

REG-GS-2324-ASM-SET 1-MATH**Suggested solutions****Multiple Choice Questions**

1. A	2. A	3. C	4. C	5. A
6. A	7. C	8. D	9. D	10. C
11. A	12. A	13. C	14. B	15. B
16. D	17. D	18. B	19. C	20. A
21. A	22. B	23. B	24. B	25. C
26. D	27. B	28. B	29. B	30. B

1. AWe have $b - a = c - b$.

I. ✓. $\frac{2^b}{2^a} = 2^{b-a}$ and $\frac{2^c}{2^b} = 2^{c-b} = 2^{b-a}$.

II. ✗. Consider when $a = 1$, $b = 2$ and $c = 3$.III. ✗. The terms are all undefined when $k = 0$.2. A

I. ✓. We have $\frac{y_2}{y_1} = \frac{y_3}{y_2} = \frac{y_4}{y_3} = \dots = \frac{y_n}{y_{n-1}}$.

Thus, we have $\frac{x_1 y_2}{x_1 y_1} = \frac{x_1 y_3}{x_1 y_2} = \frac{x_1 y_4}{x_1 y_3} = \dots = \frac{x_1 y_n}{x_1 y_{n-1}}$.

It is a geometric sequence.

II. ✓. We have $\frac{x_2}{x_1} = \frac{x_3}{x_2} = \dots = \frac{x_n}{x_{n-1}}$ and $\frac{y_2}{y_1} = \frac{y_3}{y_2} = \dots = \frac{y_n}{y_{n-1}}$.

Thus, we have $\frac{x_2 y_2}{x_1 y_1} = \frac{x_3 y_3}{x_2 y_2} = \dots = \frac{x_n y_n}{x_{n-1} y_{n-1}}$.

It is a geometric sequence.

III. ✗. Consider the geometric sequences 1, 3, 9, 27 and 1, 2, 4, 8.

 $x_1 - y_1 = 0$, $x_2 - y_2 = 1$, $x_3 - y_3 = 5$, $x_4 - y_4 = 19$ is not a geometric sequence.3. C

We have $\frac{b}{a} = \frac{c}{b}$.

I. ✓. $\frac{b^2}{a^2} = \left(\frac{b}{a}\right)^2 = \left(\frac{c}{b}\right)^2 = \frac{c^2}{b^2}$

II. ✗. Take $a = 1$, $b = 2$, $c = 4$ and $k = 2$.

The sequence 3, 4, 6 is not a geometric sequence.

III. ✓. $\frac{kb}{ka} = \frac{b}{a} = \frac{c}{b} = \frac{kc}{kb}$

4. C

- I. ✓. Common ratio = 3^3
- II. ✗. $\frac{33}{3} = 11$ and $\frac{333}{33} \neq 11$.
- III. ✓. Common ratio = -10

5. A

- I. ✓. $\frac{4^b}{4^a} = 4^{b-a} = 4^{c-b} = \frac{4^c}{4^b}$.
- II. ✓. $(4b+4) - (4a+4) = b-a = c-b = (4c+4) - (4b+4)$.
- III. ✗. Take $a = 10$, $b = 100$ and $c = 190$. Then $\log a = 1$, $\log b = 2$ and $\log c = \log 190 \neq 3$.

6. A

- I. ✓. $r = 2$.
- II. ✓. $r = 2$.
- III. ✗. Take $k = 2$. The sequence becomes 1, 2, 3, 4, which has no common ratio.

7. C

$$\Delta = (2b)^2 - 4ac = 0$$

$$b^2 = ac$$

$$\frac{b}{a} = \frac{c}{b}$$

a, b, c form a geometric sequence.

- I. ✗. Consider the case when $a = 1$, $b = 3$ and $c = 9$.
- II. ✓. $\log b - \log a = \log \frac{b}{a} = \log \frac{c}{b} = \log c - \log b$.
- III. ✓. $\frac{2018b}{2018a} = \frac{b}{a} = \frac{c}{b} = \frac{2018c}{2018b}$.

8. D

- I. ✗. Take $a = 1$, $b = 2$ and $c = 4$.
 $a+3, b+3, c+3$ is not a geometric sequence.
- II. ✓. We have $\frac{b}{a} = \frac{c}{b}$.
Thus, we have $\frac{a}{b} = \frac{b}{c}$ and c, b, a is a geometric sequence.
- III. ✓. We have $\frac{1}{b} \div \frac{1}{a} = \frac{a}{b}$ and $\frac{1}{c} \div \frac{1}{b} = \frac{b}{c} = \frac{a}{b}$.

9. D

I. ✗. $a = 1, b = -2, c = 4$ is a geometric sequence.

We have $b < a < c$.

II. ✓. We have $\frac{b}{a} = \frac{c}{b}$.

Thus, $b^2 = ac$.

III. ✓. We have $\frac{b}{a} = \frac{c}{b}$.

Thus, $\frac{b}{c} = \frac{a}{b}$ and c, b, a is a geometric sequence.

