REG-AOT-2324-ASM-SET 6-MATH
Suggested solutions

Conventional Questions

1. (a) $PS^2 = 24^2 + 18^2 - 2(24)(18)\cos 50°$ 1M

$PS \approx 18.6\,\text{cm}$ 1A

(b) Let M be the mid-point of QR. The angle required is $\angle SMP$. 1M

$SM = \sqrt{18^2 - \left(\frac{20}{2}\right)^2} = 4\sqrt{14}\,\text{cm}$ 1M

$PM = \sqrt{24^2 - 10^2} = 2\sqrt{119}\,\text{cm}$

$PS^2 = SM^2 + PM^2 - 2(SM)(PM)\cos\angle SMP$ 1M

$\angle SMP \approx 57.0°$ 1A

(c) $\angle PAS = \cos^{-1}\dfrac{AP^2 + AS^2 - SP^2}{2(AP)(AS)}$

By symmetry of the figure, $\angle PAS$ is the largest when A is at the mid-point of QR such that AP and AS are perpendicular to QR and are shortest.

When A moves from Q to mid-point M, $\angle PAS$ increases from $50°$ to $57.0°$. 1A

When A moves from mid-point M to R, $\angle PAS$ decreases from $57.0°$ to $50°$. 1A

2. (a) $AM = 10\sin 60° \approx 8.66\,\text{cm}$ 1A

$\angle MAP = \dfrac{60°}{4} = 15°$

$PQ = 2PM$

$= 2 \times 10\tan 15°$ 1M

$\approx 4.64\,\text{cm}$ 1A

(b) As the planes APB and AQC are identical and vertical, vertical distance from M to the ground is equal to the vertical distance from P to the ground.

With reference of the second figure, the vertical distance $= PM$. 1M

Vertical distance $= 10\sin 60° \tan 15°$

$\approx 2.32\,\text{cm}$ 1A

(c) Let P' and Q' be the projections of P and Q on the horizontal ground respectively.

$AP' = AQ' = AM \approx 8.66\,\text{cm}$ 1A

$P'Q' = PQ \approx 4.64\,\text{cm}$

In $\triangle AP'Q'$,

$P'Q'^2 = AP'^2 + AQ'^2 - 2(AP')(AQ')\cos\angle P'AQ'$ 1M

$\angle P'AQ' \approx 31.1°$

Thus, $\angle BAC \approx 31.1°$ 1A

3. (a) $\dfrac{\sin \angle ABD}{19} = \dfrac{\sin 80°}{30}$ 1M

$\angle ABD \approx 38.6°$ or $141°$ (rejected) 1A

(b) (i) $25^2 = 19^2 + 30^2 - 2(19)(30)\cos \angle CDA$ 1M

$\angle CDA \approx 56.1°$ 1A

(ii) Let E be a point on BD such that $AE \perp BD$.

Let F be a point on CD such that $EF \perp BD$.

Required angle is $\angle AEF$. 1A

$AE = 19\sin 80° \approx 18.7\,\text{cm}$ 1M

$DE = 19\cos 80° \approx 3.30\,\text{cm}$

$\angle EDF = \angle ABD \approx 38.6°$

$EF = DE \tan EDF \approx 2.63\,\text{cm}$

$DF = \dfrac{DE}{\cos \angle EDF} \approx 4.22\,\text{cm}$

$AF^2 = DF^2 + 19^2 - 2(DF)(19)\cos \angle CDA$ 1M

$AF \approx 17.0\,\text{cm}$

In $\triangle AEF$,

$$AF^2 = AE^2 + EF^2 - 2(AE)(EF)\cos \angle AEF$$

$\angle AEF \approx 46.6°$ 1A

Required angle is $46.6°$.

4. (a) $AD = 20\cos 50° \approx 12.9\,\text{cm}$ 1A

$DB = \sqrt{30^2 - (20\sin 50°)^2} \approx 25.8\,\text{cm}$ 1A

(b) (i) $AB^2 = 20^2 + 30^2 - 2(20)(30)\cos 45°$ 1M

$AB \approx 21.2\,\text{cm}$ 1A

(ii) Required angle is $\angle ADB$.

$$AB^2 = AD^2 + DB^2 - 2(AD)(DB)\cos \angle ADB$$ 1M

$\angle ADB \approx 55.1°$ 1A

(iii) $CD = 20\sin 50° \approx 15.3\,\text{cm}$

Volume of tetrahedron $= \dfrac{1}{3} \times \dfrac{1}{2}(AD)(DB)\sin \angle ADB(CD)$ 1M

$\approx 695\,\text{cm}^3 > 650\,\text{cm}^3$

The claim is disagreed. 1A

5. (a) $\dfrac{12\sqrt{3}}{\sin 120^\circ} = \dfrac{12}{\sin \angle ABC}$ 1M

$\angle ABC = 30^\circ$ or 150° (rejected) 1A

$\angle ACB = 180^\circ - 30^\circ - 120^\circ = 30^\circ$.

