REV-FG-2324-ASM-SET 2-MATH
Suggested solutions

Conventional Questions

1. (a) $3 = -3^2 + 8(3) + k$ 1M

$k = -12$ 1A

(b) $0 = -x^2 + 8x - 12$

$x = 2 \quad \text{or} \quad 6$

The coordinates of A and B are $(2,\ 0)$ and $(6,\ 0)$ respectively. 1A+1A

(c) (i) x-coordinate of mid-point of $AB = \frac{2+6}{2} = 4$.

The axis of symmetry is $x = 4$. 1A

(ii) $CP : PB = (4-3) : (6-4)$ 1M

$= 1 : 2$ 1A

2. (a) $CD = BE = \sqrt{(3x)^2 + (4x)^2}$ 1M

$= 5x$ cm 1A

(b) (i) $BC = \frac{96 - 3x - 4x - 5x}{2}$ 1M

$= (48 - 6x)$ cm

Area of rectangle $= (48 - 6x)(5x)$

$= 30x(8 - x)$ cm^2 1A

(ii) Area $= -30x^2 + 240x + 6x^2$

$= -24[x^2 - 2(x)(5) + 5^2] + 600$ 1M

$= -24(x-5)^2 + 600$ 1A

The maximum area of the figure is 600 cm^2. 1A

3. (a) We have

$$\begin{cases} 400 = 150 + A(180)(5) + 25B \\ 438 = 150 + A(200)(8) + 64B \end{cases}$$ 1M

Solving, we have $A = \dfrac{1}{2}$ and $B = -8$. 1M+1

Thus, $P = 150 + \dfrac{\theta t}{2} - 8t^2$.

(b) (i) $P = 150 + \dfrac{220(10)}{2} - 8(10)^2 = 450$ 1A

(ii) When $\theta = 160$ and $P = 450$,

$$450 = 150 + \frac{160t}{2} - 8t^2$$

$$2t^2 - 20t + 75 = 0$$

$\Delta = (-20)^2 - 4(2)(75) = -200 < 0$ 1M

There is no real solution for t.

Thus, it is not possible to achieve the same value of P in (i) by varying t. 1A

(c) $P = 150 + \dfrac{160}{2}t - 8t^2$

$= -8[t^2 - 2(5)(t) + 5^2] + 350$ 1M

$= -8(t-5)^2 + 350$

Thus, P is maximum when $t = 5$.

Required times is 5 minutes. 1A

4. (a) $0 = 3x^2 - 9x - 12$

$x = 4 \quad \text{or} \quad -1$

The coordinates of A and B are $(-1,\ 0)$ and $(4,\ 0)$ respectively. 1A+1A

The coordinates of C are $(0,\ -12)$. 1A

(b) $y = 3x^2 - 9x - 12$

$= 3\left[x^2 - 2\left(\dfrac{3}{2}\right)x + \left(\dfrac{3}{2}\right)^2\right] - \dfrac{75}{4}$ 1M

$= 3\left(x - \dfrac{3}{2}\right)^2 - \dfrac{75}{4}$ 1M

Coordinates of Q are $\left(\dfrac{3}{2},\ -\dfrac{75}{4}\right)$. 1A

(c) Area of $ABQC = \dfrac{1}{2}(1)(12) + \dfrac{1}{2}\left(\dfrac{3}{2}\right)\left(12 + \dfrac{75}{4}\right) + \dfrac{1}{2}\left(4 - \dfrac{3}{2}\right)\left(\dfrac{75}{4}\right)$ 1M

$= \dfrac{105}{2}$ 1A

5. (a) (i) $\alpha + \beta = 6$ and $\alpha\beta = p$ 1A

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta \quad \text{1M}$$
$$= (6)^2 - 2p$$
$$= 36 - 2p \quad \text{1A}$$

(ii) $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$ 1M

$$= (6)^2 - 4p$$
$$= 36 - 4p \quad \text{1A}$$

(b) $\Delta = (-6)^2 - 4(1)(p) > 0$ 1M+1A

$$p < 9 \quad \text{1A}$$

(c) (i) $y = x^2 - 6x + 2$

$$= [x^2 - 2(3)(x) + 3^2] - 7 \quad \text{1M}$$
$$= (x - 3)^2 - 7$$

The coordinates of V are $(3,\ -7)$. 1A

(ii) Let the coordinates of A and B be $(\alpha,\ 0)$ and $(\beta,\ 0)$ respectively.

$$OB - OA = \beta - \alpha$$
$$= \sqrt{(\beta - \alpha)^2} \quad \text{1M}$$

Using the result of (a)(ii), putting $p = 2$,

$$OB - OA = \sqrt{36 - 4(2)}$$
$$= 2\sqrt{7} \quad \text{1A}$$

(iii) Area of $\triangle ABV = \dfrac{(2\sqrt{7})(7)}{2}$ 1M

$$= 7\sqrt{7} \quad \text{1A}$$

6. (a) $b = 6$

Since α and β are roots of the equation $x^2 + ax + b = 0$.

