M2REG-TRIG-2324-ASM-SET 6-MATH
Suggested solutions

1. θ lies in quadrant II or quadrant III.

   When θ lies in quadrant II,

   $\csc\theta = \dfrac{23}{\sqrt{23^2 - 5^2}}$ 1M

   $= \dfrac{23}{6\sqrt{14}}$

   $\cot\theta = -\dfrac{5}{6\sqrt{14}}$

   $\csc\theta - \cot\theta = \dfrac{23}{6\sqrt{14}} - \left(-\dfrac{5}{6\sqrt{14}}\right)$

   $= \dfrac{\sqrt{14}}{3}$ 1A

   When θ lies in quadrant III,

   $\csc\theta = -\dfrac{23}{\sqrt{23^2 - 5^2}}$

   $= -\dfrac{23}{6\sqrt{14}}$

   $\cot\theta = \dfrac{5}{6\sqrt{14}}$

   $\csc\theta - \cot\theta = -\dfrac{23}{6\sqrt{14}} - \dfrac{5}{6\sqrt{14}}$

   $= -\dfrac{\sqrt{14}}{3}$ 1A

   Thus, $\csc\theta - \cot\theta = -\dfrac{\sqrt{14}}{3}$ or $\dfrac{\sqrt{14}}{3}$.

2. $\dfrac{(1 - \cot\theta)(1 + \cot\theta)}{2}$

   $= \dfrac{(1 - \cot^2\theta) - 2}{2}$ 1M

   $= \dfrac{-(\cot^2\theta + 1)}{2}$

   $= \dfrac{-\csc^2\theta}{2}$ 1M

   $= -\dfrac{1}{2\sin^2\theta}$

   $= -\dfrac{1}{2a^2}$ 1A

3. (a) $(\csc x+\cot x)(\csc x-\cot x)=\csc^2 x-\cot^2 x$

$\frac{3}{2}(\csc x-\cot x)=1$ 1M

$\csc x-\cot x=\frac{2}{3}$ 1

(b) $(\csc x+\cot x)+(\csc x-\cot x)=\frac{3}{2}+\frac{2}{3}$ 1M

$\csc x=\frac{13}{12}$ 1A

$\cot x=\frac{3}{2}-\frac{13}{12}=\frac{5}{12}$ 1A

4. $\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$ 1M

$\frac{5}{7}=\frac{\tan A+\tan B}{1-\left(-\frac{1}{6}\right)}$

$\tan A+\tan B=\frac{5}{6}$ 1M

Consider the sum and product of roots,

$\tan A$ and $\tan B$ are the roots of the equation $6x^2-5x-1=0$. 1M+1

5. (a) $\tan\left(2\cdot\frac{\pi}{8}\right)=\frac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}}$ 1M

$1=\frac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}}$

$1-\tan^2\frac{\pi}{8}=2\tan\frac{\pi}{8}$

$\tan^2\frac{\pi}{8}+2\tan\frac{\pi}{8}-1=0$

Thus, $\tan\frac{\pi}{8}$ is a root of the equation $x^2+2x-1=0$. 1

(b) $x^2+2x-1=0$

$x=\frac{-2\pm\sqrt{2^2-4(1)(-1)}}{2(1)}$ 1M

$=-1\pm\sqrt{2}$

Note that $\tan\frac{\pi}{8}>0>-1-\sqrt{2}$. 1M

Thus, $\tan\frac{\pi}{8}=\sqrt{2}-1$. 1A

6.

$$\sec 4A = 4$$

$$\cos 4A = \frac{1}{4}$$

$$2\cos^2 2A - 1 = \frac{1}{4} \qquad \text{1M}$$

$$\cos^2 2A = \frac{5}{8}$$

$$\sin^2 2A = 1 - \frac{5}{8} \qquad \text{1M}$$

$$= \frac{3}{8}$$

$$\cos^4 A + \sin^4 A = (\cos^2 A + \sin^2 A)^2 - 2\sin^2 A\cos^2 A \qquad \text{1M}$$

$$= 1^2 - 2\left(\frac{\sin 2A}{2}\right)^2$$

$$= 1 - \frac{1}{2}\sin^2 2A$$

$$= 1 - \frac{1}{2}\left(\frac{3}{8}\right)$$

$$= \frac{13}{16} \qquad \text{1A}$$

7. (a) When $n = 1$,

L.H.S. $= \cos x$

R.H.S. $= \dfrac{\sin 2x}{2\sin x} = \dfrac{2\sin x \cos x}{2\sin x} = \cos x =$ L.H.S.

