

REG-COT-2324-ASM-SET 1-MATH

Suggested solutions

Multiple Choice Questions

- | | | | | |
|-------|-------|-------|-------|-------|
| 1. B | 2. C | 3. D | 4. C | 5. B |
| 6. C | 7. B | 8. A | 9. D | 10. A |
| 11. A | 12. C | 13. A | 14. A | 15. B |
| 16. B | 17. D | 18. D | 19. D | 20. D |
| 21. D | 22. C | 23. B | 24. C | 25. A |
| 26. B | 27. D | 28. D | 29. D | 30. A |

1. B

- I. ✗. Orthocentre is at B .
 II. ✓. Centroid always lies inside the triangle.
 III. ✗. Incentre always lies inside the triangle.

2. C

No steps required.

3. D

- I. ✓. AD is the perpendicular bisector of BC .
 II. ✓. AD is the altitude through A .
 III. ✓. AD is the median through A .

4. C

I is the incentre of $\triangle QRS$.

$$\angle IRQ = \angle IRS = 12^\circ$$

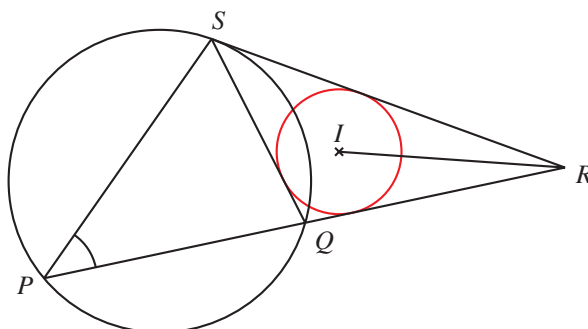
$$\angle SRQ = 12^\circ + 12^\circ = 24^\circ$$

$$\angle QPS = \angle SQR$$

In $\triangle PSR$,

$$70^\circ + 2\angle QPS + 24^\circ = 180^\circ$$

$$\angle QPS = 43^\circ$$



5. B

$$BC = 2BL = 26 \text{ cm}$$

$$AB = 2BN = 10 \text{ cm}$$

$$AC = 2CM = 24 \text{ cm}$$

$$\text{Since } 10^2 + 24^2 = 26^2, \angle BAC = 90^\circ.$$

$$\text{Required area} = \frac{(10)(24)}{2} = 120 \text{ cm}^2$$

6. C

Let the centre of circle be O .

BO is a median of $\triangle ABC$. So, BEO is a straight line.

$$\angle BAC = \angle ABE = 27^\circ$$

$$\angle CBD = \angle BAC = 27^\circ$$

$$\angle ABC = 90^\circ$$

In $\triangle ABD$,

$$x + 27^\circ + (90^\circ + 27^\circ) = 180^\circ$$

$$x = 36^\circ$$

7. B

$$\text{Let } AC = 2. \text{ Then } BC = 2, CD = 1 \text{ and } AE = \frac{AB}{2} = \sqrt{2}$$

$$\text{Note that } CE \perp AB, \angle ACE = \sin^{-1} \frac{\sqrt{2}}{2} = 45^\circ$$

$$\angle CAD = \tan^{-1} \frac{1}{2} \approx 26.6^\circ$$

$$\theta = 180^\circ - 45^\circ - \tan^{-1} \frac{1}{2} \approx 108^\circ$$

$$\sin \theta \approx 0.948683.$$

$$\text{By checking all options, we see that } \sin \theta = \frac{3\sqrt{10}}{10}$$

8. A

I. $\checkmark$. Let the radius of inscribed circle of $\triangle ABC$ be r .

Both angles are equal to $\tan^{-1} \frac{OV}{r}$.

II. $\checkmark$. Let the radius of circumcircle ABC be R .

Then $OB = OC = R$ and both angles are equal to $\tan^{-1} \frac{OV}{R}$.

III. $\times$. If $\angle ABC = 90^\circ$, then points O and B coincide.

Angle between plane VAB and plane ABC is 90° , while angle between plane VAC and plane ABC is not 90° .

9. D

$$y\text{-coordinate of circumcentre} = \frac{(-2) + (8)}{2} = 3$$

Let the coordinates of the circumcentre be $(h, 3)$.

$$\sqrt{(h-2)^2 + (3-8)^2} = \sqrt{(h-10)^2 + (14-3)^2}$$

$$h^2 - 4h + 29 = h^2 - 20h + 221$$

$$16h = 192$$

$$h = 12$$

x -coordinate of the circumcentre is 12.

