

M2REG-LIM-2324-ASM-SET 2-MATH

建議題解

$$\begin{aligned}
 1. \quad & \lim_{x \rightarrow \infty} \frac{(x-1)(x-2)(x-3)}{2x^3} \\
 &= \lim_{x \rightarrow \infty} \frac{\left(1 - \frac{1}{x}\right) \left(1 - \frac{2}{x}\right) \left(1 - \frac{3}{x}\right)}{2} \\
 &= \frac{(1-0)(1-0)(1-0)}{2} \\
 &= \frac{1}{2}
 \end{aligned}$$

1M
1A

$$\begin{aligned}
 2. \quad & \lim_{x \rightarrow \infty} \frac{3^{2x+1} - 1}{9^x} \\
 &= \lim_{x \rightarrow \infty} \frac{3 - \frac{1}{9^x}}{1} \\
 &= \frac{3-0}{1} \\
 &= 3
 \end{aligned}$$

1M
1A

$$\begin{aligned}
 3. \quad (a) \quad & \lim_{x \rightarrow \infty} \sqrt{\frac{x+5}{9x-4}} \\
 &= \lim_{x \rightarrow \infty} \sqrt{\frac{1 + \frac{5}{x}}{9 - \frac{4}{x}}} \\
 &= \sqrt{\frac{1+0}{9-0}} \\
 &= \frac{1}{3}
 \end{aligned}$$

1M
1A

$$\begin{aligned}
 (b) \quad & \lim_{x \rightarrow \infty} \sqrt{\frac{8x - 25x^2 - 4}{7x + 9 - 16x^2}} \\
 &= \lim_{x \rightarrow \infty} \sqrt{\frac{\frac{8}{x} - 25 - \frac{4}{x^2}}{\frac{7}{x} + \frac{9}{x^2} - 16}} \\
 &= \sqrt{\frac{0 - 25 - 0}{0 + 0 - 16}} \\
 &= \frac{5}{4}
 \end{aligned}$$

1M
1A

4. (a) $\lim_{x \rightarrow \infty} \frac{\sqrt{4x^2 - 5x + 1}}{2x}$
 $= \lim_{x \rightarrow \infty} \frac{\sqrt{4 - \frac{5}{x} + \frac{1}{x^2}}}{2}$ 1M
 $= \frac{\sqrt{4 - 0 + 0}}{2}$
 $= 1$ 1A
- (b) $\lim_{x \rightarrow \infty} \frac{\sqrt{9x^2 - 8x - 6}}{5x}$
 $= \lim_{x \rightarrow \infty} \frac{\sqrt{9 - \frac{8}{x} - \frac{6}{x^2}}}{5}$ 1M
 $= \frac{\sqrt{9 - 0 - 0}}{5}$
 $= \frac{3}{5}$ 1A
5. $\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{12x^2 - 5x}{hx^2 + kx}$
 $\frac{5}{4} = \lim_{x \rightarrow 0} \frac{12x - 5}{hx + k}$ 1M
 $\frac{5}{4} = \frac{-5}{k}$
 $k = -4$ 1A
- $\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{12x^2 - 5x}{hx^2 + kx}$
 $-4 = \lim_{x \rightarrow \infty} \frac{12 - \frac{5}{x}}{h + \frac{k}{x}}$ 1M
 $-4 = \frac{12 - 0}{h + 0}$
 $h = -3$ 1A
6. (a) $\lim_{x \rightarrow \infty} \frac{4e^x - 3e^{-x}}{2e^x + e^{-x}}$
 $= \lim_{x \rightarrow \infty} \frac{4 - 3e^{-2x}}{2 + e^{-2x}}$ 1M
 $= \frac{4 - 0}{2 + 0}$
 $= 2$ 1A
- (b) $\lim_{x \rightarrow \infty} \frac{6e^{-x} + e^{-3x}}{8e^{-4x} - 2e^{-x}}$
 $= \lim_{x \rightarrow \infty} \frac{6 + e^{-2x}}{8e^{-3x} - 2}$ 1M
 $= \frac{6 + 0}{0 - 2}$
 $= -3$ 1A

$$\begin{aligned}
7. \quad & \lim_{x \rightarrow -\infty} \frac{5^x + 5^{-x}}{2 \cdot 5^x - 5^{-x}} \\
&= \lim_{x \rightarrow -\infty} \frac{5^{2x} + 1}{2 \cdot 5^{2x} - 1} && 1M \\
&= \frac{0 + 1}{0 - 1} \\
&= -1 && 1A
\end{aligned}$$

$$\begin{aligned}
8. \quad & \lim_{x \rightarrow \infty} \frac{(2^x - 1)^3}{8^x + 1} = \lim_{x \rightarrow \infty} \frac{(1 - 2^{-x})^3}{1 + 2^{-3x}} && 1M \\
&= \frac{(1 - 0)^3}{1 + 0} \\
&= 1 && 1A
\end{aligned}$$

$$\begin{aligned}
9. \quad & \lim_{x \rightarrow \infty} \frac{3^{2x+1} - 1}{9^x} \\
&= \lim_{x \rightarrow \infty} \frac{3 - \frac{1}{9^x}}{1} && 1M \\
&= \frac{3 - 0}{1} \\
&= 3 && 1A
\end{aligned}$$

