REG-GOF-2223-ASM-SET 4-MATH
Suggested solutions

Conventional Questions

1. (a) $f(x) = x^2 - 4x + 42$

$= [x^2 - 2(x)(2) + (2)^2] + 38$ 1M

$= (x - 2)^2 + 38$

Required coordinates are $(2,\ 38)$. 1A

(b) $A = 6(7 - x) + [9 - (7 - x)]x$ 1M

$= x^2 - 4x + 42$

$= (x - 2)^2 + 38$

$\geq 38 > 35$

The claim is disagreed. 1A

2. (a) $f(x) = 2x^2 - 4kx + 5k^2 + 3$

$= 2[x^2 - 2(x)(k) + (k)^2] + 3k^2 + 3$ 1M

$= 2(x - k)^2 + 3k^2 + 3$

Required coordinates are $\left(k,\ 3k^2 + 3\right)$. 1A

(b) The coordinates of the vertex of the graph of $y = g(x)$ are $(k,\ 0)$.

$g(x) = f(x) - (3k^2 + 3)$ 1M

$= 2(x - k)^2$ 1A

(c) $-2x^2 + 4kx - 5k^2 - 3 = -(2x^2 - 4kx + 5k^2 + 3)$

$= -f(x)$ 1M

The graph of $y = f(x)$ is reflected about the x-axis. 1A

3. (a) $f(x) = 2x^2 - 4x + 3k + 8$

$= 2[x^2 - 2(x)(1) + (1)^2] + 3k + 6$ 1M

$= 2(x - 1)^2 + 3k + 6$

Required coordinates are $(1,\ 3k + 6)$. 1A

(b) $(-k - 1,\ 3k + 6)$ 1A

(c) Area of $\triangle POQ$

$= \frac{1}{2}[1 - (-k - 1)](3k + 6)$ 1M

$= \frac{1}{2}(k + 2)(3k + 6)$

$> \frac{1}{2}(2)(6) = 6 \quad (\text{since } k > 0)$ 1M

The claim is agreed. 1A

4. (a) $f(x) = x^2 + 4kx + 7k^2 - 4$

$= [x^2 + 2(x)(2k) + (2k)^2] + 3k^2 - 4$ 1M

$= (x + 2k)^2 + 3k^2 - 4$

Required coordinates are $\left(-2k,\ 3k^2 - 4\right)$. 1A

(b) The coordinates of U are $\left(-2k - 4,\ 3k^2 - 4\right)$.

The coordinates of V are $\left(-2k,\ 10 - 3k^2\right)$. 1A

$$\sqrt{[(-2k - 4) - 2]^2 + [(3k^2 - 4) - 0]^2} = \sqrt{[(-2k) - 2]^2 + [(10 - 3k^2) - 0]^2} \quad \text{1M}$$

$$(2k + 6)^2 + (3k^2 - 4)^2 = (2k + 2)^2 + (10 - 3k^2)^2$$

$$36k^2 + 16k - 52 = 0$$

$$k = 1 \quad \text{or} \quad \frac{13}{9} \text{ (rejected)} \quad \text{1A}$$

The coordinates of U and V are $(-6,\ -1)$ and $(-2,\ 7)$ respectively.

Required equation is

$$(x - 2)^2 + (y - 0)^2 = (-6 - 2)^2 + (-1 - 0)^2$$

$$(x - 2)^2 + y^2 = 65 \quad \text{1A}$$

5. (a) $f(2) = \dfrac{1}{k}[2^2 + (2k + 8)(2) + (25k - 20)]$

$= \dfrac{1}{k}(29k)$

$= 29$

The graph of $y = f(x)$ passes through F. 1

(b) (i) $g(x) = f(-x) + 8$ 1M

$= \dfrac{1}{k}[x^2 - (2k + 8)x + (33k - 20)]$

$= \dfrac{1}{k}\left[x^2 - 2(x)(k + 4) + (k + 4)^2\right] + \dfrac{-k^2 + 25k - 36}{k}$ 1M

$= \dfrac{1}{k}[x - (x + 4)]^2 + \dfrac{-k^2 + 25k - 36}{k}$ 1M

Required coordinates are $\left(k + 4,\ \dfrac{-k^2 + 25k - 36}{k}\right)$. 1A

(ii) $\dfrac{-k^2 + 25k - 36}{k} = 12$ 1M

$-k^2 + 13k - 36 = 0$ 1M

$k = 4 \quad \text{or} \quad 9$ 1A

(iii) The coordinates of G are $(-2,\ 37)$. 1A

Note that $\angle FVO = 90°$.

(slope of GO)(slope of GF)

$= \dfrac{37 - 0}{-2 - 0} \times \dfrac{37 - 29}{-2 - 2}$ 1M

$= 37 \neq -1$

We have $\angle FGO \neq 90°$. 1A

Thus, F, G, O and V are not concyclic. 1A

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