REG-LOG-2223-ASM-SET 2-MATH
Suggested solutions

Multiple Choice Questions

1. A	2. C	3. C	4. A	5. C
6. B	7. C	8. A	9. B	10. D
11. B	12. D	13. D	14. C	15. B
16. B	17. D	18. B	19. B	20. C
21. D	22. A	23. A	24. C	25. C
26. A	27. C	28. B	29. B	30. B

1. A

   A. $\log_4 10 \approx 1.66$

   B. $\log_8 10 \approx 1.11$

   C. $\log_6 15 \approx 1.51$

   D. $\log_{15} 4 \approx 0.512$

   The answer is A.

2. C

$$\begin{aligned}\log_{2\sqrt{2}} 64 &= \frac{\log 64}{\log 2\sqrt{2}} \\ &= \frac{\log 2^6}{\log 2^{1+\frac{1}{2}}} \\ &= \frac{6\log 2}{\frac{3}{2}\log 2} \\ &= 4\end{aligned}$$

3. C

$$\begin{aligned}\log_2 32 \times \log_2 \sqrt{128} &= \log_2 2^5 \times \log_2 2^{\frac{7}{2}} \\ &= 5 \times \frac{7}{2} \\ &= 17.5\end{aligned}$$

4. A

$$\begin{aligned}\log_{\frac{1}{\sqrt{5}}} 25 &= \frac{\log 25}{\log \frac{1}{\sqrt{5}}} \\ &= \frac{2\log 5}{-\frac{1}{2}\log 5} \\ &= -4\end{aligned}$$

5. C

$$(\log_4 27)(\log_3 2) = \frac{3\log 3}{2\log 2} \times \frac{\log 2}{\log 3}$$
$$= \frac{3}{2}$$

6. B

$$9^{\log_3 a} = (3^2)^{\log_3 a}$$
$$= (3^{\log_3 a})^2$$
$$= a^2$$

7. C

We have $\log b > \log a > 0$.
Therefore, $\log_a b = \frac{\log b}{\log a} > 1$.

8. A

$$(\log_b a^3)(\log_a b)^2 = \frac{3\log a}{\log b} \times \left(\frac{\log b}{\log a}\right)^2$$
$$= 3 \times \frac{\log b}{\log a}$$
$$= 3\log_a b$$

9. B

$$\frac{\log_a b}{\log_b a} = \frac{\log b}{\log a} \times \frac{\log a}{\log b}$$
$$= 1$$

10. D

$$\frac{\log_3 \frac{16x^2}{5}}{2\log_3(4x) - \log_3 5} = \frac{\log_3 16 + 2\log_3 x - \log_3 5}{2(\log_3 4 + \log_3 x) - \log_3 5}$$
$$= \frac{2\log_3 4 + 2\log_3 x - \log_3 5}{2\log_3 4 + 2\log_3 x - \log_3 5}$$
$$= 1$$

11. B

I. ✗. When $x = 5$, $\log_5 x^2 = \log_5 5^2 = 2$ and $\dfrac{1}{2\log_x 5} = \dfrac{1}{2\log_5 5} = \dfrac{1}{2} \neq 2$.

II. ✓. $$\begin{aligned}\frac{\log_3 x}{3\log_3 2} &= \frac{\log x}{\log 3} \div \frac{3\log 2}{\log 3} \\ &= \frac{\log x}{3\log 2} \\ &= \frac{\log x}{\log 8} \\ &= \log_8 x\end{aligned}$$

III. ✗. When $x = 10$, $\log_x 10 = \log 10 = 1$ and $\log x^{-1} = \log 10^{-1} = -1 \neq 1$.

12. D

$$\begin{aligned}a &= \log_3 \frac{1}{b} \\ &= -\log_3 b \\ \frac{1}{a} &= -\frac{1}{\log_3 b}\end{aligned}$$

13. D

$$\begin{aligned}\log_a b^3 \cdot \log_b c^3 &= \frac{3\log b}{\log a} \times \frac{3\log c}{\log b} \\ &= 9 \times \frac{\log c}{\log a} \\ &= 9\log_a c\end{aligned}$$

14. C

$$\begin{aligned}\frac{1}{y} &= \log_2 x \\ &= \frac{\log x}{\log 2} \\ y &= \frac{\log 2}{\log x} \\ &= \log_x 2\end{aligned}$$

15. B

$$\begin{aligned}\frac{\log_x \sqrt{x^3} + 2\log_y \sqrt[5]{y}}{3\log_x \sqrt{x} - \log_y \sqrt{y^5}} &= \frac{\log_x x^{\frac{3}{2}} + 2\log_y y^{\frac{1}{5}}}{3\log_x x^{\frac{1}{2}} - \log_y y^{\frac{5}{2}}} \\ &= \frac{\frac{3}{2} + 2 \times \frac{1}{5}}{3 \times \frac{1}{2} - \frac{5}{2}} \\ &= -\frac{19}{10}\end{aligned}$$

