REG-AOT-2324-ASM-SET 1-MATH
Suggested solutions

Multiple Choice Questions

1. D	2. C	3. B	4. C	5. B
6. B	7. B	8. B	9. C	10. B
11. D	12. B	13. C	14. D	15. B
16. C	17. B	18. C	19. A	20. A
21. D	22. C	23. C	24. B	

1. $\boxed{\text{D}}$

$BD = \sqrt{12^2 + 5^2} = 13\,\text{cm}$
$BH = \sqrt{5^2 + 8^2} = \sqrt{89}\,\text{cm}$
$\tan \angle FBG = \dfrac{12}{8}$ and $\tan \angle DEB = \dfrac{13}{8}$
$\angle FBG \approx 56.3°$ $\angle DEB \approx 58.4°$
$\tan \angle EBH = \dfrac{12}{\sqrt{89}}$ and $\tan \angle ACB = \dfrac{12}{5}$
$\angle EBH \approx 51.8°$ $\angle ACB \approx 67.4°$

2. $\boxed{\text{C}}$

$BD = \sqrt{(\sqrt{3})^2 + (\sqrt{3})^2} = \sqrt{6}\,\text{m}$
$DH = \sqrt{(\sqrt{3})^2 + 1^2} = 2\,\text{m}$
$BH = \sqrt{(\sqrt{3})^2 + 1^2} = 2\,\text{m}$
$BD^2 = DH^2 + BH^2 - 2(DH)(BH)\cos \angle BHD$
$\angle BHD \approx 76°$

3. $\boxed{\text{B}}$

Since BV is perpendicular to the plane VAC, $\angle BVA = \angle BVC = 90°$.
$AB = BC = \sqrt{6^2 + 8^2} = 10\,\text{cm}$ and $BM = BN = 5\,\text{cm}$
$\triangle BMN$ is equilateral. So, $MN = 5\,\text{cm}$.
$\angle VBM = \tan^{-1} \dfrac{8}{6}$
$VM^2 = 6^2 + 5^2 - (2)(6)(5)\cos \angle VBM$
$VM = 5$

So, $VM = VN = MN = 5\,\text{cm}$, and the required area is $\dfrac{1}{2}(5)^2 \sin 60° = \dfrac{25\sqrt{3}}{4}\,\text{cm}^2$.

4. $\boxed{\text{C}}$

Let the length of edges be 2. Consider the triangular pyramid at a corner.
Length of bold edges $= \sqrt{1^2+1^2} = \sqrt{2}$. Volume $= \dfrac{1}{3}\left(\dfrac{1}{2}(1)(1)\right)(1) = \dfrac{1}{6}$.
Let the height of pyramid with respect to bold edge base be h.

$$\frac{1}{6} = \frac{1}{3}\left[\frac{1}{2}(\sqrt{2})^2 \sin 60°\right] h$$

$$h = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

Distance between opposite corners of the cube $= \sqrt{2^2+2^2+2^2} = \sqrt{12} = 2\sqrt{3}$
Required distance $= 2\sqrt{3} - 2h = \dfrac{4\sqrt{3}}{3}$

5. $\boxed{\text{B}}$

Let $VA = 2\,\text{cm}$.
Note that $VA = VC$ and $\angle AVC = 60°$. $\triangle VAC$ is an equilateral triangle.

$$\begin{aligned} AB &= AC \cos 45° \\ &= \sqrt{2}\,\text{cm} \end{aligned}$$

$$AB^2 = VA^2 + VB^2 - 2(VA)(VB)\cos\angle AVB$$

$$\angle AVB \approx 41.4°$$

6. $\boxed{\text{B}}$

Let the length of the cube be 2.

