

REG-AOT-2324-ASM-SET 3-MATH

Suggested solutions

Multiple Choice Questions

- | | | | | |
|-------|-------|-------|-------|-------|
| 1. C | 2. D | 3. B | 4. C | 5. D |
| 6. D | 7. A | 8. B | 9. B | 10. D |
| 11. A | 12. B | 13. C | 14. B | 15. C |
| 16. B | 17. D | 18. C | 19. B | 20. D |
| 21. C | 22. B | 23. D | 24. C | |

1. C

Let the length of cube be 2. M and N be mid-points of BD and PQ respectively.

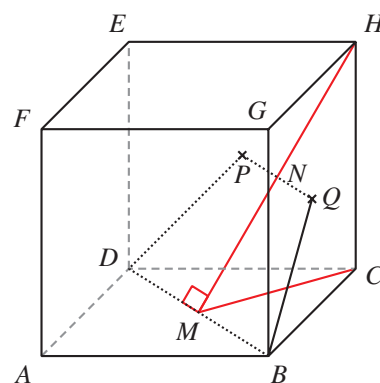
Note that since D, P, H are collinear and B, Q, H are collinear, we have M, N and H are collinear.

Note that $MH \perp BD$ and $CM \perp BD$.

The angle required is $\angle CMH$.

$$CM = \frac{1}{2}\sqrt{2^2 + 2^2} = \sqrt{2}.$$

$$\angle CMH = \tan^{-1} \frac{2}{\sqrt{2}} \approx 55^\circ.$$



2. D

Let E be a point on AC such that $VE \perp AC$.

Required angle is $\angle VED$.

Since the pyramid is symmetric about the plane VAC ,

$$\text{we have } \angle VEB = \angle VED = \frac{180^\circ}{2} = 90^\circ.$$

3. B

Let M be a point on BC such that $AM \perp BC$.

Note that AMD is a plane of reflection of the tetrahedron and it is perpendicular to the plane BCD .

We have $\angle AMD = 80^\circ$ and the required angle is $\angle ADM$.

Consider $\triangle ABC$.

$$AM = 56 \sin 60^\circ = 28\sqrt{3} \text{ cm}$$

Consider $\triangle BCD$.

$$DM = \sqrt{60^2 - 28^2} = \sqrt{2816} \text{ cm}$$

Consider $\triangle AMD$.

$$AD^2 = AM^2 + DM^2 - 2(AM)(DM) \cos 80^\circ$$

$$AD \approx 65.4 \text{ cm}$$

$$AM^2 = AD^2 + DM^2 - 2(AD)(DM) \cos \angle ADM$$

$$\angle ADM \approx 47^\circ$$

4. C

Let M and N be mid-point of AB and CD respectively.

Let X be the mid-point of MN such that VX is perpendicular to the plane $ABCD$.

We have $\theta = \angle MVN$ and $\frac{\theta}{2} = \angle VNX$.

$$DX = \frac{1}{2} \sqrt{4^2 + 2^2} = \sqrt{5} \text{ cm}$$

$$VX = \sqrt{7^2 - DX^2} = 2\sqrt{11} \text{ cm}$$

$$\begin{aligned} \tan \frac{\theta}{2} &= \frac{MX}{VX} \\ &= \frac{2}{2\sqrt{11}} \\ &= \frac{\sqrt{11}}{11} \end{aligned}$$

5. D

Let E be a point on CD such that $BE \perp CD$. Then $\theta = \angle AEB$.

$CD = \sqrt{24^2 + 7^2} = 25 \text{ cm}$. By considering the area of $\triangle BCD$,

$$\frac{(25)(BE)}{2} = \frac{(24)(7)}{2}$$

$$BE = \frac{168}{25}$$

$$\tan \theta = \frac{7}{BE} = \frac{25}{24}$$

6. D

Let D be a point on BC such that $PD \perp BC$.