$AB^2 = 12^2 + (12\sqrt{3})^2 - 2(12)(12\sqrt{3})\cos 30^\circ$

$AB = 12\,\text{cm}$ 1A

(b) Let M be the mid-point of BC. The angle required is $\angle VMA$. 1A

$VM = \sqrt{15^2 - (6\sqrt{3})^2} = 3\sqrt{13}\,\text{cm}$

$AM = \sqrt{12^2 - (6\sqrt{3})^2} = 6\,\text{cm}$

$15^2 = 6^2 + (3\sqrt{13})^2 - 2(6)(3\sqrt{13})\cos \angle VMA$ 1M

$\angle VMA \approx 124^\circ$ 1A

(c) (i) $QR = \sqrt{12^2 + 9^2} = 15\,\text{cm}$ 1A

Consider the circumcircle PQR. Since $\angle QPR = 90^\circ$, QR is a diameter and M is the centre. 1M

Thus, $PM = \dfrac{QR}{2} = 7.5\,\text{cm}$ 1A

(ii) $\triangle QRV$ is equilateral. $VM = VR\sin 60^\circ = \dfrac{15\sqrt{3}}{2}$ cm. 1A

$15^2 = VM^2 + (7.5)^2 - 2(VM)(7.5)\cos \angle PMV$ 1M

$\angle PMV = 90^\circ$

Since $VM \perp QR$ and $VM \perp PM$, VM is perpendicular to the plane PQR.

The claim is agreed. 1A

6. (a) $\angle BDC = \dfrac{360° - 150°}{2} = 105°$ 1A

$50^2 = 32^2 + BD^2 - 2(32)(BD)\cos 105°$ 1M

$0 = BD^2 - 64(BD)\cos 105° - 1476$

$BD \approx 31.0\text{ cm}$ or -47.6 cm (rejected) 1A

(b) (i) $AC^2 = 32^2 + 32^2 - 2(32)^2\cos 40°$ 1M

$AC \approx 21.9\text{ cm}$

Required distance is 21.9 cm. 1A

(ii) Let E be the mid-point of AC.

Required angle is $\angle BED$. 1A

$DE = 32\cos\dfrac{40°}{2} \approx 30.1\text{ cm}$ 1M

$BE = \sqrt{50^2 - \left(\dfrac{AC}{2}\right)^2} \approx 48.8\text{ cm}$

$BD^2 = DE^2 + BE^2 - 2(DE)(BE)\cos\angle BED$

$\angle BED \approx 37.7°$ 1A

(iii) Area of $\triangle ACD = \dfrac{1}{2}(32)(32)\sin 40°$ 1M

$= 512\sin 40°\text{ cm}^2$

Volume $= \dfrac{1}{3}(512\sin 40°)(BE\sin\angle BED)$ 1M

$\approx 3270\text{ cm}^3$ 1A

Alternative method:

Volume $= \dfrac{1}{3}\left(\dfrac{1}{2}(AC)(BE)\right)(DE\sin\angle BED)$ 1M+1M

$\approx 3270\text{ cm}^3$ 1A

(c) Let D' and P' be the projections of D and P on plane ABC respectively.

Since $\triangle BDD' \sim \triangle BPP'$, we have $\dfrac{PP'}{DD'} = \dfrac{BP}{BD} = \dfrac{1}{2}$. 1M

$$\frac{\text{volume of } ABCP}{\text{volume of } ABCD} = \frac{\frac{1}{3}(\text{area of } \triangle ABC)(PP')}{\frac{1}{3}(\text{area of } \triangle ABC)(DD')} = \frac{PP'}{DD'} = \frac{1}{2}$$

The claim is agreed. 1A

7. (a) $BD^2 = 20^2 + 25^2 - 2(20)(25)\cos 120°$ 1M

$BD = 5\sqrt{61}$ cm

$20^2 = 25^2 + BD^2 - 2(25)(BD)\cos\angle BDC$

$\angle BDC \approx 26.3°$ 1A

$\frac{AD}{BD} = \frac{BD}{25}$ 1M

$AD = 61$ cm 1A

(b) (i) Let E be a point on DC produced such that $BE \perp CD$.

Let F be a point on AD such that $EF \perp CD$. Required angle is $\angle BEF$. 1A

$BE = 20\sin(180° - 120°) = 10\sqrt{3}$ cm

$\frac{25}{\sin\angle DAC} = \frac{61}{\sin 140°}$ 1M

$\angle DAC \approx 15.3°$

$\angle ADC = 180° - 140° - \angle DAC \approx 24.7°$

$DE = CD + 20\cos(180° - 120°) = 35$ cm

$EF = DE\tan\angle ADC \approx 16.1$ cm 1A

$DF = \frac{DE}{\cos\angle ADC} \approx 38.5$ cm

$BF^2 = BD^2 + DF^2 - 2(BD)(DF)\cos\angle BDA$

$= BD^2 + DF^2 - 2(BD)(DF)\cos\angle BDC$

$BF \approx 17.7$ cm

$BF^2 = BE^2 + EF^2 - 2(BE)(EF)\cos\angle BEF$

$\angle BEF \approx 63.7°$ 1A

Required angle is $63.7°$.

(ii) Let G be a point on the ground such that BG is perpendicular to the plane ACD.

Required angle is $\angle BDG$. 1A

$BG = BE\sin\angle BEF \approx 15.5$ cm 1A

$\sin\angle BDG = \frac{BG}{BD}$

$\angle BDG \approx 23.4°$ 1A

Required angle is $23.4°$.

(iii) Let the required distance be h cm.

Area of $\triangle BCD = \frac{1}{2}(20)(25)(\sin 120°) = 125\sqrt{3}\ \text{cm}^2$.

Area of $\triangle ACD = \frac{1}{2}(AD)(25)(\sin\angle ADC) \approx 319\ \text{cm}^2$

By considering the volume of the tetrahedron $ABCD$,

$\frac{1}{3}(125\sqrt{3})(h) = \frac{1}{3}(\text{area of } \triangle ACD)(BG)$ 1M

$h \approx 22.9\ \text{cm} < 23\ \text{cm}$

The claim is disagreed. 1A

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