$MN = \sqrt{1^2+1^2} = \sqrt{2}$

$MP = DE = \sqrt{2^2+2^2} = \sqrt{8}$

$NP = \sqrt{NH^2+HP^2} = \sqrt{1^2+(1^2+2^2)} = \sqrt{6}$

$$MN^2 = MP^2 + NP^2 - 2(MP)(NP)\cos\angle MPN$$

$$\angle MPN = 30°$$

7. $\boxed{\text{B}}$

$$\begin{aligned} AC &= AD \cos\angle DAC \\ &= x\cos\beta \\ BC &= AC \sin\angle CAB \\ &= x \sin\alpha \cos\beta \end{aligned}$$

8. $\boxed{\text{B}}$

$$\begin{aligned} ME = MF &= \frac{1}{2}\sqrt{4^2+3^2+12^2} \\ &= 6.5\,\text{cm} \end{aligned}$$

$$4^2 = ME^2 + MF^2 - 2(ME)(MF)\cos\theta$$

$$\theta \approx 36°$$

9. $\boxed{\text{C}}$

Let $PQ = 1$. Then $PR = \dfrac{1}{\sin 48°}$, $QR = \dfrac{1}{\tan 48°}$, $PS = \dfrac{1}{\sin 60°}$ and $QS = \dfrac{1}{\tan 60°}$.

$$RS^2 = QR^2 + QS^2 - 2(QR)(QS)\cos 105° = PR^2 + PS^2 - 2(PR)(PS)\cos \angle RPS$$

$$\angle RPS \approx 56°$$

10. $\boxed{\text{B}}$

$BC = AC \sin 60°$

$\quad = 4\sqrt{3}$ m

$\tan\theta = \dfrac{6}{4\sqrt{3}}$

$\quad \theta \approx 41°$

11. $\boxed{\text{D}}$

Let the lengths of edges be 1. Let R be the mid-point of CD and Q be the mid-point of AC such that $PQ \perp QR$. Let S be the mid-point of HE such that $PS \perp HE$.

$QR = 0.5$ and $PR = \sqrt{1^2 - 0.5^2} = \sqrt{0.75}$

So, $PQ = \sqrt{(\sqrt{0.75})^2 - 0.5^2} = \sqrt{0.5}$.

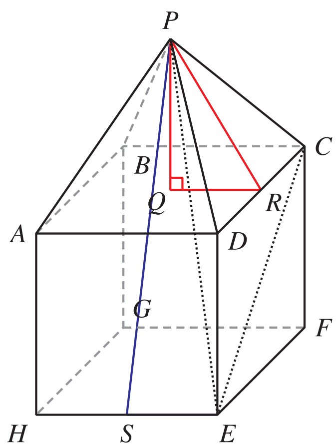
$PS = \sqrt{(\sqrt{0.5}+1)^2 + 0.5^2} \approx 1.78$

$PE = \sqrt{PS^2 + 0.5^2} \approx 1.85$ $CE = \sqrt{1^2 + 1^2} = \sqrt{2}$

In $\triangle PCE$,

$$1^2 = PE^2 + CE^2 - 2(PE)(CE)\cos\angle PEC$$

$\angle PEC \approx 32.4°$



12. $\boxed{\text{B}}$

$PE = \dfrac{1}{2}\sqrt{4^2 + 4^2}$

$\quad = \sqrt{8}$ cm

$PD = \sqrt{PE^2 + 4^2}$

$\quad = 2\sqrt{6}$ cm

13. $\boxed{\text{C}}$

$BM = MG = \sqrt{4^2 + 6^2 + 5^2} = \sqrt{77}$ cm

$$10^2 = BM^2 + MG^2 - 2(BM)(MG)\cos\angle BMG$$

$\cos\angle BMG = \dfrac{27}{77}$

14. $\boxed{\text{D}}$

$\angle DAC = \angle ACB = \alpha$
$AD = 2\cos\alpha$ cm
$CD = 2\sin\alpha$ cm
$CE = CD\cos\beta = 2\sin\alpha\cos\beta$ cm

$$\text{Required volume} = \left(\frac{1}{2}(CD)(CE)\sin\beta\right)(AD)$$
$$= 4\sin^2\alpha\cos\alpha\sin\beta\cos\beta \text{ cm}$$

15. $\boxed{\text{B}}$

Let $BG = 1$.

