**REG-VAR-2223-ASM-SET 4-MATH**
**Suggested solutions**

**Conventional Questions**

1. (a) Let $g(x) = ax + bx^2$, where $a$ and $b$ are non-zero constants. 1A

$$\begin{cases} -93 = -3a + 9b \\ 2 = 2a + 4b \end{cases}$$ 1M

Solving, $a = 13$ and $b = -6$ 1A

(b) $13x - 6x^2 = x + k$

$-6x^2 + 12x - k = 0$

$\Delta = 12^2 - 4(-6)(-k) \geq 0$ 1M

$k \leq 6$ 1A

2. (a) Let $C = a + \dfrac{b}{n}$, where $a$ and $b$ are non-zero constants. 1A

$$\begin{cases} 4.1 = a + \dfrac{b}{25\,000} \\ 3 = a + \dfrac{b}{80\,000} \end{cases}$$ 1M

Solving, we have $a = 2.5$ and $b = 40\,000$ 1A

Thus, $C = 2.5 + \dfrac{40\,000}{n}$

(b) When $n = 50\,000$, $C = 2.5 + \dfrac{40\,000}{50\,000} = 3.3$

The profit percent $= \dfrac{6 - 3.3}{3.3} \times 100\%$ 1M

$\approx 81.8\%$ 1A

3. (a) Let $f(x) = a(3x - 5)^2 + bx^3$, where $a$ and $b$ are non-zero constants. 1A

$$\begin{cases} -266 = a(-6 - 5)^2 + b(-2)^3 \\ 49 = a(9 - 5)^2 + b(3)^3 \end{cases}$$ 1M

Solving, we have $a = -2$ and $b = 3$. 1A

Thus, we have $f(x) = -2(3x - 5)^2 + 3x^3$.

(b) $f(x) = 3x^3 - 18x^2 + 60x - 50$

$= (x^2 - 6x + 12)(3x) + 24x - 50$ 1M

Required remainder is $24x - 50$. 1A

4. (a) Let $p(x) = ax + b(x+1)^2$, where $a$ and $b$ are non-zero constants. 1A

$$\begin{cases} 7 = -3a + 4b \\ 3 = a + 4b \end{cases}$$ 1M

Solving, we have $a = -1$ and $b = 1$. 1A

Thus, $p(x) = -x + (x+1)^2$.

(b) $-x + (x+1)^2 = 7 - x^2$ 1M

$2x^2 + x - 6 = 0$

$x = -2 \quad \text{or} \quad \dfrac{3}{2}$ 1A

5. (a) Let $C = a + br^2$, where $a$ and $b$ are non-zero constants. 1A

$$\begin{cases} 67 = a + b \\ 112 = a + 4^2 b \end{cases}$$ 1M

Solving, we have $a = 64$ and $b = 3$. 1A

Required cost $= 64 + 3(3)^2 = \$91$. 1A

(b) Let the radius of the larger sphere be $r$ cm.

$\left(\dfrac{r}{3}\right)^3 = \dfrac{8}{1}$

$r = 6$ 1A

Required cost $= 64 + 3(6)^2 = \$172$ 1A

6. (a) Let $V = at + bt^2$, where $a$ and $b$ are non-zero constants. 1A

$$\begin{cases} 5a + 25b = 190 \\ 15a + 225b = 510 \end{cases}$$ 1M

Solving, we have $a = 40$ and $b = -\dfrac{2}{5}$. So, $V = 40t - \dfrac{2t^2}{5}$. 1A

(b) $V = 40(25) - \dfrac{2(25)^2}{5} = 750$ 1M

Consider the cross section in the figure. When $\theta$ is maximum, the water level touches point $B$.

Let $E$ be a point on $AD$ such that $BE$ is horizontal.

