

M2REG-TRIG-2324-ASM-SET 2-MATH

Suggested solutions

$$\begin{aligned}
 1. \quad & \frac{1 + \sin \theta - \cos \theta}{1 + \sin \theta + \cos \theta} + \frac{1 + \sin \theta + \cos \theta}{1 + \sin \theta - \cos \theta} \\
 &= \frac{(1 + \sin \theta - \cos \theta)^2 + (1 + \sin \theta + \cos \theta)^2}{[(1 + \sin \theta) + \cos \theta][(1 + \sin \theta) - \cos \theta]} && 1\text{M} \\
 &= \frac{(1 + \sin \theta)^2 - 2(1 + \sin \theta) \cos \theta + \cos^2 \theta + (1 + \sin \theta)^2 + 2(1 + \sin \theta) \cos \theta + \cos^2 \theta}{(1 + \sin \theta)^2 - \cos^2 \theta} && 1\text{M} \\
 &= \frac{2(1 + \sin \theta)^2 + 2 \cos^2 \theta}{1 + 2 \sin \theta + \sin^2 \theta - \cos^2 \theta} \\
 &= \frac{2 + 4 \sin \theta + 2 \sin^2 \theta + 2 \cos^2 \theta}{2 \sin \theta + 2 \sin^2 \theta} && 1\text{M} \\
 &= \frac{4(1 + \sin \theta)}{2 \sin \theta(1 + \sin \theta)} && 1\text{M} \\
 &= \frac{2}{\sin \theta} \\
 &= 2 \csc \theta && 1
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & \sin^2 x \tan x + \cos^2 x \cot x + 2 \sin x \cos x \\
 &= (1 - \cos^2 x) \tan x + (1 - \sin^2 x) \cot x + 2 \sin x \cos x && 2\text{M} \\
 &= \tan x - \cos^2 x \left(\frac{\sin x}{\cos x} \right) + \cot x - \sin^2 x \left(\frac{\cos x}{\sin x} \right) + 2 \sin x \cos x && 1\text{M} \\
 &= \tan x - \cos x \sin x + \cot x - \sin x \cos x + 2 \sin x \cos x \\
 &= \tan x + \cot x && 1
 \end{aligned}$$

$$\begin{aligned}
 3. \quad & \left[\csc \left(\theta - \frac{7\pi}{2} \right) + \tan(3\pi - \theta) \right] \left[\tan \frac{5\pi}{4} - \cos \left(\theta - \frac{3\pi}{2} \right) \right] \\
 &= \left[\csc \left(\theta - \frac{3\pi}{2} \right) + \tan(\pi - \theta) \right] \left[1 - \cos \left(\theta - \frac{3\pi}{2} \right) \right] \\
 &= (\sec \theta - \tan \theta)(1 + \sin \theta) && 1\text{M} \\
 &= \left(\frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta} \right) (1 + \sin \theta) \\
 &= \frac{1 - \sin^2 \theta}{\cos \theta} && 1\text{M} \\
 &= \frac{\cos^2 \theta}{\cos \theta} \\
 &= \cos \theta && 1
 \end{aligned}$$

4.
$$\frac{\sec(-\theta) + \cos(\pi - \theta)}{\csc(\pi - \theta) - \sin(\pi - \theta)}$$

$$= \frac{\sec \theta - \cos \theta}{\csc \theta - \sin \theta}$$
 1M

$$= \frac{\frac{1}{\cos \theta} - \cos \theta}{\frac{1}{\sin \theta} - \sin \theta}$$

$$= \frac{1 - \cos^2 \theta}{\cos \theta} \div \frac{1 - \sin^2 \theta}{\sin \theta}$$

$$= \frac{\sin^2 \theta}{\cos \theta} \times \frac{\sin \theta}{\cos^2 \theta}$$
 1M

$$= \frac{\sin^3 \theta}{\cos^3 \theta}$$

$$= \tan^3 \theta$$
 1
5.
$$\frac{\tan^2(90^\circ + \theta)}{\csc(360^\circ + \theta) - 1} - [1 - \sec(270^\circ - \theta)]$$

$$= \frac{(-\cot \theta)^2}{\csc \theta - 1} - (1 + \csc \theta)$$
 1M+1M

$$= \frac{\cot^2 \theta - (1 + \csc \theta)(\csc \theta - 1)}{\csc \theta - 1}$$

$$= \frac{\cot^2 \theta - (\csc^2 \theta - 1)}{\csc \theta - 1}$$

$$= \frac{\cot^2 \theta - \cot^2 \theta}{\csc \theta - 1}$$
 1M

$$= 0$$

 Thus,
$$\frac{\tan^2(90^\circ + \theta)}{\csc(360^\circ + \theta) - 1} = 1 - \sec(270^\circ - \theta).$$
 1
6.
$$\tan^2 x + 3 \sec x - 3 = 0$$

$$(\sec^2 x - 1) + 3 \sec x - 3 = 0$$
 1M

$$\sec^2 x + 3 \sec x - 4 = 0$$

$$\sec x = 1 \quad \text{or} \quad -4$$

$$\cos x = 1 \quad \text{or} \quad -\frac{1}{4}$$
 1M

$$\cos x = 1 \quad \text{or} \quad \cos x = -\frac{1}{4}$$

$$x = 0 \quad \text{or} \quad 2\pi \quad x \approx 1.82 \quad \text{or} \quad 4.46$$