$BC = \frac{1}{\tan 60^\circ} = \frac{1}{\sqrt{3}}$

$AB = \frac{BC}{\tan 45^\circ} = \frac{1}{\sqrt{3}}$

$$\tan\theta = \frac{AB}{BG} = \frac{\left(\frac{1}{\sqrt{3}}\right)}{1} = \frac{1}{\sqrt{3}}$$

16. $\boxed{\text{C}}$

Let $PQ = 2$. Then $RS = 3$.

$RQ = \frac{PQ}{\tan 45^\circ} = 2$

$QS = \frac{PQ}{\tan 30^\circ} = 2\sqrt{3}$

Consider $\triangle QRS$.

$$RS^2 = RQ^2 + QS^2 - 2(RQ)(QS)\cos\angle RQS$$
$$\angle RQS \approx 60^\circ$$

17. $\boxed{\text{B}}$

Let the length of cube be 2.

$BM = 1$ and $MG = \sqrt{1^2 + 2^2} = \sqrt{5}$

$MD = \sqrt{1^2 + 2^2} = \sqrt{5}$ and $ME = \sqrt{MD^2 + 2^2} = 3$

$EG = \sqrt{2^2 + 2^2} = \sqrt{8}$

$MG^2 = ME^2 + EG^2 - 2(ME)(EG)\cos\theta$

$\cos\theta = \frac{\sqrt{2}}{2}$

18. $\boxed{\text{C}}$

Let $BC = 1$.

$AB = 1\tan 45° = 1$ and $BG = 1\tan 30° = \dfrac{1}{\sqrt{3}}$.

$$\tan\theta = \frac{AB}{AF} = \frac{1}{\left(\frac{1}{\sqrt{3}}\right)} = \sqrt{3}$$

19. $\boxed{\text{A}}$

Let $CD = 1$.

$BD = \dfrac{1}{\cos 30°} = \dfrac{2}{\sqrt{3}}$

$DH = \dfrac{1}{\sin 45°} = \sqrt{2}$

$CH = \dfrac{1}{\tan 45°} = 1$

$BC = CD\tan 30° = \dfrac{1}{\sqrt{3}}$

$BH = \sqrt{CH^2 + BC^2} = \sqrt{\dfrac{4}{3}}$

$$BH^2 = DH^2 + BD^2 - 2(DH)(BD)\cos\angle BDH$$

$\angle BDH \approx 52°$

20. $\boxed{\text{A}}$

$CM = 30 \times \dfrac{7}{3+7} = 21$ cm and $MH = 30 - 21 = 9$ cm

$MA = \sqrt{12^2 + 16^2 + 21^2} = 29$ cm and $MG = \sqrt{12^2 + 9^2} = 15$ cm

$AG = \sqrt{30^2 + 16^2} = 34$ cm

$AG^2 = AM^2 + MG^2 - 2(AM)(MG)\cos\theta$

$\cos\theta = \dfrac{-3}{29}$

21. $\boxed{\text{D}}$

Let $AD = 1$. Then $AM = \tan\theta$.

$DE = AH = \tan\beta$. $AB = EF = DE\tan\alpha = \tan\beta\tan\alpha$.

Since M is the mid-point of AB, $2AM = AB$ and $2\tan\theta = \tan\beta\tan\alpha$

22. $\boxed{\text{C}}$

$AD = BD = \dfrac{3}{\cos 30°} = 2\sqrt{3}$ m

$AB = \sqrt{AD^2 + BD^2} = 2\sqrt{6}$ m

$DC = 3\tan 30° = \sqrt{3}$ m

$AC = \sqrt{DC^2 + AD^2} = \sqrt{15}$ m

$$3^2 = AC^2 + AB^2 - 2(AC)(AB)\cos\angle BAC$$

$\angle BAC \approx 38°$

23. $\boxed{\text{C}}$

Let $OA = 1$ m.
Then $OB = \dfrac{1}{\tan 45^\circ} = 1$ m and $OC = \dfrac{1}{\tan 60^\circ} = \dfrac{1}{\sqrt{3}}$ m.

