 4^2\pi(7-3) + \frac{1}{3}\pi(4)^2(3+3) - \frac{1}{3}\pi(2)^2(3)$ 1M+1A

$= 92\pi \text{ cm}^3$ 1A

(b) New volume of cream $= 4^2\pi(7-4) + \frac{1}{2} \times \frac{4}{3}\pi(4)^3$ 1M+1A

$= \frac{272\pi}{3} \text{ cm}^3$

Percentage change

$$= \frac{\frac{272\pi}{3} - 92\pi}{92\pi} \times 100\% \quad \text{1M}$$

$$\approx -1.45\%$$

$$> -5\%$$

The claim is incorrect. 1A

7. (a) Required ratio $= 8^3 : 10^3$ 1M

$= 64 : 125$ 1A

(b) Volume of the metallic sphere $= \frac{4}{3}\pi(10)^3 = \frac{4000\pi}{3}$ cm^3

Volume of the smaller cone $= \frac{4000\pi}{3} \times \frac{64}{64+125}$ 1M

$= \frac{256000\pi}{567}$ cm^3

Let the height of the smaller cone be h cm.

$\frac{1}{3}\pi(8)^2 h = \frac{256\,000\pi}{567}$ 1M

$h = \frac{4000}{189}$

Curved surface area of the smaller cone $= \pi(8)\sqrt{8^2 + \left(\frac{4000}{189}\right)^2}$ 1M

≈ 569 cm^2 1A

8. (a) Volume of cylinder $= \pi(14)^2(182)$

$= 35672\pi$ cm^3 1A

Ratio of volume of two cones $= 9^{\frac{3}{2}} : 16^{\frac{3}{2}} = 27 : 64$. 1M

Volume of the larger cone $= 35\,672\pi \times \frac{64}{27+64}$

$= 25088\pi$ cm^3 1A

(b) Let the base radius of the larger cone be r cm.

$\frac{1}{3}\pi r^2(96) = 25\,088\pi$ 1M

$r = 28$ or -28 (rejected)

Curved surface area of the larger cone

$= \pi(28)\sqrt{28^2 + 96^2}$ 1M

$= 2800\pi$ cm^2

Required area $= 2800\pi \times \frac{9}{16}$

$= 1575\pi$ cm^2 1A

9. (a) Required radius $= \sqrt{8^2 - 4^2} = 4\sqrt{3}$ cm 1A

(b) Volume of the cone $= \frac{1}{3}\pi(4\sqrt{3})^2 h = 16\pi h$ cm^3. 1A

$$V = 16\pi h - 16\pi h \times \left(\frac{h-4}{h}\right)^3 \quad \text{1M}$$
$$= 16\pi h \times \frac{h^3 - (h-4)^3}{h^3}$$
$$= 16\pi h \times \frac{4[h^2 + h(h-4) + (h-4)^2]}{h^3}$$
$$= \frac{64\pi(3h^2 - 12h + 16)}{h^2} \quad \text{1}$$

(c) Volume of the hemisphere $= \frac{2}{3}\pi(8)^3 = \frac{1024\pi}{3}$ cm^3 1A

$$\frac{19}{48} \times \frac{1024\pi}{3} = \frac{64\pi(3h^2 - 12h + 16)}{h^2} \quad \text{1M+1A}$$
$$\frac{19h^2}{9} = 3h^2 - 12h + 16$$
$$0 = \frac{8h^2}{9} - 12h + 16$$
$$h = 12 \quad \text{or} \quad \frac{3}{2} \text{ (rejected)} \quad \text{1A}$$

Height of the cone is 12 cm.

10. (a) Median = 4 1A

Inter-quartile range = 6 − 2

= 4 1A

Standard deviation ≈ 1.99 1A

(b) (i) 8 1A

(ii) 31 1A

11. (a) $47 = (50 + c) - (10 + a)$ 1M

$c - a = 7$

$$33 = \frac{(10 + a) + 14 + 18 + \ldots + (50 + c)}{18} \quad \text{1M}$$

$a + b + c = 10$

Since $0 \le a \le 4$, $0 \le b \le 3$ and $7 \le c \le 9$, we have

$(a,\ b,\ c) = (0,\ 3,\ 7)$ or $(1,\ 1,\ 8)$. 1A

(b) Required probability $= \frac{10}{18}$ 1M

$= \frac{5}{9}$ 1A

12. (a) Median = 54 1A

Range = $74 - 30 = 44$ 1A

Inter-quartile range = $65 - 44 = 21$ 1A

(b) (i) New inter-quartile range = $70 - 45 = 25 > 21$ 1A

The distribution of the scores in second term is not less dispersed than that in the first term. 1A

(ii) There are 2 students getting Grade A in the first term.

It is possible that the scores of the top 6 students in the second term are

70 70 70 70 70 78 1M

such that the upper quartile is 70 and the maximum is 78.

Number of students who get Grade A = $6 \neq 2 + 3$.

The claim is not correct. 1A

13. (a) $96 - (60 + a) = 36$ 1M

$a = 0$ 1A

$\frac{(80 + b) + 89}{2} - 71 = 17$

$b = 7$ 1A

(b) (i) The original median of the distribution is 81.5.

The scores of the two students are lower than 81.5. 1M

The mode of the distribution will be 84, which remains unchanged.

The claim is agreed. 1A

(ii) Since the range is reduced, one of the removed datum is 60.

The mean is the least when the score of the other student is 80. 1M

Least possible mean

$= \frac{62 + 64 + \ldots + 96}{18}$

$= 80$ 1A

14. (a) Median = 70 kg 1A

$69 = \frac{(50 + a) + 57 + 58 + \ldots + (80 + b)}{15}$ 1M

$3a + b = 9$

$(80 + b) - (50 + a) = 35$ 1M

$b - a = 5$

Solving, we have $a = 1$ and $b = 6$. 1A

Inter-quartile range = $75 - 61$

$= 14$ kg 1A

(b) Weight of the new student is 75 kg. 1M

New standard deviation is 9.33 kg. 1A

15. (a) Mode of the distribution is 38.

$$38 = \frac{21 + 22 + \ldots + 69}{18} \qquad \text{1M}$$

$$2h + 2k = 20$$

The inter-quartile range is one third of the range.

$$(40 + k) - (20 + h) = \frac{69 - 21}{3} \qquad \text{1M}$$

$$k - h = -4$$

Solving, we have $h = 7$ and $k = 3$. 1A+1A

(b) (i) Original median is 38.

New median is the 10th datum, which is also 38. 1M

There is no change in the median of the distribution. 1A

(ii) Let x be the numbers of Chinese characters typed by the newly added student in one minute.

<u>Case 1: $x = 20$</u>

Standard deviation ≈ 12.9

<u>Case 2: $x = 70$</u>

Standard deviation ≈ 14.1

The least possible standard deviation is 12.9. 1M

Thus, it is impossible that the standard deviation of the distribution is less than 12.7. 1A

16. (a) $24 - (10 + a) = 9$ 1M

$$a = 5 \qquad \text{1A}$$

(b) (i) $(49 - 8) - \left[\frac{32 + (30 + b)}{2} - 15\right] < 22$ 1M

$$b > 6$$

The possible values of b are 7 and 8. 1A+1A

(ii) When $b = 7$, standard deviation ≈ 11.85. 1M

When $b = 8$, standard deviation ≈ 11.86.

The greatest possible standard deviation is 11.86. 1A

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