M2REG-LIM-2324-ASM-SET 3-MATH
Suggested solutions

1. $\lim\limits_{x\to 0}\dfrac{\sin x+\sin 3x}{x}=\lim\limits_{x\to 0}\dfrac{2\sin 2x\cos x}{x}$ 1M

$=\lim\limits_{x\to 0}\left(\dfrac{\sin 2x}{2x}\cdot 4\cos x\right)$ 1M

$=1\cdot 4$

$=4$ 1A

2. $\lim\limits_{x\to 0}\dfrac{\sin 3x+\sin 2x}{x}=\lim\limits_{x\to 0}\left(\dfrac{\sin 3x}{3x}\cdot 3+\dfrac{\sin 2x}{2x}\cdot 2\right)$ 1M

$=1(3)+1(2)$

$=5$ 1A

3. $\lim\limits_{x\to 0}\dfrac{4x\cot 2x}{\cos x}=\lim\limits_{x\to 0}\dfrac{4x\cos 2x}{\cos x\sin 2x}$

$=\lim\limits_{x\to 0}\left(\dfrac{2x}{\sin 2x}\cdot\dfrac{2\cos 2x}{\cos x}\right)$ 1M

$=1\cdot 2$

$=2$ 1A

4. $\lim\limits_{x\to 0}\dfrac{7x^2}{1-\cos 7x}=\lim\limits_{x\to 0}\left(\dfrac{7x^2}{1-\cos 7x}\cdot\dfrac{1+\cos 7x}{1+\cos 7x}\right)$ 1M

$=\lim\limits_{x\to 0}\dfrac{7x^2(1+\cos 7x)}{1-\cos^2 7x}$

$=\lim\limits_{x\to 0}\dfrac{7x^2(1+\cos 7x)}{\sin^2 7x}$

$=\lim\limits_{x\to 0}\left[\left(\dfrac{7x}{\sin 7x}\right)^2\cdot\dfrac{1+\cos 7x}{7}\right]$ 1M

$=1^2\cdot\dfrac{1+1}{7}$

$=\dfrac{2}{7}$ 1A

5. $\lim\limits_{x\to 0}\dfrac{\cos 5x-\cos 3x}{x^2}=\lim\limits_{x\to 0}\dfrac{-2\sin 4x\sin x}{x^2}$ 1M

$=\lim\limits_{x\to 0}\left(\dfrac{\sin 4x}{4x}\cdot\dfrac{\sin x}{x}\cdot(-2)(4)\right)$ 1M

$=(1)(1)(-8)$

$=-8$ 1A

6. $\lim\limits_{x\to 0}\dfrac{1-\cos x}{1-\cos 5x} = \lim\limits_{x\to 0}\dfrac{1-\left(1-2\sin^2\frac{x}{2}\right)}{1-\left(1-2\sin^2\frac{5x}{2}\right)}$ 1M

$= \lim\limits_{x\to 0}\dfrac{\sin^2\frac{x}{2}}{\sin^2\frac{5x}{2}}$

$= \lim\limits_{x\to 0}\left[\dfrac{\sin\frac{x}{2}}{\left(\frac{x}{2}\right)}\right]^2\left[\dfrac{\left(\frac{5x}{2}\right)}{\sin\frac{5x}{2}}\right]^2\cdot\dfrac{1}{25}$ 1M

$= 1^2\cdot 1^2\cdot\dfrac{1}{25}$

$= \dfrac{1}{25}$ 1A

7. $\lim\limits_{x\to 0}\dfrac{1-\cos 3x}{x\sin 3x} = \lim\limits_{x\to 0}\dfrac{1-\left(1-2\sin^2\frac{3x}{2}\right)}{x\sin 3x}$ 1M

$= \lim\limits_{x\to 0}\dfrac{2\sin^2\frac{3x}{2}}{x\sin 3x}$

$= \lim\limits_{x\to 0}\left\{\left[\dfrac{\sin\frac{3x}{2}}{\left(\frac{3x}{2}\right)}\right]^2\cdot\dfrac{3x}{\sin 3x}\cdot\dfrac{3}{2}\right\}$ 1M

$= 1^2\cdot 1\cdot\dfrac{3}{2}$

$= \dfrac{3}{2}$ 1A

8. (a) $\lim\limits_{u\to 0}\dfrac{u\sin u}{1-\cos 3u} = \lim\limits_{u\to 0}\dfrac{u\sin u}{1-\left(1-2\sin^2\frac{3u}{2}\right)}$ 1M

$= \lim\limits_{u\to 0}\dfrac{u\sin u}{2\sin^2\frac{3u}{2}}$

$= \lim\limits_{u\to 0}\dfrac{\left(\frac{3u}{2}\right)^2}{\sin^2\frac{3u}{2}}\cdot\dfrac{\sin u}{u}\cdot\dfrac{2}{9}$ 1M

$= \dfrac{2}{9}$ 1A

(b) Let $u = \dfrac{\pi}{2} - x$.