 Thus, $x = 0$ or 1.82 or 4.46 or 6.28 . 1A
7.
$$\sec^2 \theta + 14 \tan \theta + 48 = 0$$

$$(1 + \tan^2 \theta) + 14 \tan \theta + 48 = 0$$
 1M

$$\tan^2 \theta + 14 \tan \theta + 49 = 0$$

$$\tan \theta = -7$$
 1M

$$\theta \approx 98.1^\circ \quad \text{or} \quad 278^\circ$$
 1A

8. $\tan 195^\circ = \tan(60^\circ + 135^\circ)$
- $$= \frac{\tan 60^\circ + \tan 135^\circ}{1 - \tan 60^\circ \tan 135^\circ} \quad 1\text{M}$$
- $$= \frac{\sqrt{3} + (-1)}{1 - (\sqrt{3}(-1))} \quad 1\text{M}$$
- $$= \frac{\sqrt{3} - 1}{1 + \sqrt{3}} \quad 1\text{A}$$
- $$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1}$$
- $$= \frac{3 - 2\sqrt{3} + 1}{2}$$
- $$= 2 - \sqrt{3} \quad 1\text{A}$$
9. $\sin^2(x - y) - 2 \sin x \cos y \sin(x - y)$
- $$= (\sin x \cos y - \cos x \sin y)^2 - 2 \sin x \cos y (\sin x \cos y - \cos x \sin y) \quad 1\text{M}$$
- $$= \sin^2 x \cos^2 y - 2 \sin x \cos x \sin y \cos y + \cos^2 x \sin^2 y - 2 \sin^2 x \cos^2 y + 2 \sin x \cos x \sin y \cos y$$
- $$= \cos^2 x \sin^2 y - \sin^2 x \cos^2 y$$
- $$= \cos^2 x (1 - \cos^2 y) - (1 - \cos^2 x) \cos^2 y \quad 1\text{M}$$
- $$= \cos^2 x - \cos^2 y \quad 1$$
10. $\frac{\cos(A - B) - \cos(A + B)}{\cos(A - B) + \cos(A + B)}$
- $$= \frac{(\cos A \cos B + \sin A \sin B) - (\cos A \cos B - \sin A \sin B)}{(\cos A \cos B + \sin A \sin B) + (\cos A \cos B - \sin A \sin B)} \quad 1\text{M}+1\text{A}$$
- $$= \frac{2 \sin A \sin B}{2 \cos A \cos B}$$
- $$= \tan A \tan B \quad 1$$
11. $2 \cos(A - B) = 3 \cos(A + B)$
- $$2 \cos A \cos B + 2 \sin A \sin B = 3 \cos A \cos B - 3 \sin A \sin B \quad 2\text{M}$$
- $$5 \sin A \sin B = \cos A \cos B$$
- $$5 \tan A = \cot B \quad 1$$
12. $\sin(A + B) \cos C - \sin(A + C) \cos B$
- $$= (\sin A \cos B + \cos A \sin B) \cos C - (\sin A \cos C + \cos A \sin C) \sin B \quad 1\text{M}$$
- $$= \sin A \cos B \cos C + \cos A \sin B \cos C - \sin A \cos B \cos C - \cos A \cos B \sin C$$
- $$= \cos A (\sin B \cos C - \cos B \sin C)$$
- $$= \sin(B - C) \cos A \quad 1\text{M}+1$$

$$13. \tan \alpha = \frac{12}{\sqrt{13^2 - 12^2}} = \frac{12}{5} \text{ and } \tan \beta = \frac{3}{\sqrt{5^2 - 3^2}} = \frac{3}{4} \quad 1\text{M}+1\text{A}$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} \quad 1\text{M}$$

$$= \frac{\frac{12}{5} - \frac{3}{4}}{1 + \left(\frac{12}{5}\right)\left(\frac{3}{4}\right)} \quad 1\text{A}$$

$$= \frac{33}{56}$$

$$14. \quad (\text{a}) \quad \sin(x + \alpha) = 3 \cos(x - \alpha)$$

$$\sin x \cos \alpha + \cos x \sin \alpha = 3 \cos x \cos \alpha + 3 \sin x \sin \alpha \quad 1\text{M}$$

$$\frac{\sin x}{\cos x} + \frac{\sin \alpha}{\cos \alpha} = 3 + \frac{3 \sin x \sin \alpha}{\cos x \cos \alpha}$$

$$\tan x + \tan \alpha = 3 + 3 \tan x \tan \alpha \quad 1\text{M}$$

$$\tan x(1 - 3 \tan \alpha) = 3 - \tan \alpha$$

$$\tan x = \frac{3 - \tan \alpha}{1 - 3 \tan \alpha} \quad 1$$

$$(\text{b}) \quad \sin\left(x + \frac{\pi}{4}\right) = 3 \cos\left(x - \frac{\pi}{4}\right)$$

$$\tan x = \frac{3 - \tan \frac{\pi}{4}}{1 - 3 \tan \frac{\pi}{4}} \quad 1\text{M}$$

$$= \frac{3 - 1}{1 - 3(1)}$$

$$= -1$$

$$x = \frac{3\pi}{4} \quad \text{or} \quad \frac{7\pi}{4} \quad 1\text{A}$$

$$15. \tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} \quad 1\text{M}$$

$$= \frac{\frac{1}{2} + \frac{1}{5}}{1 - \frac{1}{2} \times \frac{1}{5}}$$

$$= \frac{7}{9} \quad 1\text{M}$$

$$\tan(x + y + z) = \frac{\tan(x + y) + \tan z}{1 - \tan(x + y) \tan z}$$

$$= \frac{\frac{7}{9} + \frac{1}{8}}{1 - \frac{7}{9} \times \frac{1}{8}}$$

$$= 1 \quad 1\text{A}$$

$$\text{Note that } \tan(x + y) = \frac{7}{9} < 1. \text{ We have } x + y < \frac{\pi}{4} \text{ and } x + y + z < \frac{5\pi}{4}.$$

$$\text{Thus, } x + y + z = \frac{\pi}{4}. \quad 1\text{A}$$