10. A

Point B can be obtained by rotating A through 90° anticlockwise about the origin. So, $\angle AOB = 90^\circ$.

Circumcentre is the mid-point of AB , required y -coordinate = $\frac{8-2}{2} = 3$

11. A

Since AB is parallel to the y -axis, altitude through O is parallel to the x -axis.

Let the coordinates of orthocentre H be $(x, 0)$. Since $AH \perp OB$,

$$\frac{12-0}{16-x} \times \frac{-12-0}{16-0} = -1$$

$$x = 7$$

12. C

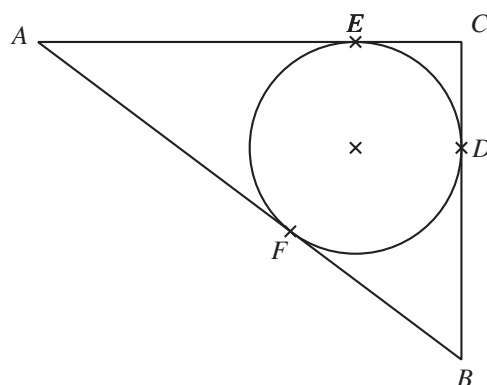
Let D, E, F be the point of contact of the inscribed circle and BC, AC, AB respectively.

Let $CE = x$. Then $AE = 12 - x$, $CD = x$ and $BD = 5 - x$.

By tangent properties, $AF = 12 - x$ and $BF = 5 - x$.

Since $AB = \sqrt{5^2 + 12^2} = 13$, we have $x = 2$.

Thus, the coordinates of incentre are $(10, 3)$.



13. A

Circumcentre lies on perpendicular bisector of OA .

$$y\text{-coordinate of the circumcentre} = \frac{0+12}{2} = 6$$

14. A

Altitude through Q is horizontal.

y-coordinate of the orthocentre = 48

Let the coordinates of the orthocentre be $(x, 48)$.

$$\frac{48 - 0}{x - 0} \times \frac{60 - 48}{0 - 96} = -1$$

$$x = 6$$

15. B

Observe that $\angle BAC = 90^\circ$.

I. ✗. Circumcentre is at the mid-point of BC .

II. ✓. x-coordinate of centroid is 1 by symmetry.

III. ✓. Orthocentre is at A (because it is a right-angled triangle).

16. B

The coordinates of the vertices of the triangle are $(0, 0)$, $(6, 0)$ and $(0, 8)$.

Let the radius of inscribed circle be r .

By considering the area of the triangle,

$$\frac{(6)(8)}{2} = \frac{(6)(r)}{2} + \frac{(8)(r)}{2} + \frac{(\sqrt{6^2 + 8^2})(r)}{2}$$

$$r = 2$$

The coordinates of the incentre are $(2, 2)$.

17. D

Denote the incentre by X .

Let D , E and F be points on OA , OB and AB respectively such that $XD \perp OA$, $XE \perp OB$ and $XF \perp AB$.

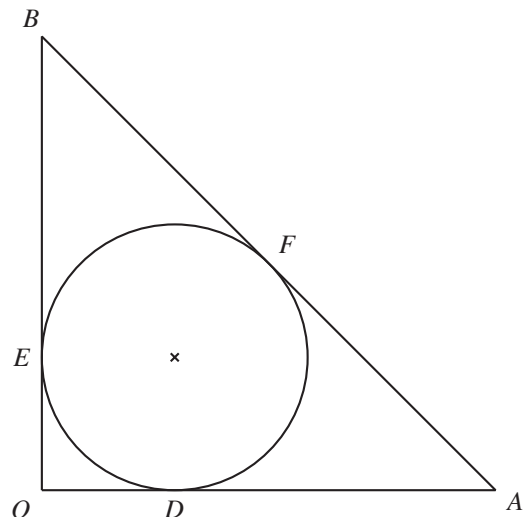
Let the radius be r .

$OD = OE = r$.

$AD = AF = 6 - r$ and $BE = BF = 6 - r$.

$$(6 - r) + (6 - r) = \sqrt{6^2 + 6^2}$$

$$r = 6 - 3\sqrt{2}$$



18. D

Altitude through B is vertical.

