

REG-TRIG-2223-ASM-SET 3-MATH**Suggested solutions****Multiple Choice Questions**

1. C	2. A	3. D	4. B	5. D
6. B	7. D	8. A	9. C	10. C
11. A	12. A	13. C	14. A	15. A
16. C	17. C	18. A	19. D	20. B
21. B	22. C	23. C	24. C	

1. CI. ✓. $x = 90^\circ$ only.II. ✗. $x = 0^\circ$ or 180° .III. ✓. $x = 90^\circ$ only.2. A

$$\sin \theta = 5 \tan \theta$$

$$\sin \theta = \frac{5 \sin \theta}{\cos \theta}$$

$$\sin \theta \left(1 - \frac{5}{\cos \theta} \right) = 0$$

$$\sin \theta = 0 \quad \text{or} \quad \cos \theta = 5 \text{ (rejected)}$$

$$\theta = 0^\circ \quad \text{or} \quad 180^\circ$$

3. D

$$3 \tan^2 x = \tan x$$

$$\tan x (3 \tan x - 1) = 0$$

$$\tan x = 0 \quad \text{or} \quad \frac{1}{3}$$

When $\tan x = 0$, $x = 0^\circ$ or 180° or 360° When $\tan x = \frac{1}{3}$, there are 2 roots (in quadrants I and III)

Altogether 5 roots.

4. B

$$2 \tan \theta = \sin \theta$$

$$2 \sin \theta = \sin \theta \cos \theta$$

$$\sin \theta (2 - \cos \theta) = 0$$

$$\sin \theta = 0 \quad \text{or} \quad \cos \theta = 2 \text{ (rejected)}$$

There are 3 roots.

5. D

We have $\sin \theta = 0$ or $\tan \theta = -3$.

When $\sin \theta = 0$, $\theta = 0^\circ$ or 180° or 360° .

When $\tan \theta = -3$, there are two roots.

Thus, there are 5 roots.

6. B

We have $2 \sin^2 x = 1$ or $\sin x = 0$.

When $\sin x = 0$, $x = 180^\circ$

When $2 \sin^2 x = 1$, $\sin x = \pm \frac{1}{\sqrt{2}}$ and there are four roots within the required range.

Thus, there are 5 roots.

7. D

$$\sin \theta = \sin^3 \theta$$

$$0 = \sin \theta (\sin^2 \theta - 1)$$

$$\sin \theta = 0 \quad \text{or} \quad \pm 1$$

With reference to the graph of sine, there are 5 roots.

8. A

$$3 \cos^2 \theta + \cos \theta - 4 = 0$$

$$\cos \theta = 1 \quad \text{or} \quad -\frac{4}{3} \text{ (rejected)}$$

$$\theta = 0^\circ$$

There is 1 root.

9. C

$$\frac{2}{1 - \sin \theta} - \frac{2}{1 + \sin \theta} = 4$$

$$2(1 + \sin \theta) - 2(1 - \sin \theta) = 4(1 - \sin \theta)(1 + \sin \theta)$$

$$4 \sin^2 \theta + 4 \sin \theta - 4 = 0$$

$$\sin \theta \approx 0.618 \quad \text{or} \quad -1.62$$

$$\theta \approx 38.2^\circ \quad \text{or} \quad 142^\circ$$

10. C

$$7 \cos^2 x = \cos x + 6$$

$$7 \cos^2 x - \cos x - 6 = 0$$

$$\cos x = 1 \quad \text{or} \quad -\frac{6}{7}$$

When $\cos x = 1$, $x = 0^\circ$ (exclude 360°)

When $\cos x = -\frac{6}{7}$, $x = 180^\circ - \cos^{-1} \frac{6}{7}$ or $180^\circ + \cos^{-1} \frac{6}{7}$

There are 3 roots.

11. A

$$9 \cos^4 \theta - 13 \cos^2 \theta + 4 = 0$$

$$\cos^2 \theta = 1 \quad \text{or} \quad \frac{4}{9}$$

$$\cos \theta = \pm 1 \quad \text{or} \quad \pm \frac{2}{3}$$

When $\cos \theta = \pm 1$, $\theta = 180^\circ$

When $\cos \theta = \pm \frac{2}{3}$, there are 3 roots (one root per quadrant).

There are total 4 roots.

12. A

$$3 \tan^2 \theta + \tan \theta - 2 = 0$$

$$\tan \theta = \frac{2}{3} \quad \text{or} \quad -1$$

There are two roots for $\tan \theta = \frac{2}{3}$ and two roots for $\tan \theta = -1$.

Thus, there are 4 roots.

