REV-POLY-2324-ASM-SET 2-MATH
Suggested solutions

Multiple Choice Questions

1. B	2. B	3. C	4. B	5. C
6. B	7. B	8. C	9. C	10. A
11. D	12. D	13. A	14. A	15. C
16. B	17. D	18. C	19. B	20. B
21. D	22. B	23. D	24. D	25. B
26. D	27. D	28. D	29. B	30. A

1. $\boxed{B}$

$$
\begin{array}{r|l}
 & x^2 + x + 3 \\
\hline
-x+3 & -x^3 + 2x^2 + 0x + 5 \\
 & \underline{-x^3 + 3x^2} \\
 & -x^2 + 0x \\
 & \underline{-x^2 + 3x} \\
 & -3x + 5 \\
 & \underline{-3x + 9} \\
 & -4
\end{array}
$$

2. $\boxed{B}$

$$
\begin{aligned}
\text{Required polynomial} &= (x^2 - 3x - 10)(3x - 1) + (3x - 2) \\
&= 3x^3 - 10x^2 - 24x + 8
\end{aligned}
$$

3. $\boxed{C}$

Let the required polynomial be $p(x)$.

$$
\begin{aligned}
2x^3 - 3x + 5 &= p(x)(2x^2 + 4x + 5) + 15 \\
2x^3 - 3x - 10 &= p(x)(2x^2 + 4x + 5) \\
p(x) &= \frac{2x^3 - 3x - 10}{2x^2 + 4x + 5} \\
&= x - 2
\end{aligned}
$$

4. $\boxed{\text{B}}$

$2x^3 + px^2 - 4x - 1 = (x^2 - 2x + q)(2x + 3) - 4$

$2x^3 + px^2 - 4x + 3 = (x^2 - 2x + q)(2x + 3)$

Compare the coefficients of x^2 and the constant terms.

$$\begin{cases} 3q = 3 \\ (1)(3) + (-2)(2) = p \end{cases}$$

Solving, we have $p = -1$ and $q = 1$.

5. $\boxed{\text{C}}$

Let $3x^3 + 2x^2 - 27x - 15 = (3x^2 + kx + 6)(Ax + B) + 3$, where A and B are constants.

$$3x^3 + 2x^2 - 27x - 18 = (3x^2 + kx + 6)(Ax + B)$$

Compare the coefficients of x^3, coefficient of x and constant term.

$$\begin{cases} 3A = 3 \\ kB + 6A = -27 \\ 6B = -18 \end{cases}$$

Solving, we have $A = 1$, $B = -3$ and $k = 11$.

6. $\boxed{\text{B}}$

$$\begin{aligned} \text{Remainder} &= \left[3\left(\frac{-2}{3}\right) - 2\right]\left[3\left(\frac{-2}{3}\right) + 5\right] + 3\left(\frac{-2}{3}\right) + 2 \\ &= -12 \end{aligned}$$

7. $\boxed{\text{B}}$

$$\text{Remainder} = \left(-\frac{1}{2}\right)^3 - \left(-\frac{1}{2}\right)^2 + 1 = \frac{5}{8}$$

8. $\boxed{\text{C}}$

$$\begin{aligned} \text{Remainder} &= (-1)^3 - 4(-1)^2 - 7(-1) + 10 \\ &= 12 \end{aligned}$$

9. $\boxed{\text{C}}$

$$\begin{aligned} \text{Remainder} &= 10\left(\frac{-1}{2}\right)^3 - 33\left(\frac{-1}{2}\right)^2 + 9\left(\frac{-1}{2}\right) - 5 \\ &= -19 \end{aligned}$$

10. $\boxed{\text{A}}$

We have $P\left(\frac{3}{2}\right) = k$.

$$\begin{aligned} \text{Remainder} &= P\left(\frac{3}{2}\right) \\ &= k \end{aligned}$$

11. $\boxed{\text{D}}$

We have $f(a) = 15$ and $g(a) = -3$.

$$\begin{aligned}\text{Remainder} &= f(a) - g(a)\\ &= 15 - (-3)\\ &= 18\end{aligned}$$

