

REG-ST-2324-ASM-SET 1-MATH**Suggested solutions****Multiple Choice Questions**

1. C	2. A	3. B	4. B	5. B
6. B	7. A	8. B	9. B	10. B
11. D	12. C	13. A	14. D	15. C
16. C	17. B	18. D	19. A	20. D
21. B	22. C	23. D	24. B	25. C
26. B	27. D	28. A	29. D	30. B

1. C

$$\begin{aligned}
 12\sqrt{3} &= 2 \times \frac{1}{2}(AB)(AD) \sin 120^\circ \\
 &= AB(6) \cdot \frac{\sqrt{3}}{2} \\
 AB &= 4
 \end{aligned}$$

2. A

$$\begin{aligned}
 \frac{1}{2}(10)(8) \sin \theta &= 20 \\
 \sin \theta &= \frac{1}{2} \\
 \theta &= 30^\circ \quad \text{or} \quad 150^\circ \text{ (rejected)}
 \end{aligned}$$

3. B

$$\begin{aligned}
 \text{Required area} &= \frac{1}{2}(4)(6) \sin 60^\circ \\
 &= 12 \left(\frac{\sqrt{3}}{2} \right) \\
 &= 6\sqrt{3} \text{ cm}^2
 \end{aligned}$$

4. B

$$\begin{aligned}
 \angle AOB &= \angle BOC = \angle COA = 120^\circ \\
 \text{Required area} &= 6^2\pi - 3 \times \frac{1}{2}(6)(6) \sin 120^\circ \\
 &\approx 66.3 \text{ cm}^2
 \end{aligned}$$

5. BLet $OA = r$ cm.

$$\begin{aligned}
 r^2\pi \times \frac{60^\circ}{360^\circ} - \frac{1}{2}r^2 \sin 60^\circ &= 10 \\
 r &\approx 10.5
 \end{aligned}$$

6. B

$$\frac{1}{2}(AB)(BC) \sin 80^\circ = 80$$

$$\frac{1}{2}(AB) \left(\frac{3AB}{2} \right) \sin 80^\circ = 80$$

$$AB \approx 10.4 \text{ cm}$$

7. A

Note that $\sin 70^\circ = \sin 110^\circ$.

$$\text{Area of } \triangle ABD = \frac{1}{2}(EA)(EB) \sin 110^\circ + \frac{1}{2}(EA)(ED) \sin 70^\circ$$

$$= \frac{1}{2}(EA)(EB) \sin 70^\circ + \frac{1}{2}(EA)(ED) \sin 70^\circ$$

$$= \frac{1}{2}(EA)(EB + ED) \sin 70^\circ$$

$$= \frac{1}{2}(EA)(BD) \sin 70^\circ$$

$$\text{Area of } \triangle BCD = \frac{1}{2}(EB)(EC) \sin 70^\circ + \frac{1}{2}(EC)(ED) \sin 110^\circ$$

$$= \frac{1}{2}(EC)(EB + ED) \sin 70^\circ$$

$$= \frac{1}{2}(EC)(BD) \sin 70^\circ$$

$$\text{Required area} = \frac{1}{2}(EA)(BD) \sin 70^\circ + \frac{1}{2}(EC)(BD) \sin 70^\circ$$

$$= \frac{1}{2}(EA + EC)(BD) \sin 70^\circ$$

$$= \frac{1}{2}(AC)(BD) \sin 70^\circ$$

$$\approx 19.7 \text{ cm}^2$$

8. B

Note that $EA = EC = 3 \text{ cm}$.

$$\text{Required area} = \frac{1}{2}(5)(3) \sin 120^\circ$$

$$= \frac{15}{2} \left(\frac{\sqrt{3}}{2} \right)$$

$$= \frac{15\sqrt{3}}{4} \text{ cm}^2$$

9. B

$$\angle BAC = 180^\circ - 90^\circ - 60^\circ = 30^\circ$$

$$AE = 6 \cos 30^\circ = 3\sqrt{3} \text{ cm}$$

$$AD = BC = \frac{6}{\tan 60^\circ} = 2\sqrt{3} \text{ cm}$$

$$\angle DAE = \angle ACB = 60^\circ$$

$$\text{Required area} = \frac{1}{2}(AD)(AE) \sin 60^\circ$$

$$= \frac{9\sqrt{3}}{2} \text{ cm}^2$$

10. B

$$BC = 36 - 8 - 16$$

$$= 12 \text{ cm}$$

$$\text{Let } s = \frac{8 + 12 + 16}{2} = 18 \text{ cm.}$$

$$\begin{aligned} \text{Required area} &= \sqrt{18(18 - 8)(18 - 12)(18 - 16)} \\ &= 12\sqrt{15} \text{ cm}^2 \end{aligned}$$