10. C

$$\frac{x-4}{3x} = \frac{x-1}{5} \div (x-4)$$

$$5(x-4)^2 = 3x(x-1)$$

$$2x^2 - 37x + 80 = 0$$

$$x = \frac{5}{2} \quad \text{or} \quad 16$$

11. A

$$\frac{x}{x-4} = \frac{x+12}{x}$$

$$x^2 = (x-4)(x+12)$$

$$0 = 8x - 48$$

$$x = 6$$

$$\text{Common ratio} = \frac{x}{x-4} = 3$$

12. A

$$\frac{4n-3}{2n+3} = \frac{8n+1}{4n-3}$$

$$(4n-3)^2 = (2n+3)(8n+1)$$

$$0 = 50n - 6$$

$$n = \frac{3}{25}$$

13. C

$$\frac{x+2}{x-2} = \frac{18}{x+2}$$

$$(x+2)^2 = 18(x-2)$$

$$x^2 - 14x + 40 = 0$$

$$x = 4 \quad \text{or} \quad 10$$

$$\text{When } x = 4, \text{ common ratio} = \frac{18}{4+2} = 3.$$

$$\text{When } x = 10, \text{ common ratio} = \frac{18}{10+2} = \frac{3}{2}.$$

14. B

$$\frac{a-2}{a-7} = \frac{a+2}{a-2}$$

$$(a-2)^2 = (a-7)(a+2)$$

$$0 = -a - 18$$

$$a = -18$$

$$\text{Common ratio} = \frac{-18-2}{-18-7} = \frac{4}{5}$$

15. B

$$k-5 = 5-h \quad \text{and} \quad \frac{4}{h} = \frac{k}{4}$$

$$h+k = 10 \qquad \qquad \qquad hk = 16$$

$$h^2 + k^2 = (h+k)^2 - 2hk$$

$$= 10^2 - 2(16)$$

$$= 68$$

16. D

$$\frac{-h}{h-7} = \frac{h+8}{-h}$$

$$(-h)^2 = (h-7)(h+8)$$

$$0 = h - 56$$

$$h = 56$$

$$\text{Common ratio} = \frac{-56}{56-7} = -\frac{8}{7}$$

17. D

$$\frac{ar^4}{ar^2} = \frac{36}{12}$$

$$r^2 = 3$$

$$r = \pm\sqrt{3}$$

$$\text{Common ratio is } \sqrt{3} \text{ or } -\sqrt{3}.$$

18. B

We have $(ar)(ar^2) = 18$ and $(ar^2)(ar^3) = 72$.

$$\frac{a^2r^5}{a^2r^3} = \frac{72}{18}$$

$$r^2 = 4$$

$$T_5 \times T_6 = (ar^4)(ar^5)$$

$$= (a^2r^3)(r^6)$$

$$= 18(4)^3$$

$$= 1152$$

19. C

We have $a + ar = 2$ and $ar^5 + ar^6 = -486$.

$$\frac{ar^5 + ar^6}{a + ar} = \frac{-486}{2}$$

$$\frac{ar^5(1+r)}{a(1+r)} = -243$$

$$r^5 = (-3)^5$$

$$r = -3$$

$$\text{First term} = \frac{2}{1+r} = -1$$

20. A

First term $= -\frac{1}{4}$ and common ratio $= \frac{2}{-1} = -2$.

$$\begin{aligned} n\text{th term} &= -\frac{1}{4}(-2)^{n-1} \\ &= -(-2)^{-2}(-2)^{n-1} \\ &= -(-2)^{n-3} \end{aligned}$$

21. A

Common ratio $= \frac{y}{x}$

$$\begin{aligned} \text{First term} &= \frac{x}{\left(\frac{y}{x}\right)^3} \\ &= \frac{x^4}{y^3} \end{aligned}$$

22. B

First term $= 3$ and common ratio $= \frac{1}{2}$.

$$\begin{aligned} (2n+1)\text{th term} &= 3\left(\frac{1}{2}\right)^{2n} \\ &= \frac{3}{2^{2n}} \end{aligned}$$

23. B

First term $= -2$ and common ratio $= \frac{-1}{2}$.

$$\begin{aligned} \text{General term} &= -2\left(-\frac{1}{2}\right)^{n-1} \\ &= (-2)(-2)^{1-n} \\ &= (-2)^{2-n} \end{aligned}$$

24. B

$$\frac{ar^6}{ar^3} = \frac{-1458}{2}$$

$$r^3 = -729$$

$$r = -9$$

25. C

Let the common ratio be r .

$$abc = 64$$

$$\left(\frac{b}{r}\right)b(br) = 64$$

$$b^3 = 64$$

$$b = 4$$

$$a + c = (a + b + c) - b = 14 - 4 = 10$$

26. D

$$3r^4 = 27$$

$$r = \sqrt[4]{3} \quad \text{or} \quad -\sqrt[4]{3} \text{ (rejected)}$$

$$xyz = (3r)(3r^2)(3r^3)$$

$$= 3r^6$$

$$= 81$$

27. B

$$1(k^3)^{n-1} = k^{3m+9}$$

$$k^{3n-3} = k^{3m+9}$$

$$3n - 3 = 3m + 9$$

$$n = m + 4$$

There are $m + 4$ terms.

