REV-NSCN-2324-ASM-SET 1-MATH
Suggested solutions

Multiple Choice Questions

1. B	2. A	3. D	4. B	5. A
6. A	7. B	8. D	9. C	10. B
11. B	12. A	13. D	14. A	15. C
16. C	17. D	18. C	19. A	20. C
21. D	22. D	23. A	24. C	25. A
26. B	27. B	28. B	29. A	30. C

1. $\boxed{\text{B}}$

$$\begin{aligned}(3+5i)+(-2+6i)-(7-2i) &= (3-2-7)+(5+6+2)i \\ &= -6+13i\end{aligned}$$

2. $\boxed{\text{A}}$

I. ✗. $\dfrac{1}{i^3} = \dfrac{i^4}{i^3} = i \neq -i$.

II. ✗. $\sqrt{-8} = \sqrt{8}i = 2\sqrt{2}i \neq 8i$.

III. ✓. $5 = 5 + 0i$ is a complex number.

3. $\boxed{\text{D}}$

$(i^3)^5 = i^{15} = (i^4)^3 i^3 = i^3 = -i$

4. $\boxed{\text{B}}$

Using calculator CMPLX mode, $\dfrac{1-i}{1+i} = -i$.

$\left(\dfrac{1-i}{1+i}\right)^6 = (-i)^6 = i^6 = i^2 = -1$.

5. $\boxed{\text{A}}$

$i^{10} - \dfrac{1}{i^3} = i^2 - \dfrac{i^4}{i^3} = -1 - i$

6. $\boxed{\text{A}}$

Using calculator CMPLX mode, $\dfrac{1+i}{1-i} = i$.

$\left(\dfrac{1+i}{1-i}\right)^{2011} = i^{2011} = i^3 = -i$.

7. $\boxed{\text{B}}$

$$\frac{6i^6+7i^7+8i^8+9i^9+10i^{10}}{1+i}=\frac{-6-7i+8+9i-10}{1+i}$$
$$=-3+5i$$

The real part is -3.

8. $\boxed{\text{D}}$

$$\frac{4i^{2020}+5i^{2019}+6i^{2018}+7i^{2017}+8i^{2016}}{1+i}$$
$$=\frac{4+5i^3+6i^2+7i+8}{1+i}$$
$$=\frac{6+2i}{1+i}$$
$$=4-2i$$

Imaginary part $=-2$

9. $\boxed{\text{C}}$

Put $k=2$.

$$\frac{i^{2020}}{k+i^{2019}}=\frac{1}{2+i^3}$$
$$=\frac{1}{2-i}$$
$$=\frac{2}{5}+\frac{1}{5}i$$

Imaginary part $=\frac{1}{5}$

Only option C gives $\frac{1}{5}$ when $k=2$.

10. $\boxed{\text{B}}$

$i-2i^2+3i^3-4i^4=i+2-3i-4=-2-2i$

Imaginary part $=-2$

11. $\boxed{\text{B}}$

Using calculator CMPLX mode, $\frac{4-i}{2+i}=\frac{7}{5}-\frac{6}{5}i$.

12. $\boxed{\text{A}}$

By calculator, $\frac{5}{2-i}=2+i$.

13. $\boxed{\text{D}}$

Using calculator CMPLX mode, $\frac{1}{i}+\frac{1}{i+1}=\frac{1}{2}-\frac{3}{2}i$

We have $a=\frac{1}{2}$ and $b=-\frac{3}{2}$.

14. $\boxed{\text{A}}$

$z = (k-4)i^{-2017} + (k+1)i^{2018}$.

Since i^{2018} is real and i^{-2017} is purely imaginary, we have $k = 4$.

Then $z = (4+1)i^{2018} = 5(-1) = -5$.

15. $\boxed{\text{C}}$

$$\frac{3+ai}{2-i} = 2 - bi$$

$$\begin{aligned} 3 + ai &= (2-bi)(2-i) \\ &= (4-b) + (-2b-2)i \end{aligned}$$

Compare real parts,

$$\begin{aligned} 3 &= 4 - b \\ b &= 1 \end{aligned}$$

16. $\boxed{\text{C}}$

Using calculator CMPLX mode, $\dfrac{i}{1+2i} = \dfrac{2}{5} + \dfrac{1}{5}i$.

$\dfrac{i}{1+2i} + ai = \dfrac{2}{5} + \left(\dfrac{1}{5} + a\right)i$ is a real number.

$$\begin{aligned} \frac{1}{5} + a &= 0 \\ a &= -\frac{1}{5} \end{aligned}$$

17. $\boxed{\text{D}}$

$z = (x-4) + 3i$ is purely imaginary.

$$\begin{aligned} x - 4 &= 0 \\ x &= 4 \end{aligned}$$

18. $\boxed{\text{C}}$

$$\begin{aligned} (2-i)z &= (2-i)(1+i)(2x-yi) \\ &= (3+i)(2x-yi) \\ &= (6x+y) + (2x-3y)i \end{aligned}$$

Since x and y are positive integers such that $6x + y = 25$ and $x > y$,

we have $x = 4$ and $y = 1$.

Thus, $x - y = 4 - 1 = 3$.