Required angle is $\angle ADP$.

$$\frac{BC = \sqrt{4^2 + 3^2} = 5 \text{ m}}{\frac{(PD)(BC)}{2}} = \frac{(PB)(PC)}{2}$$

$$PD = 2.4 \text{ m}$$

$$\begin{aligned} \tan \angle ADP &= \frac{5}{2.4} \\ &= \frac{25}{12} \end{aligned}$$

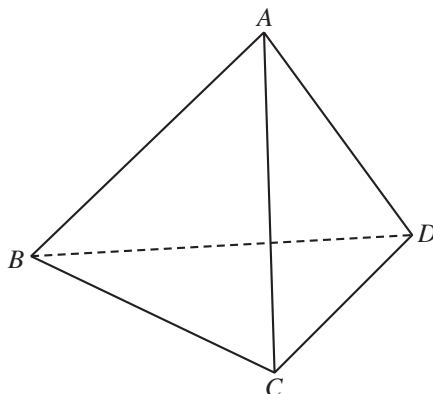
7. A

Note that $\alpha = \angle GFH = 45^\circ$, $\beta = 90^\circ$.

Thus, $\alpha < 60^\circ < \beta$.

8. B

Refer to the figure. Let E be the mid-point of BC and length of each side be x cm.



In $\triangle AED$, $AE = DE = x \sin 60^\circ = \frac{\sqrt{3}x}{2}$ cm.

$$x^2 = AE^2 + DE^2 - 2(AE)(DE) \cos \angle AED$$

$$\angle AED = \cos^{-1} \frac{1}{3}$$

Consider the volume of the tetrahedron.

$$\frac{1}{3} \left(\frac{x^2}{2} \sin 60^\circ \right) (AE \sin \angle AED) = 576$$

$$x \approx 17.0$$

Let X be the projection of A on the plane BCD .

Note that D, X, E are collinear.

Consider $\triangle AXE$.

Required height = $AE \sin \angle AED$

$$\approx 13.9 \text{ cm}$$

9. B

Let M and N be mid-points of BC and AD respectively.

Required angle is $\angle MVN$.

$$MV = NV = \sqrt{8^2 - \left(\frac{4}{2}\right)^2} = \sqrt{60} \text{ cm}$$

$$6^2 = MV^2 + NV^2 - 2(MV)(NV) \cos \angle MVN$$

$$\angle MVN \approx 46^\circ$$

10. D

Let K be a point on EG such that $AK \perp EG$.

Required angle is $\angle AKF$.

Consider the area of $\triangle EFG$.

$$\frac{1}{2}(8)(6) = \frac{1}{2}(EG)(FK)$$

$$24 = \frac{1}{2}\sqrt{8^2 + 6^2}(EK)$$

$$EK = 4.8 \text{ cm}$$

$$\tan \theta = \frac{12}{4.8}$$

$$\theta \approx 68^\circ$$

11. A

Let M and N be mid-points of BD and AD respectively.

Required angle is $\angle MVN$.

$$MV = NV = \sqrt{15^2 - 4^2} = \sqrt{209} \text{ cm}$$

$$MN = 6 \text{ cm}$$

$$MN^2 = MV^2 + NV^2 - 2(MV)(NV) \cos \angle MVN$$

$$\angle MVN \approx 24^\circ$$

12. B

Required angle is $\angle CBH$ (or $\angle DAE$).

$$\angle CBH = 45^\circ$$

13. C

$$15^2 = 9^2 + 12^2 \Rightarrow \angle BDC = 90^\circ.$$

Since $\triangle ABD$ is vertical and CD is horizontal, $AD \perp CD$.

Therefore, $\theta = \angle ADB = \tan^{-1} \frac{12}{9}$ and so $\sin \theta = \frac{4}{5}$.

14. B

Let $BC = 1$.

$$AE = BF = 1 \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$AC = \frac{AE}{\sin 30^\circ} = \sqrt{3}$$

$$\cos \angle ACB = \frac{BC}{AC} = \frac{1}{\sqrt{3}}$$

15. C

Let K be a point on AC such that $DK \perp AC$.

We have $x = \angle DKE$.

Consider the area of $\triangle ACD$.

$$\begin{aligned} \frac{(AD)(CD)}{2} &= \frac{(AC)(DK)}{2} \\ \frac{(3)(4)}{2} &= \frac{\sqrt{3^2 + 4^2}(DK)}{2} \\ DK &= 2.4 \text{ cm} \end{aligned}$$

Consider $\triangle DKE$.

$$\begin{aligned} \tan x &= \frac{DE}{DK} \\ &= \frac{2}{2.4} \\ &= \frac{5}{6} \end{aligned}$$

16. B

Note that $\angle BCT = \angle BCA = 90^\circ$.

We have BC is perpendicular to the plane ACT .

Let D be a point on AT such that $CD \perp AT$.

Then $\theta = \angle BDC$.