$$\begin{aligned}
10. \quad & \lim_{x \rightarrow -\infty} \left(\sqrt{\frac{x^3}{x-2}} + x \right) \\
&= \lim_{x \rightarrow -\infty} \left(\sqrt{\frac{(-x)^3}{2-x}} + x \right) && 1M \\
&= \lim_{x \rightarrow -\infty} \frac{\sqrt{(-x)^3} + x\sqrt{2-x}}{\sqrt{2-x}} \\
&= \lim_{x \rightarrow -\infty} \frac{(-x)\sqrt{-x} + x\sqrt{2-x}}{\sqrt{2-x}} && 1M \\
&= \lim_{x \rightarrow -\infty} \frac{x(-\sqrt{-x} + \sqrt{2-x})(\sqrt{-x} + \sqrt{2-x})}{\sqrt{2-x}(\sqrt{-x} + \sqrt{2-x})} && 1M \\
&= \lim_{x \rightarrow -\infty} \frac{2x}{\sqrt{2-x}(\sqrt{-x} + \sqrt{2-x})} \\
&= \lim_{x \rightarrow -\infty} \frac{2}{-\frac{\sqrt{2-x}}{\sqrt{-x}} + \frac{\sqrt{-x} + \sqrt{2-x}}{\sqrt{-x}}} && 1A \\
&= \lim_{x \rightarrow -\infty} \frac{2}{-\sqrt{\frac{2}{-x}} + 1 \left(1 + \sqrt{\frac{2}{-x}} + 1 \right)} \\
&= \frac{2}{-\sqrt{0+1}(1 + \sqrt{0+1})} \\
&= -2 && 1A
\end{aligned}$$

$$\begin{aligned}
11. \quad \lim_{x \rightarrow -\infty} \frac{2x}{\sqrt{5x^2 - 3x + 10}} &= \lim_{x \rightarrow -\infty} \frac{2(-\sqrt{x^2})}{\sqrt{5x^2 - 3x + 10}} \\
&= \lim_{x \rightarrow -\infty} \frac{-2}{\sqrt{5 - \frac{3}{x} + \frac{10}{x^2}}} && 1M \\
&= \frac{-2}{\sqrt{5 - 0 + 0}} \\
&= -\frac{2}{\sqrt{5}} && 1A
\end{aligned}$$

$$\begin{aligned}
12. \quad \lim_{x \rightarrow \infty} (\sqrt{x+3} - \sqrt{x+4}) & \\
&= \lim_{x \rightarrow \infty} \frac{(\sqrt{x+3} - \sqrt{x+4})(\sqrt{x+3} + \sqrt{x+4})}{\sqrt{x+3} + \sqrt{x+4}} && 1M \\
&= \lim_{x \rightarrow \infty} \frac{(x+3) - (x+4)}{\sqrt{x+3} + \sqrt{x+4}} \\
&= \lim_{x \rightarrow \infty} \frac{\frac{-1}{\sqrt{x}}}{\sqrt{1 + \frac{3}{x}} + \sqrt{1 + \frac{4}{x}}} && 1M \\
&= \frac{0}{\sqrt{1+0} + \sqrt{1+0}} \\
&= 0 && 1A
\end{aligned}$$

$$\begin{aligned}
13. \quad \lim_{x \rightarrow \infty} (\sqrt{9x^2 + 4x - 5} - 3x) & \\
&= \lim_{x \rightarrow \infty} \frac{(\sqrt{9x^2 + 4x - 5} - 3x)(\sqrt{9x^2 + 4x - 5} + 3x)}{\sqrt{9x^2 + 4x - 5} + 3x} && 1M \\
&= \lim_{x \rightarrow \infty} \frac{9x^2 + 4x - 5 - 9x^2}{\sqrt{9x^2 + 4x - 5} + 3x} \\
&= \lim_{x \rightarrow \infty} \frac{4 - \frac{5}{x}}{\sqrt{9 + \frac{4}{x} - \frac{5}{x^2}} + 3} && 1M \\
&= \frac{4 - 0}{\sqrt{9 + 0 - 0} + 3} \\
&= \frac{2}{3} && 1A
\end{aligned}$$

14. (a) $\lim_{x \rightarrow -\infty} (\sqrt{9x^2 - 1} - \sqrt{(3x + 1)^2})$
- $$= \lim_{x \rightarrow -\infty} \frac{(9x^2 - 1) - (3x + 1)^2}{\sqrt{9x^2 - 1} + \sqrt{(3x + 1)^2}} \quad 1\text{M}$$
- $$= \lim_{x \rightarrow -\infty} \frac{-6x - 2}{\sqrt{9x^2 - 1} - (3x + 1)}$$
- $$= \lim_{x \rightarrow -\infty} \frac{-6 - \frac{2}{x}}{-\sqrt{9 - \frac{1}{x^2}} - 3 - \frac{1}{x}} \quad 1\text{M}$$
- $$= \frac{-6}{-\sqrt{9} - 3}$$
- $$= 1 \quad 1\text{A}$$
- (b) $\lim_{x \rightarrow \infty} \frac{e^x - 3^x}{3^x - 2^x} = \lim_{x \rightarrow \infty} \frac{\left(\frac{e}{3}\right)^x - 1}{1 - \left(\frac{2}{3}\right)^x} \quad 1\text{M}$
- $$= \frac{0 - 1}{1 - 0}$$
- $$= -1 \quad 1\text{A}$$