

REG-AOT-2324-ASM-SET 4-MATH**Suggested solutions****Conventional Questions**

1. (a) In
- $\triangle VBC$
- ,

$$VD = 1.2 \sin 60^\circ$$

$$= \frac{3\sqrt{3}}{5} \text{ m}$$

In $\triangle ABC$,

$$AD = 1.2 \sin 60^\circ$$

$$= \frac{3\sqrt{3}}{5} \text{ m}$$

In $\triangle VAD$,

$$1.2^2 = \left(\frac{3\sqrt{3}}{5}\right)^2 + \left(\frac{3\sqrt{3}}{5}\right)^2 - 2\left(\frac{3\sqrt{3}}{5}\right)^2 \cos \angle VDA \quad 1\text{M}$$

$$\angle VDA \approx 70.52877937^\circ$$

In $\triangle VGD$,

$$VG = \frac{3\sqrt{3}}{5} \sin \angle VDA \quad 1\text{M}$$

$$\approx 0.980 \text{ m} \quad 1\text{A}$$

- (b) Required angle is also equal to the angle between VBC and ABC , i.e., $\angle VDA$. 1M+1A
 Required angle is 70.5° . 1A

2. (a) $\frac{\sin \angle AVB}{22} = \frac{\sin 115^\circ}{36} \quad 1\text{M}$

$$\angle AVB \approx 33.6^\circ$$

$$\angle VBA = 180^\circ - 115^\circ - \angle AVB \approx 31.4^\circ \quad 1\text{A}$$

- (b) In
- $\triangle VAB$
- ,

$$VA^2 = 22^2 + 36^2 - 2(22)(36) \cos \angle VBA \quad 1\text{M}$$

$$VA \approx 20.7 \text{ cm}$$

$$MP = NQ = \frac{1}{2}VA \approx 10.3 \text{ cm} \quad 1\text{M}$$

$$PQ = MN = \frac{BC}{2} = 7 \text{ cm}$$

Since the solid is symmetric, $PQNM$ is a rectangle.

$$\text{Required area} = (PQ)(MP) \quad 1\text{M}$$

$$\approx 72.4 \text{ cm}^2 \quad 1\text{A}$$

3. (a) $\frac{AB}{\sin 65^\circ} = \frac{15}{\sin 58^\circ}$ 1M
 $AB \approx 16.0 \text{ cm}$ 1A
 $AC^2 = AB^2 + 17^2 - 2(AB)(17) \cos 116^\circ$ 1M
 $AC \approx 28.0 \text{ cm}$ 1A
- (b) $\angle BAD = 180^\circ - 58^\circ - 65^\circ = 57^\circ$
 $AK = AB \cos 57^\circ \approx 8.73 \text{ cm}$ 1M
 $27^2 = AC^2 + 15^2 - 2(AC)(15) \cos \angle CAD$
 $\angle CAD \approx 70.5^\circ$
 $AC \cos \angle CAD \approx 9.36 \text{ cm} \neq AK$
 So, CK is not perpendicular to AD . 1M
 The claim is disagreed. 1A
4. (a) The paper card is symmetric about AC .
 So, $\angle BAC = 60^\circ$ and $\angle ACB = 180^\circ - 60^\circ - 70^\circ = 50^\circ$. 1A
- $\frac{AC}{\sin 70^\circ} = \frac{10}{\sin 50^\circ}$ 1M
 $AC \approx 12.3 \text{ cm}$ 1A
- $BC = \frac{10 \sin 60^\circ}{\sin 50^\circ} \approx 11.3 \text{ cm}$ 1A
- (b) (i) $AE = \sqrt{AB^2 - BE^2}$ and $CE^2 = \sqrt{BC^2 - BE^2}$
 $AC^2 = AE^2 + CE^2$
 $= BC^2 + AB^2 - 2BE^2$ 1M
 $BE \approx 6.22 \text{ cm}$ 1A
- (ii) Let h cm be the required distance.
 $AE = \sqrt{10^2 - BE^2} \approx 7.83 \text{ cm}$ and $CE = \sqrt{BC^2 - BE^2} \approx 9.4 \text{ cm}$
 $\frac{1}{3} \times \frac{(AE)(DB)}{2} \times CE = \frac{1}{3} \times \frac{(AD)(AC) \sin 60^\circ}{2} \times h$ 1M+1M
 $h \approx 8.66$ 1A
- Required distance is 8.66 cm.