$\dfrac{(AE)(10)}{2}(10) = 1000 - 750$ 1M

$AE = 5\,\text{cm}$

$\theta = \tan^{-1}\dfrac{AE}{10} \approx 26.6^\circ$ 1A

7. (a) Let $I = a + b\sqrt{S}$, where $a$ and $b$ are non-zero constants. 1A

$$\begin{cases} 7400 = a + 200b \\ 8600 = a + 300b \end{cases}$$ 1M

Solving, we have $a = 5000$ and $b = 12$. 1A

Thus, $I = 5000 + 12\sqrt{S}$.

(b) When $S = 640\,000$, $I = 5000 + 12\sqrt{640\,000} = 14\,600$ 1A

His monthly income is \$14 600.

(c) Percentage change $= \dfrac{(5000 + 12\sqrt{4 \times 640\,000}) - 14\,600}{14\,600} \times 100\%$ 1M+1A

$\approx +65.8\%$ 1A

8. (a) Let $x = a + bt$, where $a$ and $b$ are non-zero constants. 1A

$$\begin{cases} 44 = a + 2b \\ 56 = a + 3b \end{cases}$$ 1M

Solving, we have $a = 20$ and $b = 12$. 1A

Thus, $x = 20 + 12t$.

(b) (i) Let $y = kt$, where $k$ is a non-zero constant. 1A

$$28 = 2k$$

$$k = 14$$

Thus, $y = 14t$. 1A

(ii) $20 + 12t = 14t$ 1M

$$t = 10$$

Required time is 10 hours. 1A

9. (a) Let $C = a + \frac{b}{n}$, where $a$ and $b$ are non-zero constants. 1A

$$\begin{cases} 12 = a + \frac{b}{100} \\ 10.5 = a + \frac{b}{400} \end{cases}$$ 1M

Solving, we have $a = 10$ and $b = 200$. 1A

Thus, $C = 10 + \frac{200}{n}$.

(b) $10.2 = 10 + \frac{200}{n}$

$n = 1000$

1000 books are printed. 1A

(c) Profit per cent $= \dfrac{2000(80\%)(13) - 2000\left(10 + \frac{200}{2000}\right)}{2000\left(10 + \frac{200}{2000}\right)} \times 100\%$ 1M+1A

$\approx 2.97\%$ 1A

10. (a) Let $n = a + bx$, where $a$ and $b$ are non-zero constants. 1A

$$\begin{cases} 630 = a + 150b \\ 330 = a + 250b \end{cases}$$ 1M

By solving, we have $a = 1080$ and $b = -3$.

Weekly profit $= nx$

$= (1080 - 3x)(x)$ 1M

$= \$(-3x^2 + 1080x)$ 1

(b) Let the profit of each shirt sold in the previous week be $\$x$.

$$-3(x + 100)^2 + 1080(x + 100) = -3x^2 + 1080x$$ 1M+1A

$$-3x^2 - 600x - 30\,000 + 1080x + 108000 = -3x^2 + 1080x$$

$$-600x + 78\,000 = 0$$

$$x = 130$$ 1A

Required profit is \$130.

11. (a) Let $h = ma + na^2$, where $m$ and $n$ are non-zero constants. 1A

$$\begin{cases} 100 = m + n \\ 280 = 2m + 4n \end{cases}$$ 1M

Solving, $m = 60$ and $n = 40$. 1A

Therefore, $h = 60a + 40a^2$.

(b) $300a = 60a + 40a^2$ 1M

$0 = 40a^2 - 240a$

$a = 0 \quad \text{or} \quad 6$

Therefore, $(a,\ h) = (6,\ 1800)$ or $(0,\ 0)$ (rejected). 1A

(c) $h = 40a^2 + 60a$

$$= 40\left[a^2 + 2\left(\frac{3}{4}\right)a + \left(\frac{3}{4}\right)^2\right] - \frac{45}{2}$$ 1M

$$= 40\left(a + \frac{3}{4}\right)^2 - \frac{45}{2}$$

Minimum value of $h$ is $-\frac{45}{2}$, the corresponding value of $a$ is $-\frac{3}{4}$. 1A+1A

 **END OF PAPER**