11. D

$$\text{Let } s = \frac{5 + 6 + 7}{2} = 9 \text{ cm.}$$

$$\begin{aligned} \text{Base area} &= \sqrt{9(9 - 5)(9 - 6)(9 - 7)} \\ &= \sqrt{216} \\ &= 6\sqrt{6} \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Required area} &= 2(6\sqrt{6}) + (5)(8) + (6)(8) + (7)(8) \\ &= (144 + 12\sqrt{6}) \text{ cm}^2 \end{aligned}$$

12. C

$$\begin{aligned} AC &= \sqrt{10^2 + 24^2} \\ &= 26 \text{ cm} \end{aligned}$$

$$\text{Let } s = \frac{11 + 25 + 26}{2} = 31 \text{ cm.}$$

$$\begin{aligned} \text{Required area} &= \sqrt{31(31 - 25)(31 - 11)(31 - 26)} \\ &\approx 136.4 \text{ cm}^2 \end{aligned}$$

13. A

$$\text{Let } s = \frac{9 + 9 + 12}{2} = 15 \text{ cm.}$$

$$\begin{aligned} \text{Required area} &= \sqrt{15(15 - 9)^2(15 - 12)} \\ &= 18\sqrt{5} \text{ cm}^2 \end{aligned}$$

14. D

$$a = 36 \times \frac{3}{3 + 4 + 5} = 9 \text{ cm}$$

$$b = 9 \times \frac{4}{3} = 12 \text{ cm and } c = 9 \times \frac{5}{3} = 15 \text{ cm}$$

$$\text{Let } s = \frac{a + b + c}{2} = 18 \text{ cm.}$$

$$\begin{aligned} \text{Area} &= \sqrt{18(18 - 9)(18 - 12)(18 - 15)} \\ &= 54 \text{ cm}^2 \end{aligned}$$

15. C

$$\frac{\sin \angle BAC}{14} = \frac{\sin 35^\circ}{10}$$

$$\angle BAC \approx 53.4^\circ \text{ or } 127^\circ$$

2 triangles can be constructed.

16. C

$$\frac{BP}{\sin x} = \frac{AB}{\sin \angle APB} \quad \text{and} \quad \frac{PC}{\sin y} = \frac{AC}{\sin \angle APC}$$

$$BP = \frac{AB \sin x}{\sin \angle APB} \quad PC = \frac{AC \sin y}{\sin \angle APC}$$

Note that $\sin \angle APB = \sin \angle APC$.

$$\begin{aligned} \frac{BP}{PC} &= \frac{AB \sin x}{\sin \angle APB} \div \frac{AC \sin y}{\sin \angle APC} \\ &= \frac{\sin x}{\sin y} \end{aligned}$$

$$BP : PC = \sin x : \sin y$$

17. B

Note that $AD \parallel BC$ and $\angle APB = \beta$. In $\triangle ABP$,

$$\frac{AP}{\sin \alpha} = \frac{AB}{\sin \beta}$$

$$\frac{AP}{\sin \alpha} = \frac{2AP}{\sin \beta}$$

$$\frac{\sin \beta}{\sin \alpha} = \frac{2AP}{AP}$$

$$\sin \alpha : \sin \beta = 1 : 2$$

18. D

When $b = 20$, $c = 14$, $\angle C = 50^\circ$,

$$\frac{\sin B}{20} = \frac{\sin 50^\circ}{14}$$

$$\sin B \approx 1.09 > 1$$

The equation has no solution.

Thus, the triangle in D cannot exist.

19. A

$$\angle BAC = 180^\circ - 62^\circ - 74^\circ = 44^\circ$$

$$\frac{BC}{\sin 44^\circ} = \frac{4}{\sin 62^\circ}$$

$$BC = \frac{4 \sin 44^\circ}{\sin 62^\circ}$$

20. D

$$a^2 = b^2 + c^2 - 2bc \cos 120^\circ$$

$$a^2 = b^2 + c^2 - 2bc \left(-\frac{1}{2}\right)$$

$$a^2 = b^2 + c^2 + bc$$

21. B

$$\cos B = \sqrt{\cos^2 B} = \sqrt{1 - \sin^2 B} = \sqrt{1 - \left(\frac{3}{5}\right)^2} = \frac{4}{5}$$

$$AC^2 = 16^2 + 15^2 - 2(16)(15) \cos B$$

$$AC = \sqrt{97} \text{ cm}$$

22. C

Let O be the mid-point of BD .