$\tan \angle OCB = \dfrac{OB}{OC}$

$\angle OCB = 60^\circ$

24. $\boxed{\text{B}}$

Let $AB = 1$.
In $\triangle ABD$, $AD = \dfrac{1}{\sin 30^\circ} = 2$ and $BD = \dfrac{1}{\tan 30^\circ} = \sqrt{3}$.
In $\triangle ABC$, $AC = \dfrac{1}{\sin 45^\circ} = \sqrt{2}$ and $BC = \dfrac{1}{\tan 45^\circ} = 1$.

$CD^2 = 1^2 + (\sqrt{3})^2 - 2(1)(\sqrt{3}) \cos 150^\circ$

$CD = \sqrt{7}$

$CD^2 = AD^2 + AC^2 - 2(AD)(AC) \cos \angle CAD$

$\angle CAD \approx 100^\circ$

Conventional Questions

25. (a) $BG = \sqrt{4^2 + 2^2}$ 1M

$= 2\sqrt{5}$ cm 1A

$GD = BG = 2\sqrt{5}$ cm 1A

$DF = \sqrt{4^2 + 4^2}$

$= \sqrt{32}$ cm

$BD = \sqrt{4^2 + DF^2}$

$= 4\sqrt{3}$ cm 1A

(b) $BD^2 = BG^2 + DG^2 - 2(BG)(DG)\cos\angle BGD$ 1M

$\angle BGD \approx 102°$ 1A

26. (a) $EI = \sqrt{90^2 + 60^2}$ 1M

$= \sqrt{11\,700}$

≈ 108 cm 1A

$BE = \sqrt{90^2 + 60^2 + 60^2}$ 1M

$= \sqrt{15\,300}$

≈ 124 cm 1A

(b) (i) $\angle BDC = 45°$ 1A

$\angle BDI = 45° + 50° = 95°$ 1A

(ii) $BD = \sqrt{60^2 + 60^2}$ 1M

$= \sqrt{7200}$ cm

$BI^2 = BD^2 + 60^2 - 2(BD)(60)\cos\angle BDI$ 1M

$BI \approx 108$ cm 1A

(c) Required angle is $\angle BEI$. 1A

$BI^2 = BE^2 + EI^2 - 2(BE)(EI)\cos\angle BEI$ 1M

$\angle BEI \approx 55.1°$ 1A

27. (a) Let K be a point on EF such that $CK \perp EF$.

Consider $\triangle CFK$.

$$\cos\angle CFK = \frac{\left(\frac{60-30}{2}\right)}{50}$$

$$\angle CFK \approx 72.5^\circ$$

$\angle DCF = 180^\circ - \angle CFK \approx 107^\circ$ 1A

Consider $\triangle DCF$.

$$DF^2 = 30^2 + 50^2 - 2(30)(50)\cos\angle DCF$$

$$DF \approx 65.6\,\text{cm}$$ 1A

(b) Let J be a point on FH such that $CJ \perp FH$.

$AC = \sqrt{30^2 + 30^2} = 30\sqrt{2}\,\text{cm}$

$HF = \sqrt{60^2 + 60^2} = 60\sqrt{2}\,\text{cm}$

Consider $\triangle CFJ$.

$$CJ = \sqrt{50^2 - \left(\frac{HF - AC}{2}\right)^2}$$ 1M

$$= 5\sqrt{82}\,\text{cm}$$

Height of the frustum is $5\sqrt{82}$ cm. 1A

(c) $BD = AC = 30\sqrt{2}\,\text{cm}$

$BF = DF \approx 65.6\,\text{cm}$

Consider $\triangle BDF$.

Let $s = \dfrac{BD + BF + DF}{2}$.

Area of $\triangle BDF = \sqrt{s(s - BD)(s - DF)(s - BF)}$ 1M

$\approx 1320\,\text{cm}^2$

Let h cm be the required distance.

Consider the volume of the tetrahedron $CBDF$.

$$\frac{1}{3}(\text{area of } \triangle BDF)(h) = \frac{1}{3}\left[\frac{(30)(30)}{2}\right](5\sqrt{82})$$ 1M

$$h \approx 15.5$$ 1A

Required distance is 15.5 cm.

END OF PAPER