It is true for $n = 1$. 1

Assume $\cos x \cos 2x \cos 4x \ldots \cos\left(2^{k-1}x\right) = \dfrac{\sin(2^k x)}{2^k \sin x}$, where $\sin x \neq 0$ and $k \in \mathbb{Z}^+$. 1M

$\cos x \cos 2x \cos 4x \ldots \cos\left(2^{k-1}x\right)\cos\left(2^k x\right)$

$= \dfrac{\sin(2^k x)}{2^k \sin x}\cos\left(2^k x\right)$ 1M

$= \dfrac{1}{2^k \sin x}\left(\dfrac{2\sin(2^k x)\cos(2^k x)}{2}\right)$

$= \dfrac{\sin(2^{k+1}x)}{2^{k+1}\sin x}$ 1

It is true for $n = k + 1$.

By M.I., it is true $\forall n \in \mathbb{Z}^+$. 1

(b) $\cos\dfrac{\pi}{3}\cos\dfrac{2\pi}{3}\cos\dfrac{4\pi}{3}\ldots\cos\dfrac{256\pi}{3}$

$= \cos\dfrac{\pi}{3}\cos\dfrac{2\pi}{3}\cos\dfrac{4\pi}{3}\ldots\cos\dfrac{2^{9-1}\pi}{3}$

$= \dfrac{\sin\frac{2^9\pi}{3}}{2^9\sin\frac{\pi}{3}}$ 1M

$= \dfrac{\sin\left(170\pi + \frac{2\pi}{3}\right)}{512\sin\frac{\pi}{3}}$

$= \dfrac{\sin\frac{2\pi}{3}}{512\sin\frac{\pi}{3}}$

$= \dfrac{1}{512}$ 1A

8. $\sin 3x + 2\sin 5x + \sin 7x = (\sin 3x + \sin 7x) + 2\sin 5x$

$= 2\sin 5x\cos 2x + 2\sin 5x$ 1M

$= 2\sin 5x(\cos 2x + 1)$

$= 2\sin 5x[(2\cos^2 x - 1) + 1]$ 1M

$= 4\cos^2 x\sin 5x$ 1

9. $\cos 4\theta\cos 2\theta - \sin 8\theta\sin 6\theta$

$= \dfrac{1}{2}(\cos 6\theta + \cos 2\theta) - \dfrac{1}{2}(\cos 2\theta - \cos 14\theta)$ 1M

$= \dfrac{1}{2}(\cos 6\theta + \cos 14\theta)$

$= \dfrac{1}{2}\cdot 2\cos\dfrac{6\theta + 14\theta}{2}\cos\dfrac{14\theta - 6\theta}{2}$ 1M

$= \cos 10\theta\cos 4\theta$ 1

10. (a) $\cos 3\theta \cos^3 \theta + \sin 3\theta \sin^3 \theta$

$= \cos 3\theta \cos\theta \cos^2 \theta + \sin 3\theta \sin\theta \sin^2 \theta$

$= \frac{1}{2}(\cos 4\theta + \cos 2\theta) \cdot \frac{1}{2}(1 + \cos 2\theta) + \frac{1}{2}(\cos 2\theta - \cos 4\theta) \cdot \frac{1}{2}(1 - \cos 2\theta)$ 1M

$= \frac{1}{4}[(\cos 4\theta + \cos 2\theta + \cos 4\theta \cos 2\theta + \cos^2 2\theta) - (\cos 4\theta - \cos 2\theta - \cos 4\theta \cos 2\theta + \cos^2 2\theta)]$