5. (a) (i) $QR^2 = 25^2 + 30^2 - 2(25)(30) \cos 95^\circ$ 1M
 $QR \approx 40.7 \text{ cm}$ 1A
(ii) $25^2 = 30^2 + QR^2 - 2(30)(QR) \cos \angle PQR$ 1M
 $\angle PQR \approx 37.7^\circ$ 1A
(b) Let R' and M' be the feet of perpendicular of R and M on the horizontal ground respectively.
 $RR' = RP \sin 70^\circ \approx 23.5 \text{ cm}$
Since $\triangle QMM' \sim \triangle QRR'$, $MM' = \frac{RR'}{2} \approx 11.7 \text{ cm}$ 1M
 $PM^2 = 30^2 + \left(\frac{QR}{2}\right)^2 - 2(30)\left(\frac{QR}{2}\right) \cos \angle PQR$
 $PM \approx 18.7 \text{ cm}$
Angle between PM and the horizontal ground is $\angle MPM'$.
 $\sin \angle MPM' = \frac{MM'}{PM}$ 1M
 $\angle MPM' \approx 39.0^\circ < 40^\circ$
The claim is not correct. 1A

6. (a) Let M be the mid-point of AC .
 $BM = 20 \sin 60^\circ = 10\sqrt{3} \text{ cm}$
 $BE = BM \sin 60^\circ = 15 \text{ cm}$ 1A
 $\angle BEC = 90^\circ$. So, BC is a diameter of the circumcircle of $\triangle BCE$. 1M
Thus, $DE = DB = DC = \frac{20}{2} = 10 \text{ cm}$ 1A
(b) $AE = \sqrt{AB^2 - BE^2}$ 1M
 $= \sqrt{175} \text{ cm}$
 $AD = BM = 10\sqrt{3} \text{ cm}$
 $AE^2 = AD^2 + DE^2 - 2(AD)(DE) \cos \angle ADE$ 1M
 $\angle ADE \approx 49.5^\circ \neq 90^\circ$
The claim is disagreed. 1A

7. (a) (i) $17^2 = 23^2 + 10^2 - 2(23)(10) \cos \angle CAB$ 1M
 $\angle CAB \approx 42.3^\circ$ 1A
- (ii) $\frac{AF}{\sin(180^\circ - 58^\circ - \angle CAB)} = \frac{10}{\sin 58^\circ}$ 1M
 $AF \approx 11.6 \text{ cm}$ 1A
- (b) (i) $BF = 23 - AF \approx 11.4 \text{ cm}$
 $AB^2 = AF^2 + BF^2 - 2(AF)(BF) \cos 75^\circ$ 1M
 $AB \approx 14.0 \text{ cm}$ 1A
- (ii) $AP = AF \cos \angle FAC \approx 8.57 \text{ cm}$ 1M
 $17^2 = 10^2 + (AB)^2 - 2(10)(AB) \cos \angle BAC$
 $\angle BAC \approx 88.6^\circ$
 $BP^2 = AP^2 + AB^2 - 2(AP)(AB) \cos \angle BAC$
 $BP \approx 16.2 \text{ cm}$ 1M
 $AB^2 = AP^2 + BP^2 - 2(AP)(BP) \cos \angle APB$
 $\angle APB \approx 59.6^\circ \neq 90^\circ$
The claim is disagreed. 1A