$\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\left(x-\frac{\pi}{2}\right)\cos x}{1+\sin 3x} = \lim\limits_{u\to 0}\dfrac{-u\cos\left(\frac{\pi}{2}-u\right)}{1+\sin\left(\frac{3\pi}{2}-3u\right)}$

$= \lim\limits_{u\to 0}\dfrac{-u\sin u}{1-\cos 3u}$ 1M

$= -\dfrac{2}{9}$ 1A

9. Let $y = \dfrac{1}{x}$, then $x = \dfrac{1}{y}$.

$$\lim_{x\to\infty}(2x^2+x-5)\sin^2\frac{3}{x} = \lim_{y\to 0}\left(\frac{2}{y^2}+\frac{1}{y}-5\right)\sin^2 3y \qquad \text{1M}$$

$$= \lim_{y\to 0}\left(\frac{2\sin^2 3y}{y^2}+\frac{\sin^2 3y}{y}-5\sin^2 3y\right)$$

$$= \lim_{y\to 0}\left[\left(\frac{\sin 3y}{3y}\right)^2\cdot 18+\frac{\sin 3y}{3y}\cdot 3\sin 3y-5\sin^2 3y\right] \qquad \text{1M}$$

$$= 1(18)+1(0)-5(0)$$

$$= 18 \qquad \text{1A}$$

10. $\displaystyle\lim_{x\to 0}\frac{x\left[1+\tan\left(\frac{\pi}{3}+x\right)\right]}{\sqrt{3}-\tan\left(\frac{\pi}{3}+x\right)}$

$$= \lim_{x\to 0}\frac{x\left(1+\frac{\sqrt{3}+\tan x}{1-\sqrt{3}\tan x}\right)}{\sqrt{3}-\frac{\sqrt{3}+\tan x}{1-\sqrt{3}\tan x}} \qquad \text{1M}$$

$$= \lim_{x\to 0}\left[x\left(\frac{(1-\sqrt{3})\tan x+(1+\sqrt{3})}{1-\sqrt{3}\tan x}\right)\right]\div\left(\frac{(-3-1)\tan x+(\sqrt{3}-\sqrt{3})}{1-\sqrt{3}\tan x}\right)$$

$$= \lim_{x\to 0}\frac{x[(1-\sqrt{3})\tan x+(1+\sqrt{3})]}{-4\tan x}$$

$$= \lim_{x\to 0}\left(\frac{x}{\sin x}\cdot\frac{\cos x[(1-\sqrt{3})\tan x+(1+\sqrt{3})]}{-4}\right) \qquad \text{1M}$$

$$= 1\cdot\frac{1[0+(1+\sqrt{3})]}{-4}$$

$$= -\frac{1+\sqrt{3}}{4} \qquad \text{1A}$$

11. (a) $\displaystyle\lim_{x\to\infty}\frac{3x^2+2x}{x^2} = \lim_{x\to\infty}\frac{3+\frac{2}{x}}{1}$ 1M

$$= 3 \qquad \text{1A}$$

(b) $\displaystyle\lim_{x\to\infty}\left(\frac{x^2+3x+2}{x^2}\right)^{2x} = \lim_{x\to\infty}\left(1+\frac{3x+2}{x^2}\right)^{2x}$ 1M

$$= \lim_{x\to\infty}\left(1+\frac{3x+2}{x^2}\right)^{\frac{x^2}{3x+2}\cdot\frac{2(3x^2+2x)}{x^2}}$$

$$= \lim_{x\to\infty}\left(1+\frac{3x+2}{x^2}\right)^{\frac{x^2}{3x+2}\cdot\left(6+\frac{4}{x}\right)} \qquad \text{1M}$$

$$= e^6 \qquad \text{1A}$$

12. $\lim\limits_{x\to 0}(1+5x+6x^2)^{\frac{1}{x}} = \lim\limits_{x\to 0}\left[(1+2x)^{\frac{1}{x}}(1+3x)^{\frac{1}{x}}\right]$

$= \lim\limits_{x\to 0}\left\{\left[(1+2x)^{\frac{1}{2x}}\right]^2\left[(1+3x)^{\frac{1}{3x}}\right]^3\right\}$ 1M

$= e^2 \cdot e^3$

$= e^5$ 1A

13. $\lim\limits_{x\to 0}\dfrac{e^{2x}-1}{xe^{2x}} = \lim\limits_{x\to 0}\left(\dfrac{e^{2x}-1}{2x}\cdot\dfrac{2}{e^{2x}}\right)$ 1M

$= 1 \cdot \dfrac{2}{1}$

$= 2$ 1A

14. $\lim\limits_{x\to 0}\dfrac{e^{2x}-1}{x(e^x+1)} = \lim\limits_{x\to 0}\left(\dfrac{e^{2x}-1}{2x}\cdot\dfrac{2}{e^x+1}\right)$ 1M

$= 1 \cdot \dfrac{2}{e^0+1}$

$= 1$ 1A

15. $\lim\limits_{x\to 0}\dfrac{e^{2x}+e^{-2x}-2}{x\sin 4x} = \lim\limits_{x\to 0}\dfrac{e^{4x}+1-2e^{2x}}{xe^{2x}\sin 4x}$

$= \lim\limits_{x\to 0}\dfrac{(e^{2x}-1)^2}{xe^{2x}\sin 4x}$ 1M

$= \lim\limits_{x\to 0}\left[\left(\dfrac{e^{2x}-1}{2x}\right)^2\cdot\dfrac{4x}{\sin 4x}\cdot\dfrac{1}{e^{2x}}\right]$ 1M

$= 1^2 \cdot 1 \cdot \dfrac{1}{e^0}$

$= 1$ 1A