REG-MAE-2223-ASM-SET 2-MATH
Suggested solutions

Conventional Questions

1. (a)
$$\left(x+\frac{1}{x}\right)^2 = a^2$$
$$x^2+2+\frac{1}{x^2} = a^2$$
$$x^2+\frac{1}{x^2} = a^2-2 \qquad \text{1A}$$

(b) Let $x+\frac{1}{x} = a$.
$$6\left(x^2+\frac{1}{x^2}\right)-5\left(x+\frac{1}{x}\right)-38 = 0$$
$$6(a^2-2)-5(a)-38 = 0 \qquad \text{1M}$$
$$6a^2-5a-50 = 0$$
$$a = \frac{10}{3} \quad \text{or} \quad -\frac{5}{2} \qquad \text{1A}$$

When $a = \frac{10}{3}$,
$$x+\frac{1}{x} = \frac{10}{3}$$
$$x^2-\frac{10}{3}x+1 = 0$$
$$x = 3 \quad \text{or} \quad \frac{1}{3} \qquad \text{1A}$$

When $a = -\frac{5}{2}$,
$$x+\frac{1}{x} = -\frac{5}{2}$$
$$x^2+\frac{5}{2}x+1 = 0$$
$$x = -0.5 \quad \text{or} \quad -2 \qquad \text{1A}$$

Therefore, $x = -2$ or -0.5 or $\frac{1}{3}$ or 3.

2. (a) $\sqrt{2x}+\dfrac{4}{\sqrt{2x}}=4$

$(\sqrt{2x})^2-4\sqrt{2x}+4=0$ 1M

$\sqrt{2x}=2$ 1M

$2x=2^2$

$x=2$ 1A

(b) $x^5-4x=3x^3$

$x(x^4-3x^2-4)=0$ 1M

$x(x^2-4)(x^2+1)=0$

$x(x+2)(x-2)(x^2+1)=0$ 1M

$x=0$ or ± 2 1A

3. $\sqrt{a}+\sqrt{2a+1}=5$

$\sqrt{a}=5-\sqrt{2a+1}$

$a=25-10\sqrt{2a+1}+2a+1$ 1M

$10\sqrt{2a+1}=26+a$

$100(2a+1)=a^2+52a+676$ 1M

$0=a^2-148a+576$

$a=4$ or 144 (rejected) 2A

4. $(2-\sqrt{x})(3+\sqrt{x})=6x$

$-x-\sqrt{x}+6=6x$

$-7x-\sqrt{x}+6=0$ 1A

$\sqrt{x}=\dfrac{6}{7}$ or -1 (rejected) 1M+1A

$x=\dfrac{36}{49}$ 1A

5. $6(\sqrt{x}-3)^2+13(\sqrt{x}-3)-5=0$

$\sqrt{x}-3=\dfrac{1}{3}$ or $-\dfrac{5}{2}$ 1A+1A

$\sqrt{x}=\dfrac{10}{3}$ or $\dfrac{1}{2}$

$x=\dfrac{100}{9}$ or $\dfrac{1}{4}$ 1A

6. $x^{\frac{4}{5}}-13x^{\frac{2}{5}}+36=0$

$(x^{\frac{2}{5}})^2-13x^{\frac{2}{5}}+36=0$ 1M

$x^{\frac{2}{5}}=4$ or 9 1A

$x=4^{\frac{5}{2}}$ or $9^{\frac{5}{2}}$

$x=32$ or 243 1A

7.
$$\log_4 x + \log_x 16 = 3$$
$$\frac{\log x}{2\log 2} + \frac{4\log 2}{\log x} = 3 \qquad \text{1M}$$
$$(\log x)^2 + 8(\log 2)^2 = 6(\log 2)(\log x)$$
$$(\log x)^2 - 6(\log x)(\log x) + 8(\log 2)^2 = 0 \qquad \text{1A}$$
$$(\log x - 4\log 2)(\log x - 2\log 2) = 0 \qquad \text{1M}$$
$$\log x = 4\log 2 \quad \text{or} \quad 2\log 2 \qquad \text{1A}$$
$$x = 16 \quad \text{or} \quad 4 \qquad \text{1A}$$

8.
$$\log_2 \sqrt{x+4} = \log_{16}(2x^2 + 12x + 11)$$
$$\frac{\frac{1}{2}\log(x+4)}{\log 2} = \frac{\log\left(2x^2 + 12x + 11\right)}{\log 16} \qquad \text{1M}$$
$$\frac{\log(x+4)}{2\log 2} = \frac{\log\left(2x^2 + 12x + 11\right)}{4\log 2} \qquad \text{1M}$$
$$2\log(x+4) = \log\left(2x^2 + 12x + 11\right)$$
$$(x+4)^2 = 2x^2 + 12x + 11 \qquad \text{1M}$$
$$x^2 + 4x - 5 = 0$$
$$(x+5)(x-1) = 0$$
$$x = 1 \quad \text{or} \quad -5 \text{ (rejected)} \qquad \text{1A}$$

