

REG-GS-2324-ASM-SET 4-MATH**Suggested solutions****Multiple Choice Questions**

1. B	2. A	3. D	4. D	5. D
6. A	7. C	8. B	9. B	10. A
11. C	12. D	13. B	14. D	15. C
16. D	17. C	18. A	19. Cancelled	20. A
21. D	22. C	23. D	24. A	

1. **B**

- I. ✓. Common ratio = 2^m .
 II. ✗. $\frac{2m^2}{m} = 2m$ and $\frac{3m^4}{2m^2} = \frac{3m^2}{2} \neq 2m$.
 III. ✓. Sequence: $\log m, 2 \log m, 4 \log m, 8 \log m$. Common ratio = 2.

2. **A**

$$b^2 = ac$$

$$\frac{b}{a} = \frac{c}{b}$$

- I. ✓. We have $\frac{\sqrt{b}}{\sqrt{a}} = \sqrt{\frac{b}{a}}$ and $\frac{\sqrt{c}}{\sqrt{b}} = \sqrt{\frac{c}{b}}$.
 Since $\frac{b}{a} = \frac{c}{b}$, $\frac{\sqrt{b}}{\sqrt{a}} = \frac{\sqrt{c}}{\sqrt{b}}$.
 II. ✗. Take $a = 1, b = 2$ and $c = 4$.
 $\frac{2^b}{2^a} = \frac{2^2}{2^1} = 2$ and $\frac{2^c}{2^b} = \frac{2^4}{2^2} = 4$.
 $2^a, 2^b, 2^4 c$ is not a geometric sequence.
 III. ✗. Take $a = 10, b = 100$ and $c = 1000$.
 $\frac{\log b}{\log a} = \frac{\log 100}{\log 10} = 2$ and $\frac{\log c}{\log b} = \frac{\log 1000}{\log 100} = \frac{3}{2}$.
 $\log a, \log b, \log c$ is not a geometric sequence.

3. **D**

$$\frac{x+1}{2} = \frac{x^2-7}{x+1}$$

$$(x+1)^2 = 2(x^2-7)$$

$$0 = x^2 - 2x - 15$$

$$x = 5 \quad \text{or} \quad -3$$

4. D

$$5 - h = k - 5 \quad \text{and} \quad \frac{3}{h} = \frac{k}{3}$$

$$h + k = 10 \qquad hk = 9$$

$$h^2 + k^2 = (h + k)^2 - 2hk$$

$$= 10^2 - 2(9)$$

$$= 82$$

5. D

$$\frac{x+7}{4} = \frac{x^2+11}{x+7}$$

$$(x+7)^2 = 4(x^2+11)$$

$$0 = 3x^2 - 14x - 5$$

$$x = -\frac{1}{3} \quad \text{or} \quad 5$$

6. A

Let the common ratio be R .

$$\begin{cases} p + r = p + pR^2 = 2 \\ q + s = pR + pR^3 = 32 \end{cases}$$

$$\frac{pR + pR^3}{p + pR^2} = \frac{32}{2}$$

$$\frac{pR(1 + R^2)}{p(1 + R^2)} = 16$$

$$R = 16$$

The common ratio is 16.

7. C

Let a and r be the first term and common ratio respectively.

$$\begin{cases} (a)(ar) = 48 \\ (ar)(ar^3) = 1296 \end{cases}$$

$$\frac{(ar)(ar^3)}{(a)(ar)} = \frac{1296}{48}$$

$$r^3 = 27$$

$$r = 3$$

When $r = 3$, $a^2 = \frac{48}{r} = 16$.

Required product $= (ar^2)(ar^4)$

$$= a^2 r^6$$

$$= 16(3^6)$$

$$= 11\,664$$

8. B

Let n be the number of terms.

$$(a^4)(a^3)^{n-1} = a^{145}$$

$$a^{3n-3} = a^{141}$$

$$3n - 3 = 141$$

$$n = 48$$

There are 48 terms.

9. B

Let r be the common ratio.

$$\begin{cases} a_1 + a_1 r + a_1 r^2 = 1 \\ a_1 r^3 + a_1 r^4 + a_1 r^5 = 8 \end{cases}$$

$$\frac{a_1 r^3 (1 + r + r^2)}{a_1 (1 + r + r^2)} = \frac{8}{1}$$

$$r^3 = 8$$

$$r = 2$$

When $r = 2$, $a_1 = \frac{1}{1 + r + r^2} = \frac{1}{7}$.