28. B

$$972 \left(\frac{1}{3}\right)^{n-1} = \frac{4}{27}$$

$$\left(\frac{1}{3}\right)^{n-1} = \frac{1}{6561}$$

$$(n-1) \log \frac{1}{3} = \log \frac{1}{6561}$$

$$n = 9$$

There are 9 terms.

29. B

$$(-8) \left(\frac{3}{2}\right)^{n-1} > -300$$

$$\left(\frac{3}{2}\right)^{n-1} < \frac{75}{2}$$

$$(n-1) \log \frac{3}{2} < \log \frac{75}{2}$$

$$n < 9.94$$

There are 9 terms.

30. B

$$4(3)^{n-1} > 1800$$

$$3^{n-1} > 450$$

$$(n-1) \log 3 > \log 450$$

$$n > 6.56$$

$$\text{Required term} = 4(3)^{7-1}$$

$$= 2916$$

Conventional Questions

31. (a) $\frac{-16}{32-2k} = \frac{-k}{4} \div (-16)$ 1M

$$256 = \frac{k^2}{2} - 8k$$

$$0 = \frac{k^2}{2} - 8k - 256$$

$$k = 32 \quad \text{or} \quad -16$$
 1A

(b) $k = -16$ and positive terms in the sequence are 64, 4, ...

Let the required number be n .

$$64 \left(\frac{4}{64} \right)^{n-1} > 1 \times 10^{-30}$$
 1M

$$(n-1) \log \frac{1}{16} > \log \frac{1 \times 10^{-30}}{64}$$
 1M

$$n < 27.4$$

Required number is 27. 1A

32. (a) Let the first term and common ratio be a and r respectively.

$$\frac{ar^4}{ar^2} = \frac{400}{900}$$
 1M

$$r^2 = \frac{4}{9}$$

$$\text{First term} = \frac{400}{r^4}$$

$$= \frac{400}{\left(\frac{4}{9}\right)^2}$$

$$= 2025$$
 1A

(b) Common ratio = $\sqrt{\frac{4}{9}} = \frac{2}{3}$. 1A

$$2025 \left(\frac{2}{3} \right)^n - 2025 \left(\frac{2}{3} \right)^{2n} < 2 \times 10^{-5}$$

$$-2025 \left(\frac{2}{3} \right)^{2n} + 2025 \left(\frac{2}{3} \right)^n - 2 \times 10^{-5} < 0$$

$$\left(\frac{2}{3} \right)^n < 9.8765 \times 10^{-9} \quad \text{or} \quad \left(\frac{2}{3} \right)^n > 1.00 \text{ (rejected)} \quad 1A$$

$$n \log \frac{2}{3} < \log 9.8765 \times 10^{-9}$$
 1M

$$n > 45.5$$

The least value of n is 46. 1A

33. (a) $(x+2)(x-2) = 8(x-1)$

$$x^2 - 8x + 4 = 0$$

Thus, $p = 8$ and $q = 4$.

1A+1A

(b) Common ratio $= \frac{\log 8}{\log 4} = \frac{3 \log 2}{2 \log 2} = \frac{3}{2}$.

1M

$$(\log 4) \left(\frac{3}{2}\right)^\alpha + (\log 4) \left(\frac{3}{2}\right)^{2\alpha} < \log 2^{2020}$$

1M

$$2(1.5)^\alpha + 2(1.5)^{2\alpha} < 2020$$

$$(1.5)^{2\alpha} + 1.5^\alpha - 1010 < 0$$

$$\frac{-1 - \sqrt{4041}}{2} < 1.5^\alpha < \frac{-1 + \sqrt{4041}}{2}$$

Since $1.5^\alpha > 0$,

$$0 < 1.5^\alpha < \frac{-1 + \sqrt{4041}}{2}$$

$$\alpha \log 1.5 < \log \frac{-1 + \sqrt{4041}}{2}$$

1M

$$\alpha < 8.49$$

The greatest value of α is 8.

1A

34. (a) $T(n) = 1701 \left(-\frac{1}{3}\right)^{n-1}$

1A

(b) $1701 \left(-\frac{1}{3}\right)^{k-1} = -\frac{7}{6561}$

1M

$$\left(-\frac{1}{3}\right)^{k-1} = \left(-\frac{1}{3}\right)^{13}$$

1M

$$k = 14$$

1A

(c) Consider only the negative terms $-567, -63, -7, \dots$

Suppose the required number is n .

$$-567 \left(\frac{1}{9}\right)^{n-1} < -\frac{1}{100}$$

1M

$$\frac{1}{9^{n-1}} > \frac{1}{56700}$$

$$(n-1) \log \frac{1}{9} > \log \frac{1}{56700}$$

1M

$$n < 5.98$$

There are 5 terms.

1A