$\alpha\beta = b = 6$. 1A

(b) (i) $y = x^2 + ax + 6$

$= \left[x + ax + \left(\frac{a}{2}\right)^2\right] + 6 - \frac{a^2}{4}$ 1M

$= \left(x + \frac{a}{2}\right)^2 + 6 - \frac{a^2}{4}$ 1A

If the area of $\triangle ABC$ is twice the area of $\triangle ABP$,

$2\left[\frac{a^2}{4} - 6\right] = 6$ 1M+1M

$\frac{a^2}{2} - 9 = 0$

$a = 6$ or -6 (rejected) 1A

(ii) By (b)(i), coordinates of P are $\left(\frac{-6}{2},\ 6 - \frac{6^2}{4}\right) = (-3,\ -3)$. 1A

(c) $x^2 + 6x + 6 = 0$

$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(6)}}{2(1)}$ 1M

$= -3 \pm \sqrt{3}$

Therefore, $\alpha = -3 + \sqrt{3}$ and $\beta = -3 - \sqrt{3}$.

$OA = 3 - \sqrt{3}$ and $AB = 2\sqrt{3} \neq 2OA$. 1A

The claim is disagreed. 1A

7. (a) Vertex of the graph of $y = -\frac{1}{2}(x - h)^2 + 8$ is $(h,\ 8)$. 1M

The two graphs have the same vertex. Since $f(2) = 8$, we have $h = 2$. 1A

(b) (i) Required area $= \frac{(AB + d)k}{2} = \frac{(12 - k)k}{2}$ 1A

(ii) Area of $OABD = -\frac{1}{2}k^2 + 6k$

$= -\frac{1}{2}[k^2 - 2(6)(k) + 6^2] + 18$ 1M

$= -\frac{1}{2}(k - 6)^2 + 18$

Area of $OABD$ is maximum when $k = 6$. 1A

When $y = 6$,

$6 = -\frac{1}{2}(x - 2)^2 + 8$ 1M

$(x - 2)^2 = 4$

$x = 0$ or 4

So, $AB = 4$ 1A

$4 + d + 6 = 12$

$d = 2$

Thus, $D = (2,\ 0)$. 1A

8. (a) $y = x^2 - 8x + 12$

$= (x^2 - 8x + 16) - 4$ 1M

$= (x - 4)^2 - 4$

Coordinates of V are $(4,\ -4)$. 1A

(b) $C\ (0,\ 12)$. 1A

$0 = x^2 - 8x + 12$

$x = 2 \quad \text{or} \quad 6$

So, $A\ (2,\ 0)$ and $B\ (6,\ 0)$. 1A

Area of $AVBC = \dfrac{(6-2)(12)}{2} + \dfrac{(6-2)(4)}{2}$

$= 32$ 1A

(c) Slope of $AV = \dfrac{0+4}{2-4} = -2$, slope of $AC = \dfrac{12-0}{0-2} = -6$ 1M

Slope of $BV = \dfrac{0+4}{6-4} = 2$, slope of $BC = \dfrac{12-0}{0-6} = -2$

$AV // BC$ while AC is not parallel to BV.

So, $AVBC$ is a trapezium. 1

(d) Let the required height be h.

$$32 = \frac{(\sqrt{12^2 + 6^2} + \sqrt{4^2 + (2-4)^2})h}{2}$$ 1M+1M

$h \approx 3.58$ 1A

Required height is 3.58.

9. (a) Slope of $L_1 = \frac{4-1}{2+1} = 1$. Equation of L_1 is

$$y - 4 = 1(x-2) \qquad \text{1M}$$

$$y = x + 2 \qquad \text{1A}$$

(b) (i) Slope of $L_2 = -1$. Equation of L_2 is

$$y - a^2 = -(x-a) \qquad \text{1M}$$

$$y = -x + a + a^2 \qquad \text{1A}$$

(ii) Solve $\begin{cases} y = x + 2 \\ y = -x + a + a^2 \end{cases}$.

$$x + 2 = -x + a + a^2 \qquad \text{1M}$$

$$x = \frac{a^2 + a - 2}{2}$$

When $x = \frac{a^2+a-2}{2}$, $y = \frac{a^2+a-2}{2} + 2 = \frac{a^2+a+2}{2}$.

So, $D\left(\frac{a^2+a-2}{2},\ \frac{a^2+a+2}{2}\right)$. 1A

$$DP^2 = \left(\frac{a^2+a-2}{2} - a\right)^2 + \left(\frac{a^2+a+2}{2} - a^2\right)^2 \qquad \text{1M}$$

$$= \frac{(a^2-a-2)^2}{4} + \frac{(-a^2+a+2)^2}{4}$$

$$= \frac{2(a^2-a-2)^2}{4}$$

$$DP = \frac{1}{\sqrt{2}}\sqrt{(a^2-a-2)^2}$$

Since $a^2 - a - 2 = (a-2)(a+1) \le 0$ for $-1 \le a \le 2$, 1M

$$DP = \frac{-1}{\sqrt{2}}(a^2-a-2) \qquad \text{1}$$

(iii) $AB = \sqrt{(4-1)^2 + (2+1)^2} = 3\sqrt{2}$

Area of $\triangle ABP = \frac{1}{2}(3\sqrt{2})\left(\frac{-1}{\sqrt{2}}(a^2-a-2)\right)$ 1M

$$= -\frac{3}{2}\left[\left(a^2 - a + \frac{1}{4}\right) - \frac{9}{4}\right] \qquad \text{1M}$$

$$= -\frac{3}{2}\left(a - \frac{1}{2}\right)^2 + \frac{27}{8}$$

Maximum area of $\triangle ABP$ is $\frac{27}{8}$ when $P\left(\frac{1}{2},\ \frac{1}{4}\right)$. 1A+1A

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