The x -coordinate of the orthocentre is 24.

Let the coordinates of the orthocentre be $(24, y)$.

$$\frac{y-0}{24-0} \times \frac{18-0}{24-48} = -1$$

$$y = 32$$

19. D

Since OA is vertical, orthocentre lies on the horizontal line through B

Thus, y -coordinate of orthocentre $= -12$.

Let the coordinates of orthocentre H be $(x, -12)$. As $AH \perp BO$,

$$\frac{36+12}{0-x} \times \frac{12}{16} = -1$$

$$x = 36$$

20. D

Let $H(h, k)$ be the orthocentre.

$$\begin{array}{l} AH \perp BC \quad \text{and} \quad BH \perp AC \\ \frac{k+19}{h+38} \times \frac{9+1}{-10+2} = -1 \quad \frac{k-9}{h+10} \times \frac{-1+19}{-2+38} = -1 \\ \frac{k+19}{h+38} = \frac{4}{5} \quad \frac{k-9}{h+10} = -2 \end{array}$$

Solving, we have $h = -8$ and $k = 5$.

21. D

$$x\text{-coordinate of vertex} = \frac{-k}{2(1)} = -\frac{k}{2}$$

$$\text{mid-point of } PR = \left(-\frac{k}{2}, 0\right)$$

Consider the x -coordinate of centroid,

$$-2 = \frac{0(1) + \left(-\frac{k}{2}\right)(2)}{1+2}$$

$$k = 6$$

22. C

Let the coordinates of Q be (p, q) . Denote the orthocentre of $\triangle OPQ$ by H .

$$OP \perp QH \quad \text{and} \quad PH \perp OQ$$

$$\frac{-18}{26} \times \frac{q-21}{p+3} = -1 \quad \frac{-18+3}{26-21} \times \frac{q}{p} = -1$$

$$13p - 9q = 300 \quad p - 3q = 0$$

Solving, we have $p = 30$ and $q = 10$.

Thus, y-coordinate of Q is 10.

23. B

$\triangle OAB$ is a right-angled triangle. So, orthocentre of $\triangle OAB$ is at point O .

$$\text{Area of } \triangle OAB = \frac{1}{2}(12)(16) = 96.$$

$AB = \sqrt{12^2 + 16^2} = 20$. Let the required distance be d . By considering the area of $\triangle OAB$,

$$\frac{1}{2}(AB)(d) = 96$$

$$d = 9.6$$

24. C

The straight line $x - 2y + 10 = 0$ is perpendicular to the straight line $2x + y + a = 0$.

The triangle formed is a right-angled triangle. The orthocentre lies at the right-angled vertex.

When $x = -6$,

$$(-6) - 2y + 10 = 0$$

$$y = 2$$

Substitute $(-6, 2)$ into $2x + y + a = 0$,

$$2(-6) + (2) + a = 0$$

$$a = 10$$

25. A

$$P\left(\frac{k}{2}, 0\right), Q(0, -k) \text{ and } R(0, k).$$

Circumcentre lies on perpendicular bisector of QR , i.e., the x -axis.

Circumcentre is at $(-3, 0)$.

$$\frac{k}{2} - (-3) = \sqrt{3^2 + k^2}$$

$$0 = \frac{3k^2}{4} - 3k$$

$$k = 4 \quad \text{or} \quad 0$$

y-coordinate of $R = 4$

26. B

$$A = \left(\frac{k}{3}, 0\right) \text{ and } B = \left(0, \frac{k}{4}\right)$$

Circumcentre lies on perpendicular bisector of AC . So $h = \frac{k + \frac{k}{3}}{2} = \frac{2k}{3}$.