12. $\boxed{\text{D}}$

$$\begin{aligned}\text{Remainder} &= (-1)^4 - 2(-1)^3 + 3(-1)^2 - 4(-1) + 5\\ &= 15\end{aligned}$$

13. $\boxed{\text{A}}$

$$\text{Remainder} = (-1)^{2k+1} - k + k = (-1)^{2k+1} = -1$$

14. $\boxed{\text{A}}$

$$\begin{aligned}\text{Remainder} &= (-1)^{2009} + (-1)^{2008} + (-1)^{2007} + \ldots + (-1)\\ &= (-1 + 1) + (-1 + 1) + \ldots + (-1)\\ &= -1\end{aligned}$$

15. $\boxed{\text{C}}$

$$-2 = 6\left(\frac{1}{2}\right)^3 - \left(\frac{1}{2}\right)^2 + a\left(\frac{1}{2}\right) - 1$$

$$a = -3$$

16. $\boxed{\text{B}}$

$$\begin{aligned}\text{Remainder} &= f(-1 + 1)\\ &= f(0)\\ &= (0 - 1)(0 + 3)(0 - a)\\ &= 3a\end{aligned}$$

17. $\boxed{\text{D}}$

$$\begin{aligned}\text{Remainder} &= f(-3 - 3)\\ &= f(-6)\end{aligned}$$

18. $\boxed{\text{C}}$

I. ✓. $f(-1) = -1 + 7 - 6 = 0 \quad \Rightarrow \quad x + 1$ is a factor of $f(x)$.

II. ✓. $f(-2) = -8 + 14 - 6 = 0 \quad \Rightarrow \quad x + 2$ is a factor of $f(x)$.

So, $(x + 1)(x + 2)$ is a factor of $f(x)$.

19. $\boxed{\text{B}}$

$f(x) = (2x^2 + x - 1)q(x)$, where $q(x)$ is a polynomial.
$f(x) = (2x - 1)(x + 1)q(x)$
We have $f\left(\frac{1}{2}\right) = f(-1) = 0$.

20. $\boxed{\text{B}}$

A. ✗. $(-2)^3 - 2(-2)^2 - (-2) + 2 = -12$
B. ✓. $(-2)^3 + 2(-2)^2 - (-2) - 2 = 0$
C. ✗. $2(-2)^3 - (-2)^2 - 2(-2) + 1 = -15$
D. ✗. $2(-2)^3 + (-2)^2 - 2(-2) - 1 = -9$

21. $\boxed{\text{D}}$

$$\begin{aligned}(-1)^{2n} + k(-1) - 2k &= 0\\ 1 - 3k &= 0\\ k &= \frac{1}{3}\end{aligned}$$

22. $\boxed{\text{B}}$

$$\begin{aligned}f(x) &= x^3 + x^2 - 5x + 3\\ &= (x - 1)(x^2 + 2x - 3)\\ &= (x - 1)^2(x + 3)\end{aligned}$$

23. $\boxed{\text{D}}$

We have $f(-1) = f(2) = 0$.
$$\begin{cases}0 = (-1)^3 + 2(-1)^2 - a + b\\ 0 = (2)^3 + 2(2)^2 + 2a + b\end{cases}$$
Solving, we have $a = -5$ and $b = -6$.
$$\begin{aligned}f(x) &= x^3 + 2x^2 - 5x - 6\\ &= (x + 1)(x^2 + x - 6)\\ &= (x + 1)(x - 2)(x + 3)\end{aligned}$$

24. $\boxed{\text{D}}$

We have $f(-2) = f(1) = 0$.
$$\begin{cases}0 = (-2)^3 - 4(-2)^2 - 2a + b\\ 0 = 1 - 4 + a + b\end{cases}$$
Solving, we have $a = -7$ and $b = 10$.
$$\begin{aligned}f(x) &= x^3 - 4x^2 - 7x + 10\\ &= (x + 2)(x^2 - 6x + 5)\\ &= (x + 2)(x - 1)(x - 5)\end{aligned}$$

25. $\boxed{\text{B}}$

$4^3 + k(4)^2 + 2k(4) + 8 = 0$

$$k = -3$$

$P(x) = x^3 - 3x^2 - 6x + 8$

By calculator, $P(1) = P(-2) = 0$. So, $(x-1)(x+2)$ is a factor of $P(x)$.