Consider the area of $\triangle ACT$.

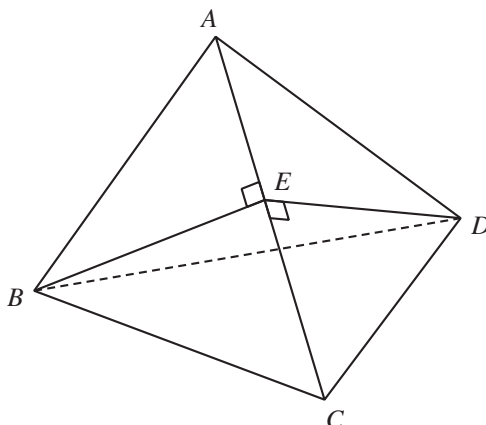
$$\begin{aligned} \frac{(40)(30)}{2} &= \frac{\sqrt{40^2 + 30^2}(CD)}{2} \\ CD &= 24 \text{ cm} \end{aligned}$$

Consider $\triangle BCD$.

$$\begin{aligned} \tan \theta &= \frac{20}{24} \\ &= \frac{5}{6} \end{aligned}$$

17. D

Since $BE \perp AC$ and $DE \perp AC$, required angle is $\angle BED$.



18. C

Let length of each edge be 2. Let K be a point on BV such that $AK \perp BV$ and $CK \perp BV$.

$$AK = CK = 2 \sin 60^\circ = \sqrt{3} \text{ and } AC = \sqrt{2^2 + 2^2} = 2\sqrt{2}$$

In $\triangle AKC$, $\angle AKC$ is the required angle.

$$(2\sqrt{2})^2 = (\sqrt{3})^2 + (\sqrt{3})^2 - 2(\sqrt{3})^2 \cos \angle AKC$$

$$\angle AKC \approx 109^\circ$$

19. B

Let F be a point on AC such that $BF \perp AC$.

Required angle is $\angle BFD$.

Let $BC = 1$ cm.

Note that $\triangle ABC$ and $\triangle ACD$ are equilateral.

$$DF = BF = 1 \sin 60^\circ = \frac{\sqrt{3}}{2} \text{ cm}$$

$$BD = \sqrt{1^2 + 1^2} = \sqrt{2} \text{ cm}$$

$$BD^2 = DF^2 + BF^2 - 2(DF)(BF) \cos \angle BFD$$

$$\angle BFD \approx 109^\circ$$

20. D

Let E be a point on CD such that $BE \perp CD$. Then $AE \perp CD$ and $\theta = \angle AEB$.

$$\angle BCD = \tan^{-1} \frac{15}{8}, \text{ and } BE = BC \sin \angle BCD = \frac{120}{17} \text{ m}$$

$$\tan \theta = \frac{8}{BE} = \frac{17}{15}$$

21. C

Let K be a point on AM such that $DK \perp AM$.

Then $\alpha = \angle EMD$ and $\beta = \angle EKD$.

I. ✓. Since $\angle DKM = 90^\circ$, we have $DK < DM$ and $\alpha < \beta$.

II. ✗. Since $\alpha < \beta$, we have $\cos \alpha > \cos \beta$.

III. ✓. $\angle DAM = \angle BMA = \tan^{-1} \frac{5}{2.5} \approx 63.4^\circ$
 $DK = AD \sin \angle DAM \approx 4.47 \text{ cm}$
 $\tan \beta = \frac{DE}{DK} = \frac{4}{\sqrt{5}}$

22. B

Refer to the figure. Let E be the mid-point of BC and length of each side be 12 cm.

In $\triangle AED$, $AE = DE = 12 \sin 60^\circ = 6\sqrt{3} \text{ cm}$

$$12^2 = AE^2 + DE^2 - 2(AE)(DE) \cos \angle AED$$

$$\angle AED = \cos^{-1} \frac{1}{3}$$

Let X be the projection of A on the plane BCD . Then

X is the centroid of $\triangle BCD$ and it lies on DE .