Note that $\triangle ABO$, $\triangle AEO$ and $\triangle DEO$ are equilateral, we have $BD = 16$ cm.

$$16^2 = 10^2 + 10^2 - 2(10)(10) \cos \angle BCD$$

$$\angle BCD \approx 106^\circ$$

23. D

Let r cm be the radius of the circle.

$$8^2 = r^2 + r^2 - 2r^2 \cos 120^\circ$$

$$64 = 3r^2$$

$$r \approx 4.62$$

$$\begin{aligned} \text{Required area} &= r^2 \pi - \frac{1}{2}(8)(8) \sin 60^\circ \\ &\approx 39.3 \text{ cm}^2 \end{aligned}$$

24. B

$$10^2 = 11^2 + 3^2 - 2(11)(3) \cos \angle CAB$$

$$\angle CAB \approx 63.0^\circ$$

$$CD = 11 \sin \angle CAB$$

$$\approx 9.80 \text{ cm}$$

25. C

$$10^2 = 9^2 + 12^2 - 2(9)(12) \cos \theta$$

$$\theta \approx 55^\circ$$

26. B

Consider $\triangle BCD$.

$$10^2 = 5^2 + 6^2 - 2(5)(6) \cos \angle BCD$$

$$\angle BCD \approx 131^\circ$$

$$\angle ABC = 180^\circ - \angle BCD \approx 49^\circ$$

27. D

$$3^2 = 5^2 + 7^2 - 2(5)(7) \cos A \quad \text{and} \quad 5^2 = 7^2 + 3^2 - 2(7)(3) \cos B$$

$$\cos A = \frac{13}{14}$$

$$\cos B = \frac{11}{14}$$

$$\cos A : \cos B = \frac{13}{14} : \frac{11}{14} = 13 : 11$$

28. A

$$x^2 = y^2 + z^2 - 2yz \cos 135^\circ$$

$$x^2 = y^2 + z^2 - 2zy \left(-\frac{\sqrt{2}}{2} \right)$$

$$x^2 = y^2 + z^2 + \sqrt{2}yz$$

29. D

$$AC^2 = 3^2 + 4^2 - 2(3)(4) \cos 120^\circ$$

$$AC \approx 6.08$$

30. B

$$4^2 = 6^2 + 4^2 - 2(6)(4) \cos \angle ACB$$

$$\angle ACB \approx 41.4^\circ$$

$$AD^2 = 4^2 + 4^2 - 2(4)(4) \cos \angle ACB$$

$$AD \approx 2.83 \text{ cm}$$

Conventional Questions

31. (a) Let $s = \frac{12 + 5 + 15}{2} = 16$ cm.
 Area of $\triangle ACD = \sqrt{16(16 - 15)(16 - 12)(16 - 5)}$ 1M
 $\approx 26.5 \text{ cm}^2$ 1A
- (b) Area of $\triangle BDC = \text{area of } \triangle ADC$
 $= \frac{1}{2}(5)(17) \sin \angle BDC$ 1M+1M
 $\angle BDC \approx 38.6^\circ \text{ or } 141^\circ \text{ (rejected)}$ 1A
32. (a) $\frac{AC}{\sin 40^\circ} = \frac{AB}{\sin 110^\circ}$ 1M
 $AB \approx 8.77 \text{ cm}$ 1A
- (b) $\angle A = 180^\circ - 40^\circ - 110^\circ = 30^\circ$
 Required area $= \frac{1}{2} \times 6 \times AB \times \sin 30^\circ$ 1M
 $\approx 13.2 \text{ cm}^2$ 1A
33. Let the length of XZ be x cm.
 $\frac{x}{\sin(180^\circ - 23^\circ - 61^\circ)} = \frac{YZ}{\sin 23^\circ}$ 1M
 $YZ = \frac{x \sin 23^\circ}{\sin 96^\circ}$ 1A
- $\frac{1}{2}(YZ)(XZ) \sin 61^\circ = 40$ 1M
 $\frac{x^2 \sin 23^\circ \sin 61^\circ}{2 \sin 96^\circ} = 40$ 1M
 $x \approx 15.3$ 2A
- Length of XZ is 15.3 cm.

34. (a) In $\triangle ABD$,

$$\angle BAD = 180^\circ - \theta$$

$$\begin{aligned} BD^2 &= 5^2 + 8^2 - 2(5)(8) \cos(180^\circ - \theta) \\ &= 89 + 80 \cos \theta \end{aligned} \quad 1\text{M}$$

In $\triangle BCD$,

$$\begin{aligned} BD^2 &= 6^2 + 7^2 - 2(6)(7) \cos \theta \\ &= 85 - 84 \cos \theta \end{aligned} \quad 1\text{M}$$

Therefore,

$$\begin{aligned} 89 + 80 \cos \theta &= 85 - 84 \cos \theta \\ (80 + 84) \cos \theta &= (85 - 89) \\ \cos \theta &= -\frac{1}{41} \end{aligned} \quad \begin{array}{l} 1\text{M} \\ 1 \end{array}$$

(b) $BD^2 = 89 + 80 \cos \theta$

$$BD^2 = 89 + 80 \left(-\frac{1}{41} \right) \quad 1\text{M}$$

$$BD \approx 9.33 \quad 1\text{A}$$