Circumcentre is equidistant from B and A ,

$$\begin{aligned}\sqrt{\left(\frac{2k}{3} - k\right)^2 + 38^2} &= \sqrt{\left(\frac{2k}{3}\right)^2 + \left(38 - \frac{k}{4}\right)^2} \\ -\frac{4k^2}{3} + k^2 &= -19k + \frac{k^2}{16} \\ k &= 48 \quad \text{or} \quad 0 \text{ (rejected)}\end{aligned}$$

$$\text{So, } h = \frac{2}{3} \times 48 = 32.$$

27. D

Circumcentre is equidistant from the two vertices.

$$\begin{aligned}\sqrt{(k+4)^2 + (-4+8)^2} &= \sqrt{(k-6)^2 + (-4-2)^2} \\ k^2 + 8k + 32 &= k^2 - 12k + 72 \\ k &= 2\end{aligned}$$

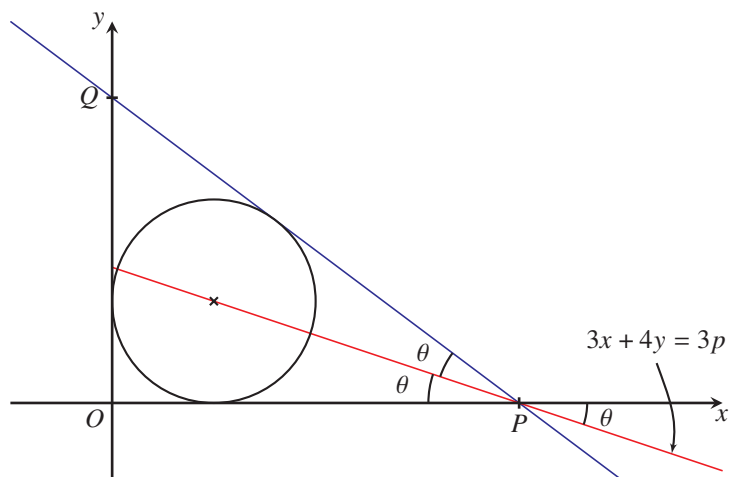
28. D

Note that $(p, 0)$ satisfies the equation $3x + 4y = 3p$.

The straight line $3x + 4y = 3p$ passes through $P(p, 0)$ and the in-centre of $\triangle OPQ$.

Therefore, it is the angle bisector of $\angle OPQ$.

Let the acute angle between the straight line and the x -axis be θ .



$$\begin{aligned} \text{Slope of the line} &= -\frac{3}{4} = -\tan \theta & \text{and} & \quad \frac{OQ}{OP} = \tan 2\theta \\ \theta &= \tan^{-1} \frac{3}{4} & \quad \frac{q}{p} &= \frac{24}{7} \\ p : q &= 7 : 24 \end{aligned}$$

29. D

The line $5x + 4y = 4b$ passes through $B(0, b)$.

Therefore, $5x + 4y = 4b$ passes through the mid-point of $OA \left(\frac{a}{2}, 0 \right)$.

$$\begin{aligned} 5 \left(\frac{a}{2} \right) + 0 &= 4b \\ \frac{a}{b} &= \frac{8}{5} \end{aligned}$$

30. A

- I. ✓. G lies inside $\triangle OAB$, which is in the second quadrant. The x - and y -coordinates are not equal (one positive and one negative).
- II. ✓. Let the radius of inscribed circle be r . Then the coordinates of G are $(-r, r)$.

$$4r + (-r) = 3kb$$

$$r = kb$$

Using tangent properties, OB is divided into two segments with lengths $b - r$ and r .
 OA is divided into two segments with lengths $10 - r$ and r .

$$(10 - r) + (b - r) = \sqrt{10^2 + b^2}$$

$$[10 + b(1 - 2k)]^2 = b^2 + 100$$

$$100 + 20b(1 - 2k) + b^2(1 - 2k)^2 = b^2 + 100$$

$$b^2(4k^2 - 4k) + 20b(1 - 2k) = 0$$

$$\begin{aligned} b &= -\frac{20(1 - 2k)}{4k^2 - 4k} \\ &= \frac{5(1 - 2k)}{k(1 - k)} \end{aligned}$$

$$\text{Required distance} = r = kb = \frac{5(1 - 2k)}{1 - k}$$

III. ✗. When $k = \frac{1}{6}$, $r = \frac{5(1 - 2k)}{1 - k} = 4$.

Equation of inscribed circle is $(x + 4)^2 + (y - 4)^2 = 4^2$.

$$(x + 4)^2 + (5 - 3x - 4)^2 = 16$$

$$10x^2 + 2x + 1 = 0$$

$$\Delta = 2^2 - 4(10)(1) = -36 < 0.$$

The straight line $3x + y = 5$ does not cut the inscribed circle of $\triangle OAB$ and hence is not a tangent.