Only option B satisfies this.

26. $\boxed{\text{D}}$

We have $f(-3) = -63$ and $f\left(\frac{1}{2}\right) = 0$.

$$\begin{cases} -63 = a(-3)^3 + 2(-3)^2 + b(-3) + 3 \\ 0 = a\left(\frac{1}{2}\right)^3 + 2\left(\frac{1}{2}\right)^2 + b\left(\frac{1}{2}\right) + 3 \end{cases}$$

Solving, we have $a = 4$ and $b = -8$.

27. $\boxed{\text{D}}$

$$\begin{aligned} 2\left(\frac{-k}{2}\right)^4 + k\left(\frac{-k}{2}\right)^3 - 4\left(\frac{-k}{2}\right) - 16 &= 0 \\ 2k - 16 &= 0 \\ k &= 8 \end{aligned}$$

28. $\boxed{\text{D}}$

Let $f(x) = (x+1)q(x)$, where $q(x)$ is a polynomial.

$$\begin{aligned} f(3x+1) &= [(3x+1)+1]q(3x+1) \\ &= (3x+2)q(3x+1) \end{aligned}$$

Thus, $3x+2$ is a factor of $f(3x+1)$.

29. $\boxed{\text{B}}$

Let $f(x) = (x+1)q(x)$, where $q(x)$ is a polynomial.

$$\begin{aligned} f(3x-4) &= [(3x-4)+1]q(3x-4) \\ &= (3x-3)q(3x-4) \\ &= 3(x-1)q(3x-4) \end{aligned}$$

Thus, $x-1$ is a factor of $f(3x-4)$.

30. $\boxed{\text{A}}$

Let $f(x) = (x-1)q(x)$, where $q(x)$ is a polynomial.

$$\begin{aligned} f(2x+1) &= [(2x+1)-1]q(2x+1) \\ &= 2xq(2x+1) \end{aligned}$$

Thus, x is a factor of $f(2x+1)$.

Conventional Questions

31. (a) We have

$$\begin{cases} -7 = [3(3)+p](3-2)^2 + q \\ 0 = (12+p)(2)^2 + q \end{cases}$$ 1M

Solving, we have $p = \dfrac{-32}{3}$ and $q = \dfrac{-16}{3}$. 1A+1A

(b) $0 = \left(3x - \dfrac{32}{3}\right)(x-2)^2 - \dfrac{16}{3}$

$0 = (9x-32)(x^2-4x+4) - 16$

$0 = 9x^3 - 68x^2 + 164x - 144$

$0 = (x-4)(9x^2-32x+36)$ 1M+1A

$x = 4$ or $9x^2 - 32x + 36 = 0$

For $9x^2 - 32x + 36 = 0$, $\Delta = 32^2 - 4(9)(36) = -272 < 0$. 1M

The equation has no real roots.

