M2REG-TRIG-2324-ASM-SET 1-MATH
Suggested solutions

1. Since $\sin\alpha > 0$ and $\sec\beta < 0$, α and β lie in 2nd quadrant.
   When $\sin\alpha = \frac{3}{5}$,

$$\cos\alpha = \frac{-\sqrt{5^2-3^2}}{5} = -\frac{4}{5} \quad \text{1M}$$

$$\tan\alpha = \frac{3}{-\sqrt{5^2-3^2}} = -\frac{3}{4} \quad \text{1A}$$

   When $\sec\beta = -\frac{13}{12}$,

$$\csc\beta = \frac{13}{\sqrt{13^2-12^2}} = \frac{13}{5}$$

$$\tan\beta = \frac{-\sqrt{13^2-12^2}}{12} = -\frac{5}{12} \quad \text{1A}$$

$$\frac{\cos\alpha + \csc\beta}{\tan\alpha - \tan\beta} = \frac{-\frac{4}{5} + \frac{13}{5}}{-\frac{3}{4} + \frac{5}{12}}$$

$$= -\frac{27}{5} \quad \text{1A}$$

2. $\tan^2\theta(\csc^2\theta + \sec^2\theta) = \frac{\sin^2\theta}{\cos^2\theta}\left(\frac{1}{\sin^2\theta} + \frac{1}{\cos^2\theta}\right)$ 1M

$$= \frac{1}{\cos^2\theta}\left(\frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}\right)$$

$$= \sec^4\theta \quad \text{1M+1A}$$

3. $\left(\cot\theta + \frac{1}{\cot\theta}\right)\cos\theta = \left(\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}\right)\cos\theta$ 1M

$$= \frac{\cos^2\theta}{\sin\theta} + \sin\theta$$

$$= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta}$$

$$= \csc\theta \quad \text{1M+1A}$$

4. $\csc^2\alpha + \csc^2\beta = (1 + \cot^2\alpha) + (1 + \cot^2\beta)$ 1M

$$= 2 + (\cot^2\alpha + \cot^2\beta)$$

$$= 3 \quad \text{1}$$

5. $\dfrac{\cos\theta+\cot\theta}{\tan\theta+\sec\theta}=\dfrac{\cos\theta+\frac{\cos\theta}{\sin\theta}}{\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}}$ 1M

$=\dfrac{\cos\theta+\frac{\cos\theta}{\sin\theta}}{\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}}\times\dfrac{\sin\theta\cos\theta}{\sin\theta\cos\theta}$ 1M

$=\dfrac{\cos^2\theta(\sin\theta+1)}{\sin\theta(\sin\theta+1)}$

$=\cos\theta\times\cot\theta$ 1

6. $\dfrac{\sin\theta}{\csc\theta-\cot\theta}-\dfrac{\tan\theta}{\csc\theta+\cot\theta}=\dfrac{\sin\theta(\csc\theta+\cot\theta)-\tan\theta(\csc\theta-\cot\theta)}{\csc^2\theta-\cot^2\theta}$ 1M

$=\dfrac{1+\cos\theta-\frac{1}{\cos\theta}+1}{1}$ 2M

$=2+\cos\theta-\dfrac{1}{\cos\theta}$

$=2-\dfrac{1-\cos^2\theta}{\cos\theta}$

$=2-\sec\theta\sin^2\theta$ 1

7. $\sin x\cos x(\sin x+\cos x)-\dfrac{1+2\sin x\cos x}{\sec x+\csc x}$

$=\dfrac{(\sin^2 x\cos x+\sin x\cos^2 x)\left(\frac{1}{\cos x}+\frac{1}{\sin x}\right)-1-2\sin x\cos x}{\sec x+\csc x}$ 1M

$=\dfrac{\sin^2 x+\sin x\cos x+\sin x\cos x+\cos^2 x-1-2\sin x\cos x}{\sec x+\csc x}$ 1M

$=0$

Thus, $\sin x\cos x(\sin x+\cos x)=\dfrac{1+2\sin x\cos x}{\sec x+\csc x}$. 1

8. $\dfrac{1+\sin x+\cos x}{1+\sin x-\cos x}-\dfrac{1+\cos x+\cot x+\csc x}{1+\sin x}$

$=\dfrac{(1+\sin x+\cos x)(1+\sin x)-(1+\cos x+\cot x+\csc x)(1+\sin x-\cos x)}{(1-\sin x-\cos x)(1+\sin x)}$

$=\dfrac{(1+2\sin x+\sin^2 x+\cos x+\sin x\cos x)-\left(1+\sin x-\cos x+\cos x+\sin x\cos x-\cos^2 x+\cot x+\cos x-\frac{\cos^2 x}{\sin x}+\csc x+1-\cot x\right)}{(1+\sin x-\cos x)(1+\sin x)}$ 1M

$=\dfrac{-1+\sin x+\sin^2 x+\cos^2 x+\frac{1-\cos^2 x}{\sin x}}{(1+\sin x-\cos x)(1+\sin x)}$

$=\dfrac{\sin x+\frac{-\sin^2 x}{\sin x}}{(1+\sin x-\cos x)(1+\sin x)}$ 1M

$=0$

Thus, $\dfrac{1+\sin x+\cos x}{1+\sin x-\cos x}=\dfrac{1+\cos x+\cot x+\csc x}{1+\sin x}$. 1

9. $\dfrac{1+\cos^2 A-\cos^2(A+C)}{\cos A\sin(A+C)}$

$= \dfrac{1+\cos^2 A-\cos^2(\pi-B)}{\cos A\sin(\pi-B)}$

$= \dfrac{1+\cos^2 A-\cos^2 B}{\cos A\sin B}$ 1M

$= \dfrac{\cos^2 A+\sin^2 B}{\cos A\sin B}$ 1M

$= \dfrac{\cos A}{\sin B}+\dfrac{\sin B}{\cos A}$

$= \cos A\csc B+\sec A\sin B$ 1

10. $(2\sin x+\sec x)(2\cos x+\csc x)$

$= 4\sin x\cos x+2+2+\dfrac{1}{\sin x\cos x}$ 1M

$= \dfrac{(2\sin x\cos x)^2+4\sin x\cos x+1}{\sin x\cos x}$ 1M

$= \dfrac{(2\sin x\cos x+1)^2}{\sin x\cos x}$ 1

11. For any real number θ,

$\csc^2\theta \geq 1$ 1M

$2\csc^2\theta+1 \geq 3$

$\dfrac{1}{2\csc^2\theta+1} \leq \dfrac{1}{3}$

$\dfrac{6}{2\csc^2\theta+1} \leq 2$ 1M

Required value is 2. 1A

12. For any real number θ,

$\sec^2\theta \geq 1$ 1M

$2\sec^2\theta-1 \geq 1$

$\dfrac{1}{2\sec^2\theta-1} \leq 1$

$-\dfrac{5}{2\sec^2\theta-1} \geq -5$

$3-\dfrac{5}{2\sec^2\theta-1} \geq -2$ 1M

Required value is -2. 1A