19. $\boxed{\text{A}}$

$$\frac{bi}{2+3i} = a - 2i$$

$$\begin{aligned} bi &= (2+3i)(a-2i) \\ &= (2a+6) + (3a-4)i \end{aligned}$$

$$\begin{cases} 2a + 6 = 0 \\ 3a - 4 = b \end{cases}$$

Solving, $a = -3$ and $b = -13$.

20. $\boxed{\text{C}}$

Put $k = 1$, using the calculator CMPLX mode,

$$\frac{5k + 10i}{1 - 2i} = \frac{5 + 10i}{1 - 2i} = -3 + 4i$$

Imaginary part = 4

Only option C gives 4 when $k = 1$.

21. $\boxed{\text{D}}$

Let $x = 1$. Using calculator CMPLX mode, $(x - 5i)(4 + i) = (1 - 5i)(4 + i) = 9 - 19i$.

Only option D gives a value of 9 when $x = 1$.

22. $\boxed{\text{D}}$

$$\begin{aligned} (a+bi)^2 - (a-bi)^2 &= [(a+bi) + (a-bi)][(a+bi) - (a-bi)] \\ &= (2a)(2bi) \\ &= 4abi \end{aligned}$$

23. $\boxed{\text{A}}$

Let $\alpha = 1$. Using calculator CMPLX mode, $\dfrac{\alpha^2 + 9}{\alpha - 3i} = 1 + 3i$.

Only option A satisfies this.

24. $\boxed{\text{C}}$

A. ✗. $(\sqrt{2}i)^2 = -2 \neq 2i$.

B. ✗. $(-\sqrt{2}i)^2 = -2 \neq 2i$.

C. ✓. $(1 + i)^2 = 1 + 2i - 1 = 2i$.

D. ✗. $(1 - i)^2 = 1 - 2i - 1 = -2i$.

25. $\boxed{\text{A}}$

$i^3(ai - 4) = -i(ai - 4) = a + 4i$

Real part = a

26. $\boxed{\text{B}}$

$6i(1-3i) = 18+6i$

$$\begin{cases} x+y=18 \\ x-2y=6 \end{cases}$$

Solving, we have $x = 14$ and $y = 4$.

27. $\boxed{\text{B}}$

Let $\alpha = 1$.

Using calculator CMPLX mode, $-i(\alpha - 8i^3) = 8 - i$.

The answer is B.

28. $\boxed{\text{B}}$

I. ✓. Calculator CMPLX mode.

II. ✓. $1 + z + z^2 = 0$ by calculator. So, $z^3 + z^4 + z^5 = z^3(1 + z + z^2) = 0$.

III. ✗. $(z^{3n} + z^{3n+1} + z^{3n+2}) + \ldots + (z^{12n-3} + z^{12n-2} + z^{12n-1}) + z^{12n} = 0 + 1 = 1 \neq 0$.

29. $\boxed{\text{A}}$

$$\begin{aligned} \frac{k+10i}{3+4i} + \frac{k-i}{2+3i} &= \frac{(k+10i)(3-4i)}{3^2+4^2} + \frac{(k-i)(2-3i)}{2^2+3^2} \\ &= \frac{(3k+40)+(30-4k)i}{25} + \frac{(2k-3)+(-3k-2)i}{13} \end{aligned}$$

$z_1 + z_2$ is a purely imaginary number.

$$\begin{aligned} \frac{3k+40}{25} + \frac{2k-3}{13} &= 0 \\ k &= -5 \end{aligned}$$

$z_1 + z_2 = \frac{50i}{25} + \frac{13i}{13} = 3i$

The imaginary part is 3.

30. $\boxed{\text{C}}$

Note that $z_1 - z_2$ is a purely imaginary number.

Let $z_1 - z_2 = xi$, where x is a real number.

$$\begin{aligned} \frac{4-ki}{1+i} - \frac{k+3i}{1-i} &= xi \\ (4-ki)(1-i) - (k+3i)(1+i) &= xi(1+i)(1-i) \\ [(4-k)+(-k-4)i] - [(k-3)+(k+3)i] &= xi(1^2 - i^2) \\ (-2k+7)+(-2k-7)i &= 2xi \end{aligned}$$

Compare real and imaginary parts, we have $-2k + 7 = 0$ and $-2k - 7 = 2x$.

Solving, we have $k = \frac{7}{2}$ and $x = -7$.

Thus, $z_1 - z_2 = -7i$.

$$z_1 = \frac{4-ki}{1+i} \times \frac{1-i}{1-i} \quad \text{and} \quad z_2 = \frac{k+3i}{1-i} \times \frac{1+i}{1+i}$$

$$= \frac{(4-k)+(-k-4)i}{1^2-i^2} \qquad = \frac{(k-3)+(k+3)i}{1^2-i^2}$$

$$= \frac{(-k+4)+(-k-4)i}{2} \qquad = \frac{(k-3)+(k+3)i}{2}$$

The real part of z_1 is equal to the real part of z_2.

$$\frac{-k+4}{2} = \frac{k-3}{2}$$

$$k = \frac{7}{2}$$

$$z_1 - z_2 = \frac{\left(-\frac{7}{2}+4\right)+\left(-\frac{7}{2}-4\right)i}{2} - \frac{\left(\frac{7}{2}-3\right)+\left(\frac{7}{2}+3\right)i}{2}$$

$$= -7i$$

END OF PAPER