

M2REG-BT-2324-ASM-SET 2-MATH

Suggested solutions

$$\begin{aligned}
 1. \text{ 7th term} &= C_6^n (ax^2)^{n-6} \left(\frac{1}{2x}\right)^6 \\
 &= C_6^n a^{n-6} 2^{-6} x^{2n-18}
 \end{aligned}$$

1A

Put $2n - 18 = 0$, we have $n = 9$.

1A

$$C_6^9 a^{9-6} 2^{-6} = \frac{567}{16}$$

$$a^3 = 27$$

$$a = 3$$

1A

$$\begin{aligned}
 2. \text{ General term} &= C_r^{12} (x^5)^{12-r} \left(-\frac{2}{x}\right)^r \\
 &= C_r^{12} (-2)^r x^{60-6r}
 \end{aligned}$$

1M

Put $60 - 6r = 0$, we have $r = 10$.

1M

$$\text{Constant term} = C_{10}^{12} (-2)^{10}$$

$$= 67\,584$$

1A

$$\begin{aligned}
 3. \text{ (a) General term} &= C_r^9 (2x^2)^{9-r} \left(\frac{1}{x}\right)^r \\
 &= C_r^9 (2)^{9-r} x^{18-3r}
 \end{aligned}$$

1A

1M

When $18 - 3r = 0$, $r = 6$.

1M

$$\text{Constant term} = C_6^9 (2)^{9-6} = 672.$$

1A

(b) When $18 - 3r = 2$, $r = \frac{16}{3}$, which is not an integer.

There is no x^2 term.

1M+1A

$$\begin{aligned}
 4. \text{ (a) General term} &= C_r^8 x^{8-r} \left(\frac{2}{x^3}\right)^r \\
 &= C_r^8 2^r x^{8-4r}
 \end{aligned}$$

1M

Put $8 - 4r = 0$, we have $r = 2$.

1M

Constant term exists.

$$\text{Constant term} = C_2^8 2^2$$

$$= 112$$

1A

(b) Put $8 - 4r = 3$, we have $r = \frac{5}{4}$.

1M

Since r should be a non-negative integer, x^3 term does not exist.

1A

$$5. \text{ General term} = C_r^{12} (x)^{12-r} \left(-\frac{y}{2}\right)^r$$

$$= C_r^{12} \left(-\frac{1}{2}\right)^r x^{12-r} y^r$$

1M

Put $r = 8$.

1M

$$\text{Required coefficient} = C_8^{12} \left(-\frac{1}{2}\right)^8$$

$$= \frac{495}{256}$$

1A

$$6. \quad (a) \quad (1 - 4x)^n = 1 + C_1^n(-4x) + C_2^n(-4x)^2 + \dots$$

1M

$$\frac{n(n-1)}{2}(-4)^2 = 240$$

1M

$$n^2 - n - 30 = 0$$

$$n = 6 \quad \text{or} \quad -5 \text{ (rejected)}$$

1A

$$(b) \quad (1 - 4x)^6 = 4096x^6 - 6144x^5 + 3840x^4 + \dots$$

$$\left(1 + \frac{2}{x}\right)^5 = 1 + \frac{10}{x} + \frac{40}{x^2} + \dots$$

1M

$$\text{Required coefficient} = 4096(40) - 6144(10) + 3840(1)$$

1M

$$= 106\,240$$

1A

$$7. \quad (x + 3)^5 = x^5 + C_1^5(3)x^4 + C_2^5(3)^2x^3 + C_3^5(3)^3x^2 + C_4^5(3)^4x + 3^5$$

1M

$$= x^5 + 15x^4 + 90x^3 + 270x^2 + 405x + 243$$

1A

$$(x + 3)^5 \left(x - \frac{4}{x}\right)^2 = (x^5 + 15x^4 + 90x^3 + 270x^2 + 405x + 243) \left(x^2 - 8 + \frac{16}{x^2}\right)$$

1M

$$\text{Required coefficient} = (1)(16) + (90)(-8) + (405)(1)$$

1M

$$= -299$$

1A

$$8. \quad (3 + x)^6 = x^6 + C_1^6x^5(3) + C_2^6x^4(3)^2 + C_3^6x^3(3)^3 + \dots$$

1M

$$= x^6 + 18x^5 + 135x^4 + 540x^3 + \dots$$

$$\left(\frac{2}{x} - 1\right)^2 (3 + x)^6 = \left(\frac{4}{x^2} - \frac{4}{x} + 1\right) (x^6 + 18x^5 + 135x^4 + 540x^3 + \dots)$$

$$\text{Required coefficient} = 4(18) + (-4)(135) + 1(540)$$

1M

$$= 72$$

1A

$$9. \quad (a) \quad \left(2x + \frac{1}{x}\right)^3 = (2x)^3 + 3(2x)^2\left(\frac{1}{x}\right) + 3(2x)\left(\frac{1}{x}\right)^2 + \left(\frac{1}{x}\right)^3$$