We have $a_2 = a_1 r = \frac{2}{7}$.

10. A

Let r be the common ratio.

$$243r^3 = 9$$

$$r^3 = \frac{1}{27}$$

$$r = \frac{1}{3}$$

$$\begin{aligned} y - x &= 243r^2 - 243r \\ &= -54 \end{aligned}$$

11. C

Let r be the common ratio.

$$\frac{3}{r} + 3r = \frac{13}{2}$$

$$3 + 3r^2 = \frac{13r}{2}$$

$$3r^2 - \frac{13r}{2} + 3 = 0$$

$$r = \frac{2}{3} \quad \text{or} \quad \frac{3}{2}$$

12. D

Let r be the common ratio.

$$6 + 6r + 6r^2 = 186$$

$$6r^2 + 6r - 180 = 0$$

$$r = -6 \quad \text{or} \quad 5$$

13. B

Let a and r be the first term and common ratio respectively.

$$\frac{ar^6}{ar^4} = \frac{6}{24}$$

$$r^2 = \frac{1}{4}$$

$$r = \pm \frac{1}{2}$$

I. ✓. $T_1 = a = \frac{24}{r^4} > 0$

II. ✓. $T_1 - T_2 = T_1(1 - r) > 0$ for $r = \pm 1$.

III. ✗. When $r = -\frac{1}{2}$,

$$\begin{aligned} T_2 + T_3 + \dots + T_{200} &= \frac{T_2(1 - r^{199})}{1 - r} \\ &= \frac{ar(1 - r^{199})}{1 - r} \end{aligned}$$

Note that $a > 0$, $r < 0$, $(1 - r^{199}) > 0$ and $(1 - r) > 0$.

The sum is therefore negative.

14. D

Common ratio = $\frac{y}{x}$

Let a be the first term.

$$a \left(\frac{y}{x} \right)^2 = x$$

$$a = \frac{x^3}{y^2}$$

$$\begin{aligned} \text{Sum to infinity} &= \frac{\left(\frac{x^3}{y^2} \right)}{1 - \frac{y}{x}} \\ &= \frac{x^3}{y^2} \div \frac{x - y}{x} \\ &= \frac{x^4}{y^2(x - y)} \end{aligned}$$

15. C

$$\begin{aligned} \text{Sum to infinity} &= \frac{-a}{1 - \left(-\frac{1}{a} \right)} \\ &= -a \div \frac{a + 1}{a} \\ &= -\frac{a^2}{1 + a} \end{aligned}$$

16. D

Common ratio = $\frac{n}{m}$

Let a be the first term.

$$a \left(\frac{n}{m} \right)^3 = m$$

$$a = \frac{m^4}{n^3}$$

$$\begin{aligned} \text{Sum to infinity} &= \frac{\left(\frac{m^4}{n^3} \right)}{1 - \frac{n}{m}} \\ &= \frac{m^4}{n^3} \div \frac{m-n}{m} \\ &= \frac{m^5}{n^3(m-n)} \end{aligned}$$

17. C

$$\begin{aligned} \text{Sum to infinity} &= \frac{18}{1 - \frac{1}{3}} \\ &= 27 \end{aligned}$$

18. A

Let r be the common ratio.

$$\text{First term} = \frac{-9}{r}$$

$$\begin{aligned} \frac{\left(\frac{-9}{r} \right)}{1-r} &= \frac{81}{4} \\ \frac{-9}{r(1-r)} &= \frac{81}{4} \end{aligned}$$

$$-36 = 81r - 81r^2$$

$$81r^2 - 81r - 36 = 0$$

$$r = -\frac{1}{3} \quad \text{or} \quad \frac{4}{3} \text{ (rejected)}$$

Note that $-1 < r < 1$ for the existence of sum to infinity.

19. Cancelled

Sum to infinity does not exist for $x = 3$.

Wrong question setting.