$= \frac{\cos 2\theta}{2}(1 + \cos 4\theta)$

$= \frac{\cos 2\theta}{2}\left[1 + (2\cos^2 2\theta - 1)\right]$ 1M

$= \cos^3 2\theta$ 1

(b) $\cos^3 \frac{2\pi}{12} = \cos \frac{3\pi}{12} \cos^3 \frac{\pi}{12} + \sin \frac{3\pi}{12} \sin^3 \frac{\pi}{12}$ 1M

$\left(\frac{\sqrt{3}}{2}\right)^3 = \frac{1}{\sqrt{2}} \cos^3 \frac{\pi}{12} + \frac{1}{\sqrt{2}} \sin^3 \frac{\pi}{12}$

$\frac{3\sqrt{3}}{8} = \frac{1}{\sqrt{2}}\left(\cos^3 \frac{\pi}{12} + \sin^3 \frac{\pi}{12}\right)$

$\frac{3\sqrt{6}}{8} = \left(\cos \frac{\pi}{12} + \sin \frac{\pi}{12}\right)\left(\cos^2 \frac{\pi}{12} - \cos \frac{\pi}{12} \sin \frac{\pi}{12} + \sin^2 \frac{\pi}{12}\right)$ 1M

$\frac{3\sqrt{6}}{8} = \left(\cos \frac{\pi}{12} + \sin \frac{\pi}{12}\right)\left(1 - \frac{1}{2}\sin \frac{\pi}{6}\right)$

$\frac{3\sqrt{6}}{8} = \left(\cos \frac{\pi}{12} + \sin \frac{\pi}{12}\right)\left[1 - \frac{1}{2}\left(\frac{1}{2}\right)\right]$

$\cos \frac{\pi}{12} + \sin \frac{\pi}{12} = \frac{\sqrt{6}}{2}$ 1A

11. (a) $\sin\frac{n\pi}{24}\sin\frac{\pi}{24} = -\frac{1}{2}\left[\cos\left(\frac{n\pi}{24}+\frac{\pi}{24}\right)-\cos\left(\frac{n\pi}{24}-\frac{\pi}{24}\right)\right]$ 1M

$= -\frac{1}{2}\left[\cos\frac{(n+1)\pi}{24}-\cos\frac{(n-1)\pi}{24}\right]$ 1

(b) $\sum_{k=1}^{5}\sin\frac{2k\pi}{24}\sin\frac{\pi}{24} = \sum_{k=1}^{5}\left[-\frac{1}{2}\left[\cos\frac{(2k+1)\pi}{24}-\cos\frac{(2k-1)\pi}{24}\right]\right]$ 1M

$= -\frac{1}{2}\left[\sum_{k=1}^{5}\cos\frac{(2k+1)\pi}{24}-\sum_{k=1}^{5}\cos\frac{(2k-1)\pi}{24}\right]$

$= -\frac{1}{2}\left(\cos\frac{11\pi}{24}-\cos\frac{\pi}{24}\right)$

$= -\frac{1}{2}\left[\cos\left(\frac{\pi}{2}-\frac{\pi}{24}\right)-\cos\frac{\pi}{24}\right]$

$= -\frac{1}{2}\left(\sin\frac{\pi}{24}-\cos\frac{\pi}{24}\right)$ 1A

$\sum_{k=1}^{5}\sin\frac{k\pi}{12} = \frac{\sum_{k=1}^{5}\sin\frac{2k\pi}{24}\sin\frac{\pi}{24}}{\sin\frac{\pi}{24}}$ 1M

$= \frac{-\frac{1}{2}\left(\sin\frac{\pi}{24}-\cos\frac{\pi}{24}\right)}{\sin\frac{\pi}{24}}$

$= -\frac{1}{2}+\frac{1}{2}\cot\frac{\pi}{24}$

$= -\frac{1}{2}+\frac{1}{2}(2+\sqrt{2}+\sqrt{3}+\sqrt{6})$

$= \frac{1+\sqrt{2}+\sqrt{3}+\sqrt{6}}{2}$ 1A

 END OF PAPER