Thus, there is 1 real root. 1A

32. (a) (i) $f(-3) = m(-3)^3 + 3(m-2)(-3)^2 - 13(-3) + 15$ 1M

$= -27m + 27m - 54 + 39 + 15$

$= 0$

Thus, $x+3$ is a factor of $f(x)$. 1

(ii) $f(x) = (x+3)(mx^2 - 6x + 5)$ 1A

(b) (i) When $f(x) = 0$,

$(x+3)(mx^2-6x+5) = 0$

$x = -3$ or $mx^2 - 6x + 5 = 0$

If the roots are real,

$\Delta = 6^2 - 4(m)(5) \geq 0$ 1M

$m \leq \dfrac{9}{5}$

Since $m \neq 0$, we have $m < 0$ or $0 < m \leq \dfrac{9}{5}$. 1A

(ii) Put $m = 1$, when $f(x) = 0$, 1A

$(x+3)(x^2-6x+5) = 0$

$(x+3)(x-1)(x-5) = 0$

$x = -3$ or 1 or 5

The claim is agreed. 1A

33. (a) $f(-1) = h = -h - 1 + 9 + k$ 1M

$2h - k = 8$

$0 = f\left(\frac{1}{2}\right) = \frac{h}{8} - \frac{1}{4} - \frac{9}{2} + k$ 1M

$h + 8k = 38$

Solving, we have $h = 6$ and $k = 4$. 1A+1A

(b) $f(x) = 6$

$6x^3 - x^2 - 9x - 2 = 0$

$(x+1)(6x^2 - 7x - 2) = 0$ 1M

$x = -1 \quad \text{or} \quad \frac{7 \pm \sqrt{7^2 - 4(6)(-2)}}{2(6)}$ 1M

$= -1 \quad \text{or} \quad \frac{7 \pm \sqrt{97}}{12}$

There are some irrational roots in $f(x) = 6$.

The claim is disagreed. 1A

34. (a) Let $f(x) = (x^2 - 4)(Ax + B) + 4x + k$, where A and B are constants. 1A

$f(2) = 0 + 4(2) + k = 0$ 1M

$k = -8$ 1A

(b) We have

$$\begin{cases} f(-4) = (16-4)(-4A+B) - 16 - 8 = 0 \\ f(0) = (-4)(B) - 8 = 40 \end{cases}$$ 1M

Solving, we have $A = -\frac{7}{2}$ and $B = -12$. 1A

$f(x) = (x^2 - 4)\left(-\frac{7}{2}x - 12\right) + 4x - 8 = 0$

$-\frac{1}{2}(x+2)(x-2)(7x+24) + 4(x-2) = 0$

$\frac{1}{2}(x-2)[-(x+2)(7x+24) + 8] = 0$ 1M

$\frac{1}{2}(x-2)(-7x^2 - 38x - 40) = 0$

$-\frac{1}{2}(x-2)(7x+10)(x+4) = 0$

$x = 2 \quad \text{or} \quad -4 \quad \text{or} \quad -\frac{10}{7}$

The claim is disagreed. 1A

35. (a) We have

$$\begin{cases} 0 = 4(3)(3+1)^2 + 3a + b \\ 2b + 165 = 4(-2)(-2+1)^2 - 2a + b \end{cases}$$ 1M

Therefore, $3a + b = -192$ and $-2a - b = 173$.

Solving, we have $a = -19$ and $b = -135$. 1A+1A

(b) $0 = 4x(x+1)^2 - 19x - 135$

$= 4x^3 + 8x^2 - 15x - 135$ 1A

$= (x-3)(4x^2 + 20x + 45)$ 1M

$x = 3 \quad \text{or} \quad \dfrac{-20 \pm \sqrt{20^2 - 4(4)(45)}}{2(4)}$ 1M

$= 3 \quad \text{or} \quad -\dfrac{5}{2} \pm i\sqrt{5}$

All roots of $f(x) = 0$ are not irrational, the claim is disagreed. 1A

36. (a) $p(x) = (x^2 + x + 1)(2x^2 - 37) + cx + c - 1$ 1M

$(5^2 + 5 + 1)(2(5)^2 - 37) + 5c + c - 1 = 0$ 1M

$c = -67$ 1A

(b) $p(-3) = (9 - 3 + 1)(18 - 37) - 67(-3) - 68$

$= 0$

Thus, $x + 3$ is a factor of $p(x)$. 1

(c) $0 = (x^2 + x + 1)(2x^2 - 37) - 67x - 68$

$= 2x^4 + 2x^3 - 35x^2 - 104x - 105$

$= (x-5)(2x^3 + 12x^2 + 25x + 21)$

$= (x-5)(x+3)(2x^2 + 6x + 7)$ 1M

$x = 5 \quad \text{or} \quad -3 \quad \text{or} \quad 2x^2 + 6x + 7 = 0$

For the equation $2x^2 + 6x + 7 = 0$, $\Delta = 6^2 - 4(2)(7) = -20 < 0$. 1M

The roots of $2x^2 + 6x + 7 = 0$ are not real.

The claim is not correct. 1A

END OF PAPER