9. (a)
$$\log_{(x+1)} 512 = 3$$
$$(x+1)^3 = 512 \qquad \text{1M}$$
$$x + 1 = \sqrt[3]{512}$$
$$x = 7 \qquad \text{1A}$$

(b)
$$\log(2x+5) + \log(4-x) - \log(x+4) = \log 5$$
$$\log[(2x+5)(4-x)] = \log[5(x+4)] \qquad \text{1M}$$
$$(2x+5)(4-x) = 5(x+4)$$
$$-2x^2 + 3x + 20 = 5x + 20$$
$$-2x^2 - 2x = 0 \qquad \text{1A}$$
$$-2x(x+1) = 0$$
$$x = -1 \quad \text{or} \quad 0 \qquad \text{1A}$$

10. $\log_3(x-3) - \log_9(x-5) = 1$

$$\frac{\log(x-3)}{\log 3} - \frac{\log(x-5)}{\log 9} = 1 \qquad \text{1M}$$

$$\frac{\log(x-3)}{\log 3} - \frac{\log(x-5)}{2\log 3} = 1$$

$$2\log(x-3) - \log(x-5) = 2\log 3$$

$$\log\frac{(x-3)^2}{x-5} = \log 9 \qquad \text{1M}$$

$$\frac{(x-3)^2}{x-5} = 9$$

$$x^2 - 6x + 9 = 9x - 45$$

$$x^2 - 15x + 54 = 0 \qquad \text{1M}$$

$$x = 6 \quad \text{or} \quad 9 \qquad \text{1A}$$

11. $\log(2x+1) + \log(3x-7) = \log(11x+1)$

$$\log[(2x+1)(3x-7)] = \log(11x+1) \qquad \text{1M}$$

$$(2x+1)(3x-7) = 11x+1 \qquad \text{1M}$$

$$6x^2 - 11x - 7 = 11x + 1$$

$$6x^2 - 22x - 8 = 0$$

$$3x^2 - 11x - 4 = 0 \qquad \text{1A}$$

$$(x-4)(3x+1) = 0$$

$$x = 4 \quad \text{or} \quad -\frac{1}{3} \text{ (rejected)} \qquad \text{2A}$$

12. (a) $z = \dfrac{5(a+i)}{a-i} \times \dfrac{a+i}{a+i}$ 1M

$$= \frac{5(a^2-1) + 10ai)}{a^2+1}$$

$$= \frac{5(a^2-1)}{a^2+1} + \frac{10a}{a^2+1}i \qquad \text{1A}$$

(b) $\dfrac{5(a^2-1)}{a^2+1} - \dfrac{10a}{a^2+1} = 1$ 1M

$$5a^2 - 10a - 5 = a^2 + 1$$

$$4a^2 - 10a - 6 = 0$$

$$a = 3 \quad \text{or} - 0.5 \text{ (rejected)} \qquad \text{1A}$$

$z = 4 + 3i$, and the two roots of the equation are $4 \pm 3i$.

$s = (4+3i)(4-3i) = 25$ 1A

13. (a) $f(x) = -x^2 + 25x$

$$= -\left[x^2 - 2(x)\left(\frac{25}{2}\right) + \left(\frac{25}{2}\right)^2\right] + \frac{625}{4} \quad \text{1M}$$

$$= -\left(x - \frac{25}{2}\right)^2 + \frac{625}{4} \quad \text{1A}$$

Thus, maximum value of $f(x)$ is $\frac{625}{4}$. 1A

(b) (i) $900 = k \times 1.5^2$

$k = 400$ 1A

(ii) $1500 = 400 \times 1.5^m$

$1.5^m = \frac{15}{4}$

$m \log 1.5 = \log \frac{15}{4}$ 1M

$m \approx 3.26$

The density reaches 1500μ g/m^3 after 3.26 hours. 1A

(c) (i) $0 = 400 \times 1.5^{n+4} - 81 \times 2.25^n$ 1M

$0 = 2025^4 \times 1.5^n - 81 \times 1.5^{2n}$

$0 = 1.5^n(2025 - 81 \times 1.5^n)$

$1.5^n = \frac{2025}{81}$ or 0 (rejected)

$n \log 1.5 = \log \frac{2025}{81}$ 1M

$n \approx 7.94$

X is removed 7.94 hours after adding the chemicals. 1A

(ii) $E = 400 \times 1.5^{n+4} - 81 \times 2.25^n$

$= 400 \times 1.5^4 \times 1.5^n - 81 \times 1.5^{2n}$

$= -81y^2 + 2025y$ 1A

(iii) By (c)(ii), $E = 81(-y^2 + 25y)$.

By (a),

$$E = 81\left[-\left(y - \frac{25}{2}\right)^2 + \frac{625}{4}\right]$$

$$= -81\left(y - \frac{25}{2}\right)^2 + \frac{50\,625}{4}$$

Thus, the greatest density of pollutant X is $12\,656\mu$ g/m^3. 1A

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