13. C

$$4 \cos^2 \theta - 7 \sin \theta - 7 = 0$$

$$4(1 - \sin^2 \theta) - 7 \sin \theta - 7 = 0$$

$$-4 \sin^2 \theta - 7 \sin \theta - 3 = 0$$

$$\sin \theta = -1 \quad \text{or} \quad -\frac{3}{4}$$

$\sin \theta = -1$ has one root and $\sin \theta = -\frac{3}{4}$ has two roots.

14. A

$$\cos x = \tan x$$

$$\cos^2 x = \sin x$$

$$-\sin^2 x - \sin x + 1 = 0$$

$$\sin x \approx -1.62 \text{ (rejected)} \quad \text{or} \quad 0.618$$

There are 2 roots and hence 2 intersections.

15. A

$$\frac{3}{\tan^2(90^\circ - \alpha)} - \frac{5}{\sin(270^\circ + \alpha)} = \frac{1}{\cos^2 \alpha}$$

$$3 \tan^2 \alpha + \frac{5}{\cos \alpha} = \frac{1}{\cos^2 \alpha}$$

$$3 \sin^2 \alpha + 5 \cos \alpha = 1$$

$$-3 \cos^2 \alpha + 5 \cos \alpha + 2 = 0$$

$$\cos \alpha = -\frac{1}{3} \quad \text{or} \quad 2 \text{ (rejected)}$$

There are two roots for $\cos \alpha = -\frac{1}{3}$. Required number of roots is 2.

16. C

$$\cos^2 x - \sin x = 1$$

$$1 - \sin^2 x - \sin x = 1$$

$$\sin x(\sin x + 1) = 0$$

$$\sin x = 0 \quad \text{or} \quad -1$$

There are three roots for $\sin x = 0$ and one root for $\sin x = -1$.

Thus, there are 4 roots.

17. C

$$5(1 - \sin^2 x) = \sin x + 1$$

$$0 = 5 \sin^2 x + \sin x - 4$$

$$\sin x = \pm \frac{2}{\sqrt{5}}$$

When $\sin x = \frac{2}{\sqrt{5}}$, there are two roots; when $\sin x = -\frac{2}{\sqrt{5}}$, there are also two roots.

There are total 4 roots.

18. A

$$3 \tan x = 2 \sin(90^\circ - x)$$

$$\frac{3 \sin x}{\cos x} = 2 \cos x$$

$$3 \sin x = 2 \cos^2 x$$

$$3 \sin x = 2(1 - \sin^2 x)$$

$$2 \sin^2 x + 3 \sin x - 2 = 0$$

$$\sin x = 0.5 \quad \text{or} \quad -2 \text{ (rejected)}$$

$$x = 30^\circ \quad \text{or} \quad 150^\circ$$

There are 2 roots.

19. D

$$2 \cos^2 \theta = 2 - \sin \theta$$

$$2(1 - \sin^2 \theta) = 2 - \sin \theta$$

$$-2 \sin^2 \theta + \sin \theta = 0$$

$$\sin \theta = 0 \quad \text{or} \quad \frac{1}{2}$$

When $\sin \theta = 0$, $\theta = 0^\circ$ or 180° or 360° .

When $\sin \theta = \frac{1}{2}$, $\theta = 30^\circ$ or 150° .

There are 5 roots.

20. B

$$4 + \cos(90^\circ + \theta) = 4 \cos^2 \theta$$

$$4 - \sin \theta = 4(1 - \sin^2 \theta)$$

$$4 \sin^2 \theta - \sin \theta = 0$$

$$\sin \theta = 0 \quad \text{or} \quad \frac{1}{4}$$

The equation $\sin \theta = 0$ has one root.

The equation $\sin \theta = \frac{1}{4}$ has two roots.

Thus, the equation $4 + \cos(90^\circ + \theta) = 4 \cos^2 \theta$ has 3 roots.

21. B

$$\frac{1 + \cos \theta}{\sin \theta} = 3 \tan \theta$$

$$\cos \theta(1 + \cos \theta) = 3 \sin^2 \theta$$

$$\cos \theta + \cos^2 \theta = 3(1 - \cos^2 \theta)$$

$$4 \cos^2 \theta + \cos \theta - 3 = 0$$

$$\cos \theta = \frac{3}{4} \quad \text{or} \quad -1$$

Note that when $\cos \theta = -1$, we have $\theta = 180^\circ$ and $\sin \theta = 0$, which should be rejected.

There are two roots for $\cos \theta = \frac{3}{4}$.

Thus, there are 2 roots.

