REG-LOG-2223-ASM-SET 4-MATH
Suggested solutions

Multiple Choice Questions

1. C	2. D	3. A	4. D	5. B
6. B	7. C	8. B	9. B	10. B
11. D	12. B	13. A	14. B	15. C
16. D	17. C	18. A		

1. C

Suppose it takes n months.

$$\begin{aligned} 4000(1-8\%)^n &< 1000 \\ 0.92^n &< 0.25 \\ n\log 0.92 &< \log 0.25 \\ n &> 16.6 \end{aligned}$$

It takes 17 months.

2. D

Suppose it takes n years.

$$\begin{aligned} 550\,000(1+3\%)^3(1+2.5\%)^{n-3} &> 800\,000 \\ 1.025^{n-3} &> \frac{16}{11(1.03)^3} \\ (n-3)\log 1.025 &> \log\frac{16}{11(1.03)^3} \\ n &> 14.6 \end{aligned}$$

It takes 15 years.

3. A

$$\begin{aligned} 2(1+2.5\%)^n &> 2.8 \\ 1.025^n &> 1.4 \\ n\log 1.025 &> \log 1.4 \\ n &> 13.6 \end{aligned}$$

It takes 14 years.

4. D

Note that when $t = 0$, $N = A$.

Suppose it takes n years.

$$\begin{aligned} A(5^{-n}) &= \frac{1}{4}A \\ 5^{-n} &= \frac{1}{4} \\ -n \log 5 &= \log \frac{1}{4} \\ n &= \frac{-\log 4}{-\log 5} \\ &= \log_5 4 \end{aligned}$$

5. B

$$\begin{aligned} 50\,000\left(1 + \frac{12\%}{12}\right)^n &> 12\,000 + 50\,000 \\ 1.01^n &> 1.24 \\ n \log 1.01 &> \log 1.24 \\ n &> 21.6 \end{aligned}$$

It takes 22 months.

6. B

$$\begin{aligned} 420\,000 \times (1 - 6\%)^t &< 250\,000 \\ 0.94^t &< \frac{25}{42} \\ t \log 0.94 &< \log \frac{25}{42} \\ t &> 8.38 \end{aligned}$$

It takes 9 years.

7. C

Take logarithm on all numbers.

Note that $(-345)^{768} = 345^{768}$.

A. $768 \log 345 \approx 1949.0$

B. $-786 \log 453 \approx -2087.7$

C. $867 \log \frac{1}{435} \approx -2287.6$

D. $876 \log \frac{2}{543} \approx -2132.0$

Number in option C is the least after taking logarithm.

The answer is C.

8. B

Take logarithm on all numbers.

A. $241 \log 124 \approx 504.5$

B. $214 \log 241 \approx 509.8$

C. $142 \log 412 \approx 371.3$

D. $124 \log 421 \approx 325.4$

Number in option B is the greatest after taking logarithm. The original number is also the greatest among all.

9. B

Take logarithms on all numbers:

A. $325 \log 352 \approx 828$

B. $352 \log 235 \approx 835$

C. $253 \log 523 \approx 688$

D. $235 \log 532 \approx 641$

The greatest number is therefore 235^{532}.

10. B

Take logarithm on the numbers.

A. $\rightarrow 251 \log 125 \approx 526.3$

B. $\rightarrow 251 \log 152 \approx 547.6$

C. $\rightarrow 215 \log 251 \approx 515.9$

D. $\rightarrow 152 \log 521 \approx 413.0$

Number in option B is the greatest.

11. D

Take logarithm on the numbers.

A. $500 \log 2010 \approx 1652$

B. $1020 \log 1500 \approx 3240$

C. $1500 \log 1020 \approx 4513$

D. $2010 \log 500 \approx 5425$

The greatest number is 500^{2010}.

12. B

Take logarithm on all numbers.

A. $600 \log 0.1 = -600$

B. $900 \log 0.2 \approx -629$

C. $1200 \log 0.3 \approx -627$

D. $1500 \log 0.4 \approx -597$

Number in option B is the least after taking logarithm.
The answer is B.

13. A

Take logarithms on the two expressions with higher indices.

A. $\log 2013^{2014} = 2014 \log 2013 \approx 6654$

B. $\log 2015^{2012} = 2012 \log 2015 \approx 6648$

2013^{2014} has the greatest value.

14. B

Take log on the 'positive' part.

A. $589 \log 111 \approx 1205$

B. $577 \log 123 \approx 1206$

C. $565 \log 135 \approx 1204$

D. $553 \log 147 \approx 1199$

The 'positive' part of the number in B is the greatest. So, it gives the least value when it is negative.

15. C

$$\begin{aligned} \log_2 6789^{2345} &= 2345 \log_2 6789 \\ &\approx 30\,000 \\ 6789^{2345} &\approx 2^{30\,000} \\ 6789^{2345} &\approx 8^{10\,000} \end{aligned}$$

16. D

$$\begin{aligned} \log 2012^{6054} &= 6054 \log 2012 \\ &\approx 20\,000 \\ 2012^{6054} &\approx 10^{20\,000} \end{aligned}$$

The answer is D.

17. C

$\log 123^{456} = 456 \log 123$

≈ 953

$123^{456} \approx 10^{953}$

The answer is C.

18. A

$400^{1000} << 5000^{1200}$.

So, $\frac{5000^{3000}}{5000^{1200} + 400^{1000}} \approx \frac{5000^{3000}}{5000^{1200}} = 5000^{1800}$.

Since $\log 5000^{1800} = 1800 \log 5000 \approx 6660$, $5000^{1800} \approx 10^{6660}$.

Conventional Questions

19. $\log_4 y = \frac{6}{2}x + 6$ 1M+1A

$y = 4^{3x+6}$ 1A

20. $\log_{16} y - 0 = -\frac{1}{4}(\log_8 x - 4)$ 1M

$\log_{16} y = \log_8 8x^{-\frac{1}{4}}$

$\frac{\log y}{4\log 2} = \frac{\log 8x^{-\frac{1}{4}}}{3\log 2}$ 1M

$y = \left(8x^{-\frac{1}{4}}\right)^{\frac{4}{3}}$

$= 16x^{-\frac{1}{3}}$ 1A

21. (a) $m = \frac{5-2}{0-6} = -\frac{1}{2}$ 1A

$c = 5$ 1A

(b) (i) $\log y = -\frac{x}{2} + 5$

$y = 10^{-\frac{x}{2}+5}$ 1M+1A

(ii) When $x = 8$, $y = 10^{-\frac{8}{2}+5} = 10$. 1A

22. Slope $= \frac{2}{4} = \frac{1}{2}$

$\log_{16} y = \frac{1}{2}x - 2$ 1M

$y = 16^{\frac{x}{2}-2}$ 1M

$= \frac{1}{256} \cdot 4^x$

So, $a = \frac{1}{256}$ and $b = 4$. 1A

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