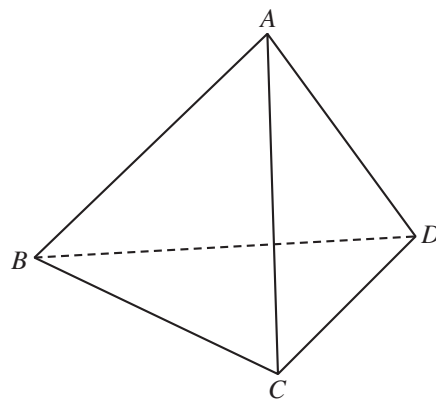
In $\triangle AEX$, $\angle AXE = 90^\circ$ and

height = $AE \sin \angle AED$

$$= \sqrt{96} \text{ cm}$$

$$\text{Volume} = \frac{1}{2} (12)^2 \sin 60^\circ (\sqrt{96}) \times \frac{1}{3}$$

$$= 144\sqrt{2} \text{ cm}^3$$



23. D

Refer to the figure. Let E be the mid-point of BC and length of each side be x cm.

In $\triangle AED$, $AE = DE = x \sin 60^\circ = \frac{\sqrt{3}x}{2} \text{ cm}$

$$x^2 = AE^2 + DE^2 - 2(AE)(DE) \cos \angle AED$$

$$\angle AED = \cos^{-1} \frac{1}{3}$$

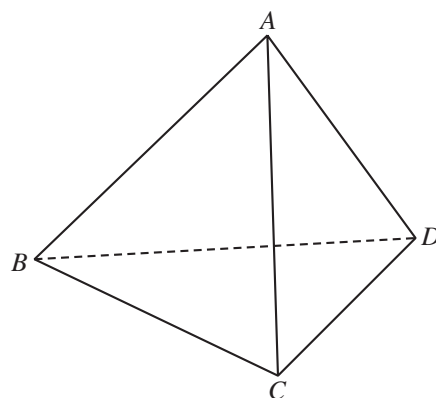
Let X be the projection of A on the plane BCD . Then

X lies on DE and $AX \perp BD$.

In $\triangle AEX$,

$$AE \sin \angle AED = 4$$

$$x = 2\sqrt{6}$$



24. C

Let K be a point on VB such that $AK \perp VB$. We have $CK \perp VB$ also.

Required angle is $\angle AKC$.

$$AC = \sqrt{12^2 + 12^2} = 12\sqrt{2} \text{ cm}$$

$$VA = VB = VC = \sqrt{8^2 + \left(\frac{12\sqrt{2}}{2}\right)^2} = \sqrt{136} \text{ cm}$$

In $\triangle VAB$,

$$\cos \angle ABV = \frac{\left(\frac{12}{2}\right)}{VB}$$

$$\angle ABV \approx 59.0^\circ$$

$$AK = CK = 12 \sin \angle ABV \approx 10.3 \text{ cm}$$

In $\triangle AKC$,

$$AC^2 = AK^2 + CK^2 - 2(AK)(CK) \cos \angle AKC$$

$$\angle AKC \approx 111^\circ$$

Conventional Questions

25. (a) In $\triangle ABC$,

$$AC^2 = 9^2 + 15^2 - 2(9)(15) \cos 120^\circ \quad 1M$$

$$AC = 21 \text{ cm} \quad 1A$$

In $\triangle ADC$, $\angle ADC = 180^\circ - 120^\circ = 60^\circ$

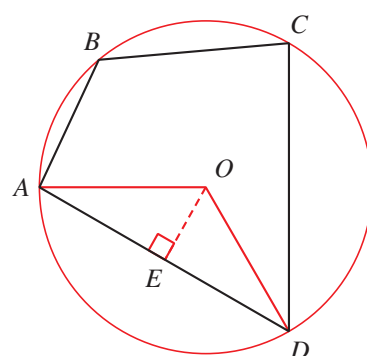
Since $AD = CD$, $\angle DAC = \angle ACD = \frac{180^\circ - 60^\circ}{2} = 60^\circ$ and hence $\triangle ACD$ is equilateral. 1M

So, $AD = AC = 21 \text{ cm}$. 1A

(b) Centre of the required circle is at the circumcentre O of $\triangle ACD$. Let E be the mid-point of AD .

$$\angle OAE = \frac{60^\circ}{2} = 30^\circ.$$

$$\begin{aligned} \text{Radius} = OA &= \frac{AE}{\cos 30^\circ} \\ &= \frac{21}{2} \div \cos 30^\circ \\ &= 7\sqrt{3} \text{ cm} \end{aligned}$$



(c) Required angle is $\angle OEV$. 1A

$$OE = OA \sin 30^\circ = \frac{7\sqrt{3}}{2} \text{ cm} \quad 1M$$

$$\tan \angle OEV = \frac{15}{OA} \quad 1M$$

$$\angle OEV \approx 68.0^\circ \quad 1A$$

Required angle is 68.0° .