20. A

Let the first term and common ratio be a and r respectively.

$$\begin{cases} \frac{a}{1-r} = \frac{81}{4} \\ ar = -9 \end{cases}$$

$$ar \div \frac{a}{1-r} = -9 \div \frac{81}{4}$$

$$r(1-r) = -\frac{4}{9}$$

$$-r^2 + r + \frac{4}{9} = 0$$

$$r = -\frac{1}{3} \quad \text{or} \quad \frac{4}{3} \text{ (rejected)}$$

21. D

First term = $3^{6-2(1)} = 81$ and second term = $3^{6-2(2)} = 9$

$$\text{Common ratio} = \frac{9}{81} = \frac{1}{9}$$

22. C

Let a and r be the first term and common ratio respectively.

$$\frac{a}{1-r} = 3a$$

$$a = 3a(1-r)$$

$$0 = a[3(1-r) - 1]$$

$$0 = a(2-3r)$$

$$r = \frac{2}{3} \quad \text{or} \quad a = 0 \text{ (rejected)}$$

23. D

$$\begin{aligned} \text{Required sum} &= \frac{-81}{1 - \left(\frac{-9}{-81}\right)} \\ &= -\frac{729}{8} \end{aligned}$$

24. A

$$\begin{aligned} \text{Required sum} &= \frac{12}{1 - \frac{3}{12}} \\ &= 16 \end{aligned}$$

Conventional Questions

25. (a) $30 + 30r + 30r^2 = 52.5$ 1M

$$30r^2 + 30r - 22.5 = 0$$

$$r = \frac{1}{2} \quad \text{or} \quad -\frac{3}{2} \text{ (rejected)} \quad \text{1A}$$

(b) Height of the balloon $\leq \frac{30}{1 - \frac{1}{2}}$ 1M

$$= 60 \text{ m} < 75 \text{ m}$$

The balloon cannot reach a height of 75 m. 1A

(c) Let the time required be n minutes.

$$\frac{30(1 - 0.5^n)}{1 - 0.5} = 58.125 \quad \text{1M}$$

$$0.5^n = 0.03125$$

$$n \log 0.5 = \log 0.03125 \quad \text{1M}$$

$$n = 5$$

The time required is 5 minutes. 1A

26. (a) Required probability $= \frac{3}{7} \times \frac{3}{8} + \frac{4}{7} \times \frac{6}{8}$ 1M

$$= \frac{33}{56} \quad \text{1A}$$

(b) Required probability $= \frac{33}{56} + \left(\frac{23}{56}\right)^2 \frac{33}{56} + \left(\frac{23}{56}\right)^4 \frac{33}{56} + \dots$ 1M

$$= \frac{\frac{33}{56}}{1 - \frac{529}{3136}} \quad \text{1M}$$

$$= \frac{56}{79} \quad \text{1A}$$

(c) Required probability $= \frac{\left(\frac{23}{56}\right) \frac{33}{56} + \left(\frac{23}{56}\right)^3 \frac{33}{56}}{1 - \frac{56}{79}}$ 1M+1A

$$\approx 0.972 \quad \text{1A}$$

27. (a) (i) Required number

$$= 120\,000(1 + 10\%)^n - 3000(1 + 10\%)^{n-1} - 3000(1 + 10\%)^{n-2} - \dots - 3000 \quad 1M$$

$$= 120\,000(1.1)^n - \frac{3000(1.1^n - 1)}{1.1 - 1} \quad 1M$$

$$= 90\,000(1.1)^n + 30\,000 \quad 1A$$

$$(ii) 90\,000(1.1)^n + 30\,000 > 200\,000 \quad 1M$$

$$1.1^n > \frac{17}{9}$$

$$n \log 1.1 > \log \frac{17}{9} \quad 1M$$

$$n > 6.67$$

The total number of applicants will exceed 20 000 in year 2020. 1A

(b) We have

$$\begin{cases} 10\,000p + q = 37\,100 \\ 10\,000p^2 + q = 39\,641 \end{cases} \quad 1M$$

$$10\,000p^2 + (37\,100 - 10\,000p) = 39\,641 \quad 1M$$

$$10\,000p^2 - 10\,000p - 2541 = 0$$

$$p = 1.21 \quad \text{or} \quad -0.21 \text{ (rejected)}$$

When $p = 1.21$, $q = 25\,000$. 1A

Let m be the number of years after 2013 such that the number of flats does not fulfil the demand.

$$90\,000(1.1)^m + 30\,000 > 10\,000(1.21)^m + 25\,000 \quad 1M$$

$$0 > 1.21^m - 9(1.1)^m - 0.5$$

$$-0.055\,216\,789\,57 < 1.1^m < 9.055\,216\,790$$

$$m \log 1.1 < \log 9.055\,216\,790$$

$$m < 23.1 \quad 1A$$

Therefore, the number of flats does not fulfil the demand from 2014 to 2036.

The claim is agreed. 1A