1A

$$= 8x^3 + 12x + \frac{6}{x} + \frac{1}{x^3}$$

$$(b) \quad (3x^2 - x - 5) \left(2x + \frac{1}{x}\right)^3 = (3x^2 - x - 5) \left(8x^3 + 12x + \frac{6}{x} + \frac{1}{x^3}\right)$$

$$\text{Coefficient of } x = (3)(6) + (-5)(12)$$

1M

$$= -42$$

1A

10. $\left(2 - \frac{x}{2}\right)^7 = 2^7 + C_1^7(2)^6\left(-\frac{x}{2}\right) + C_2^7(2)^5\left(-\frac{x}{2}\right)^2 + C_3^7(2)^4\left(-\frac{x}{2}\right)^3 + \dots$ 1M
 $= 128 - 224x + 168x^2 - 70x^3 + \dots$
 $\left(4x + \frac{9}{x}\right)^2 \left(2 - \frac{x}{2}\right)^7 = \left(\frac{81}{x^2} + 72 + 16x^2\right)(128 - 224x + 168x^2 - 70x^3 + \dots)$
Required coefficient $= 81(-70) + 72(-224)$ 1M
 $= -21\,798$ 1A
11. $(1 + 2x)^3 = 1 + 6x + 12x^2 + \dots$ 1A
 $(1 + 3x)^4 = 1 + 12x + 54x^2 + \dots$
 $(1 + 2x)^4(1 + 3x)^4 = (1 + 6x + 12x^2 + \dots)(1 + 12x + 54x^2 + \dots)$
 $= 1 + 18x + 138x^2 + \dots$ 1M+1A
12. $(1 + x)^m = 1 + C_1^m x + C_2^m x^2 + \dots$
 $= 1 + mx + \frac{m(m-1)}{2}x^2 + \dots$
 $(1 - 2x)^n = 1 + C_1^n(-2x) + C_2^n(-2x)^2 + \dots$
 $= 1 - 2nx + 2n(n-1)x^2 + \dots$
Compare the coefficients of x and x^2 , we have

$$\begin{cases} m - 2n = 0 \\ \frac{m(m-1)}{2} - 2mn + 2n(n-1) = -9 \end{cases}$$
 1M

$$\frac{2n(2n-1)}{2} - 2(2n)n + 2n(n-1) = -9$$
 1M

$$-3n = -9$$

$$n = 3$$
 1A
When $n = 3$, $m = 2(3) = 6$. 1A
13. $(1 + 4px)(1 + qx)^4 = (1 + 4px)[1 + 4qx + 6(qx)^2 + \dots]$ 1M
 $= (1 + 4px)(1 + 4qx + 6q^2x^2 + \dots)$
 $0 = 1(4q) + (4p)(1)$ 1M
 $p = -q$
 $-40 = 1(6q^2) + 4p(4q)$ 1M
 $-40 = 6q^2 + 16(-q)(q)$ 1M
 $q^2 = 4$
 $q = \pm 2$
Thus, $(p, q) = (-2, 2)$ or $(2, -2)$. 1A+1A

$$14. (a + 5x)(1 + 4x)^n = (a + 5x) [1 + C_1^n(4x) + C_2^n(4x)^2 + \dots] \quad 1M$$

$$= (a + 5x)[1 + 4nx + 8n(n-1)x^2 + \dots]$$

$$= a + (5 + 4an)x + [20n + 8an(n-1)]x^2 + \dots$$

Compare like terms, we have

$$\begin{cases} a = 11 & 1A \\ 5 + 4an = 313 & 1M \\ 20n + 8an(n-1) = b \end{cases}$$

$$5 + 4(11)n = 313 \quad 1M$$

$$n = 7 \quad 1A$$

$$20(7) + 8(11)(7)(6) = b$$

$$b = 3836 \quad 1A$$

$$15. (a) (1 - x)^4 = 1 - 4x + 6x^2 - 4x^3 + x^4 \quad 1A$$

$$(b) (1 + kx)^9(1 - x)^4 = (1 + 9kx + 36k^2x^2 + \dots)(1 - 4x + 6x^2 - 4x^3 + x^4) \quad 1A$$

$$-3 = 1(6) + 9k(-4) + 36k^2(1) \quad 1M$$

$$0 = 36k^2 - 36k + 9$$

$$k = \frac{1}{2} \quad 1A$$