26. (a) $AC^2 = 8^2 + 12^2 - 2(8)(12) \cos 100^\circ$ 1M

$AC \approx 15.5 \text{ cm}$ 1A

(b) (i) Let M be the mid-point of BD .

$$DM = 3 \text{ cm}$$

In $\triangle AMD$,

$$\begin{aligned} AM &= \sqrt{8^2 - 3^2} \\ &= \sqrt{55} \text{ cm} \end{aligned}$$

1M

In $\triangle CMD$,

$$\begin{aligned} CM &= \sqrt{12^2 - 3^2} \\ &= \sqrt{135} \text{ cm} \end{aligned}$$

In $\triangle ACM$,

$$(\sqrt{55})^2 = AC^2 + (\sqrt{135})^2 - 2(AC)(\sqrt{135}) \cos \angle ACM$$

1M

$$\angle ACM \approx 27.1^\circ$$

Required angle is 27.1° .

1A

(ii) Let Q be the foot of perpendicular from D to AC .

$$\text{Since } \sin \frac{\angle DPB}{2} = \frac{DM}{PD},$$

$\angle DPB$ attains maximum when PD is minimum, i.e., $P = Q$.

1A

When P moves from A to Q , $\angle DPB$ increases.

When P moves from Q to C , $\angle DPB$ decreases.

1A

27. (a) (i) In $\triangle ABC$,

$$28^2 = 35^2 + 21^2 - 2(35)(21) \cos \angle BCM$$

$$\angle BCM \approx 53.1^\circ$$

1A

$$(ii) \frac{CM}{\sin(180^\circ - 75^\circ - \angle BCM)} = \frac{21}{\sin 75^\circ}$$

1M

$$CM \approx 17.1 \text{ cm}$$

1A

(b) (i) In $\triangle ACM$, $AM = 35 - CM \approx 17.9 \text{ cm}$, and

$$AC^2 = AM^2 + CM^2 - 2(AM)(CM) \cos \angle AMC$$

1M

$$AC \approx 28.1 \text{ cm}$$

1A

(ii) In $\triangle CMN$, $CN = CM \cos \angle BCM \approx 10.3 \text{ cm}$

1M

In $\triangle ABC$,

$$28^2 = 21^2 + AC^2 - 2(AC)(21) \cos \angle ACB$$

$$\angle ACB \approx 67.7^\circ$$

In $\triangle ACN$,

$$AN^2 = CN^2 + AC^2 - 2(CN)(AC) \cos \angle ACB$$

$$AN \approx 26.0 \text{ cm}$$

$$AC^2 = CN^2 + AN^2 - 2(AN)(CN) \cos \angle ANC$$

$$\angle ANC \approx 89.1^\circ \neq 90^\circ$$

So, $\angle ANC$ is not a right angle.

1M

Therefore, $\angle AMN$ is not the angle between the face BCM and the horizontal ground.

The claim is disagreed.

1A

28. (a) (i) $CM = \sqrt{10^2 + 10^2}$
 $= 10\sqrt{2} \text{ cm}$ 1A
- (ii) $\tan \angle ACB = \frac{20}{10}$
 $\angle ACB \approx 63.4^\circ$ 1A
- (iii) Area of $\triangle ACM = \frac{1}{2}(20 - 10)(10)$
 $= 50 \text{ cm}^2$ 1A
- (b) (i) In $\triangle CDM$, $\angle DCM = \angle ACB - 45^\circ \approx 18.4^\circ$ 1A
- $DM^2 = CD^2 + (10\sqrt{2})^2 - 2(CD)(10\sqrt{2}) \cos \angle DCM$ 1M
- $CD \approx 7.45 \text{ cm}$ 1A
- In $\triangle BCD$, $BD = \sqrt{10^2 - CD^2} \approx 6.67 \text{ cm}$ 1A
- (ii) Let N be the foot of perpendicular from B to CM .
 Required angle is $\angle BND$.
 In $\triangle BNC$, $BN = 10 \sin 45^\circ = 5\sqrt{2} \text{ cm}$
 In $\triangle BND$,
 $\sin \angle BND = \frac{BD}{5\sqrt{2}}$ 1M
- $\angle BND \approx 70.5^\circ$ 1A
- Required angle is 70.5° .