22. C

A. 3 roots

B. 3 roots

C. 4 roots

D. 2 roots

23. C

$$3 - 6 \cos^2 \theta = 8 \sin \theta \cos \theta$$

$$3(\sin^2 \theta + \cos^2 \theta) - 6 \cos^2 \theta = 8 \sin \theta \cos \theta$$

$$3 \sin^2 \theta - 8 \sin \theta \cos \theta - 3 \cos^2 \theta = 0$$

$$\frac{3 \sin^2 \theta}{\cos^2 \theta} - \frac{8 \sin \theta \cos \theta}{\cos^2 \theta} - 3 = 0$$

$$3 \tan^2 \theta - 8 \tan \theta - 3 = 0$$

$$\tan \theta = -\frac{1}{3} \quad \text{or} \quad 3$$

24. C

$$\sin \theta \tan \theta = 1$$

$$\frac{\sin^2 \theta}{\cos \theta} = 1$$

$$1 - \cos^2 \theta = \cos \theta$$

$$-\cos^2 \theta - \cos \theta + 1 = 0$$

$$\begin{aligned}\cos \theta &= \frac{1 \pm \sqrt{1^2 - 4(-1)(1)}}{-2} \\ &= \frac{-1 \mp \sqrt{5}}{2}\end{aligned}$$

Since $\sin \theta \tan \theta = 1$, $\sin \theta$ and $\tan \theta$ are both positive or both negative.

In any of the case, $\cos \theta > 0$.

$$\text{Thus, } \cos \theta = \frac{\sqrt{5} - 1}{2}.$$

Conventional Questions

25. (a) $\sin(x - 25^\circ) = \frac{1}{2}$

$$x - 25^\circ = 30^\circ \quad \text{or} \quad 180^\circ - 30^\circ$$

$$x = 55^\circ \quad \text{or} \quad 175^\circ$$

1A

(b) $\sqrt{3} \tan(x + 26^\circ) = -1$

$$\tan(x + 26^\circ) = -\frac{1}{\sqrt{3}}$$

1M

$$x + 26^\circ = 180^\circ - 30^\circ \quad \text{or} \quad 360^\circ - 30^\circ$$

$$x = 124^\circ \quad \text{or} \quad 304^\circ$$

1A

(c) $\cos(70^\circ + x) = \cos 80^\circ$

$$70^\circ + x = 80^\circ \quad \text{or} \quad 360^\circ - 80^\circ$$

$$x = 10^\circ \quad \text{or} \quad 210^\circ$$

1A

(d) $\frac{\cos 147^\circ}{\sin(100^\circ - x)} = 1$

$$\cos 147^\circ = \sin(100^\circ - x)$$

$$\sin(x - 100^\circ) = -\cos 147^\circ$$

1M

$$x - 100^\circ = 57^\circ \quad \text{or} \quad 180^\circ - 57^\circ$$

$$x = 157^\circ \quad \text{or} \quad 223^\circ$$

1A

26. (a) $3 \cos^2 x - 10 \cos x + 3 = 0$
- $$\cos x = \frac{1}{3} \quad \text{or} \quad 3 \text{ (rejected)} \quad 1A$$
- $$x \approx 70.5^\circ \quad \text{or} \quad 289^\circ \quad 1A$$
- (b) $12 \cos^2 x = \sin x + 6$
- $$12(1 - \sin^2 x) = \sin x + 6 \quad 1M$$
- $$-12 \sin^2 x - \sin x + 6 = 0$$
- $$\sin x = \frac{2}{3} \quad \text{or} \quad -\frac{3}{4} \quad 1A$$
- $$\sin x = \frac{2}{3} \quad \text{or} \quad \sin x = -\frac{3}{4}$$
- $$x \approx 41.8^\circ \quad \text{or} \quad 138^\circ \quad x \approx 229^\circ \quad \text{or} \quad 311^\circ \quad 1A$$
- (c) $\tan x + 1 = \tan(270^\circ - x)$
- $$\tan x + 1 = \frac{1}{\tan x}$$
- $$\tan^2 x + \tan x = 1$$
- $$\tan^2 x + \tan x - 1 = 0 \quad 1M$$
- $$\tan x = \frac{-1 \pm \sqrt{5}}{2} \quad 1A$$
- $$\tan x = \frac{-1 + \sqrt{5}}{2} \quad \text{or} \quad \tan x = \frac{-1 - \sqrt{5}}{2}$$
- $$x \approx 31.7^\circ \quad \text{or} \quad 212^\circ \quad x \approx 122^\circ \quad \text{or} \quad 302^\circ \quad 1A$$
- (d) $9 \sin x - \frac{10}{\tan x} = 0$
- $$9 \sin x - \frac{10 \cos x}{\sin x} = 0$$
- $$9 \sin^2 x - 10 \cos x = 0$$
- $$9(1 - \cos^2 x) - 10 \cos x = 0 \quad 1M$$
- $$-9 \cos^2 x - 10 \cos x + 9 = 0$$
- $$\cos x = \frac{-5 + \sqrt{106}}{9} \quad \text{or} \quad \frac{-5 - \sqrt{106}}{9} \text{ (rejected)} \quad 1A$$
- $$x \approx 54.0^\circ \quad \text{or} \quad 306^\circ \quad 1A$$