16. B

$$\begin{aligned}\log_2 a + \log_{\frac{1}{2}} \sqrt{a} &= \frac{\log a}{\log 2} + \frac{\frac{1}{2}\log a}{\log \frac{1}{2}} \\ &= \frac{\log a}{\log 2} + \frac{\log a}{-2\log 2} \\ &= \frac{\log a}{2\log 2} \\ &= \frac{1}{2}\log_2 a\end{aligned}$$

17. D

$$\begin{aligned}\frac{\log_8 x^2}{\log_2 x} &= \frac{2\log x}{\log 8} \div \frac{\log x}{\log 2} \\ &= \frac{2\log x}{3\log 2} \times \frac{\log 2}{\log x} \\ &= \frac{2}{3}\end{aligned}$$

18. B

$$\begin{aligned}\log\left(10a^b b^a\right) &= \log 10 + \log a^b + \log b^a \\ &= 1 + b\log a + a\log b\end{aligned}$$

19. B

$$\begin{aligned}\frac{\log_x x^5 + \log_y \sqrt[3]{y^2}}{\log_x \sqrt{x^5} - \log_y y^3} &= \frac{5 + \log_y y^{\frac{2}{3}}}{\log_x x^{\frac{5}{2}} - 3} \\ &= \frac{5 + \frac{2}{3}}{\frac{5}{2} - 3} \\ &= -\frac{34}{3}\end{aligned}$$

20. C

We have $a = \log 3$ and $b = \log 4 = 2\log 2$.

$$\begin{aligned}\log\frac{5}{6} &= \log\frac{10}{2^2 \times 3} \\ &= \log 10 - 2\log 2 - \log 3 \\ &= 1 - a - b\end{aligned}$$

21. D

We have $a = \log_3 4$ and $b = \log_3 6$.

$$\begin{aligned}\log_3 \frac{1}{8} &= \log_3 \frac{3}{4 \times 6} \\ &= \log_3 3 - \log_3 4 - \log_3 6 \\ &= 1 - a - b\end{aligned}$$

Alternative solution

$$\begin{aligned}\log_3 \frac{1}{8} &= -3 \log_3 2 \\ &= -3 \times \frac{a}{2} \\ &= -\frac{3a}{2}\end{aligned}$$

22. A

$$\begin{aligned}\frac{1}{y} &= \log_5 x \\ \frac{1}{y} &= \frac{\log x}{\log 5} \\ y &= \frac{\log 5}{\log x} \\ &= \log_x 5\end{aligned}$$

23. A

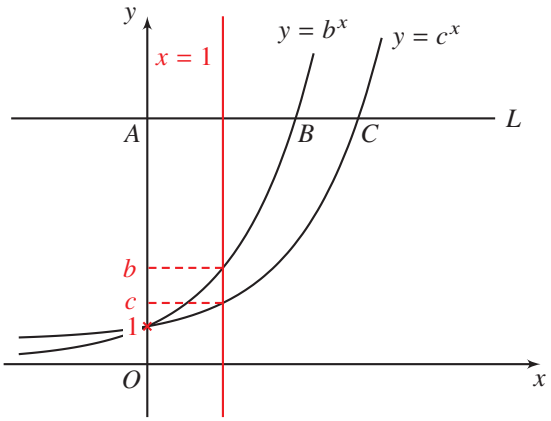
$$x = 8^n$$

$$n = \log_8 x$$

$$\begin{aligned}\log_8 \frac{x^3}{64} &= 3 \log_8 x - \log_8 64 \\ &= 3n - 2\end{aligned}$$

24. C

Draw the line $x = 1$. The intersections are $(1,\ b)$ and $(1,\ c)$.



I. ✗. $b > c$

II. ✓. $b > c > 1 \quad \Rightarrow \quad bc > 1$

III. ✓. Let L: $y = k$.

$B\left(\log_b k,\ k\right)$ and $C\left(\log_c k,\ k\right)$

$$\frac{AB}{AC} = \frac{\log_b k}{\log_c k}$$
$$= \frac{\log k}{\log b} \div \frac{\log k}{\log c}$$
$$= \frac{\log c}{\log b}$$
$$= \log_b c$$

25. C

$$y = \log_7 2x$$
$$2x = 7^y$$
$$x = \frac{1}{2}(7^y)$$

26. A

$$\log_2(x - y) = 2$$
$$x - y = 2^2$$
$$x - y = 4$$

Solve $\begin{cases} x + y = 2 \\ x - y = 4 \end{cases}$, we have $x = 3$ and $y = -1$.

27. C

$$x = 3^y \cdot 9^a$$
$$\log x = y \log 3 + a \log 9$$
$$= y \log 3 + 2a \log 3$$
$$y \log 3 = \log x - 2a \log 3$$
$$y = \frac{\log x}{\log 3} - 2a$$
$$= \log_x 3 - 2a$$

28. B

$$x = \log_4 \frac{1}{7}$$
$$4^x = \frac{1}{7}$$

29. B

$$\log_a 8 = 3$$
$$a^3 = 8$$
$$a = 2$$

30. B

$$5\log_2 \alpha + 5\log_2 \beta = \frac{10}{3}$$
$$\log_2 \alpha\beta = \frac{2}{3}$$
$$\alpha\beta = 2^{\frac{2}{3}}$$

 END OF PAPER