REG-GOF-2223-ASM-SET 2-MATH
Suggested solutions

Multiple Choice Questions

1. B	2. B	3. C	4. C	5. D
6. B	7. A	8. D	9. C	10. C
11. A	12. B	13. C	14. A	15. D
16. B	17. A	18. C	19. D	20. A
21. B	22. D	23. C	24. B	25. A
26. A	27. A	28. B	29. D	30. B

1. B

I. ✓. When $x = 0$, $4 = p + q\tan 0° = p$.

II. ✗. $y = \tan x°$ is an increasing curve. Since the curve here is also increasing, $q > 0$.

III. ✓. When $x = -\alpha$, $0 = 4 + q\tan(-\alpha)° \Rightarrow \tan\alpha° = \dfrac{4}{q} > 0$

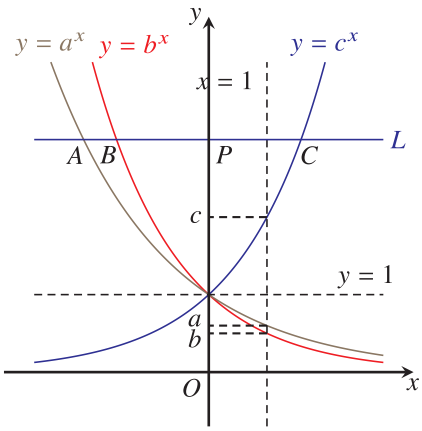
2. B

Draw the line $x = 1$ and $y = 1$.
From the graph, $0 < b < a < 1 < c$.

I. ✓. From the graph, $0 < a < 1$ and $0 < b < 1$.
So, $ab < 1$.

II. ✗. The graph $y = b^x$ and $y = c^x$ is symmetric about the y-axis. Thus, $b^{-1} = c$.
We have $ac > bc = 1$.

III. ✓.



3. C

$y = f(x) \quad \longrightarrow \quad y = -f(x) \quad \longrightarrow \quad y = -f(x-2)$
Reflect about the x-axis. Translate rightwards by 2 units.
The answer is C.

4. C

$g(x) = -\frac{1}{2} f(x)$

The graph of $y = f(x)$ is reduced along the y-axis to $\frac{1}{2}$ times the original and then is reflected about the x-axis to the graph of $y = g(x)$.

The answer is C.

5. D

I. ✓. Decreasing curve $\Rightarrow$ $0 < a < 1$ $\Rightarrow$ $a < 1$.

Remark: note that the final $\boxed{\Rightarrow}$ is not reversible.

II. ✓. Increasing curve $\Rightarrow$ $b > 1$.

III. ✓. Reflection image $\Rightarrow$ $b^{-x} = a^x$ for all real x.

Put $x = 1$, then $b^{-1} = a$ and $ab = 1$.

6. B

I. ✓. The graph $y = f(x-2)$ is 2 units on the right of $y = f(x)$, which can possibly be $y = g(x)$.

II. ✓. The graph $y = f(-x+2)$ is obtained by translating the graph $y = f(x)$ 2 units to the left [to $y = f(x+2)$]
and then reflect with respect to the y-axis, which is also possible.

III. ✗. The graph $y = f(-x-2)$ is obtained by translating the graph $y = f(x)$ 2 units to the right [to $y = f(x-2)$]
and then reflect with respect to the y-axis. The vertex should lie on the left of the y-axis.

7. A

$y = f(x) \longrightarrow y = -f(x) \longrightarrow y = -f(x) + 1$

The new graph is obtained by

(a) reflect the graph about the x-axis,

(b) translate upwards by 1 unit.

The answer is A.

8. D

y-intercept $= 0.5^{0-1} + 1 = 2 + 1 = 3$

When x is large, 0.5^{x-1} is close to 0, and so the value of y is close to 1.

Only option D satisfies these.

9. C

From $\boxed{y = f(x)}$ to $\boxed{y = f(x-2) + 1}$,

the graph is translated 2 units rightwards and 1 unit upwards.

The only possible option is C.

10. C

One graph opens upwards and the other opens downwards ⇒ involve a reflection in x-axis ⇒ option A or C

Option A will not alter the x-intercepts ⇒ only option C left

11. A

I. ✓. Sub $(0,\ 2)$ into $y = h + k\tan 2x°$,
$2 = h + k(0) \Rightarrow h = 2 > 0$

II. ✓. The graph of $y = \tan 2x°$ is increasing but the graph here is decreasing.
⇒ the "coefficient" of the $\tan 2x°$ term is negative ⇒ $k < 0$.

Alternative solution

For small positive value of x (say $x = 0.1$), $\tan 2x° > 0$ (e.g. $\tan 0.2° > 0$)

On the curve when $x = 0.1$, the y-coordinate is smaller than 2.

$$2 + k\tan 0.2° < 2$$
$$k < 0$$

III. ✗. There is no "breaking of curve" on $0 \le x \le \alpha \Rightarrow 2\alpha° < 90°$
$\Rightarrow \alpha < 45$ and $\tan\alpha° > 0$

Since $k < 0$, we have $\frac{1}{k} < 0$. And $\tan\alpha° \neq \frac{1}{k}$ (one positive and one negative)

Alternative solution

Consider the point on curve when $x = \frac{\alpha}{2}$, we have $y = 2 + k\tan\alpha°$

If $\tan\alpha° = \frac{1}{k}$,

then the y-coordinate becomes $2 + k \times \frac{1}{k} = 3$, which is a contradiction as the point lies below the line $y = 2$.

12. B

The graph passes through $(0,\ 1)$ and $(90,\ 0)$.

The answer is B.

13. C

Enlarge to 3 times of the original along the x-axis:

$y = \tan x \longrightarrow y = \tan\frac{x}{3}$

Reduce to $\frac{1}{2}$ times the original along the y-axis:

$y = \tan\frac{x}{3} \longrightarrow y = \frac{1}{2}\tan\frac{x}{3}$

14. A

The graph of exponential curve was translated b units upwards. So, $b > 0$. Exponential graph is increasing if the base is greater than 1. So, $a^{-1} > 1$ and therefore $0 < a < 1$.

15. $\boxed{\text{D}}$

Check the value of $g(0.5)$.

A. ✗. $g(0.5) = f\left(\frac{0.5}{2}\right) - 1 = f(0.25) - 1$

B. ✗. $g(0.5) = f(1+1) = f(2)$

C. ✗. $g(0.5) = f\left(\frac{0.5}{2} - 1\right) = f(-0.75)$

D. ✓. $g(0.5) = f(1) - 1 = 0 - 1 = -1$

16. $\boxed{\text{B}}$

$y = f(x) \quad \longrightarrow \quad y = f(-x) \quad \longrightarrow \quad y = f(-2x)$

Reflect about y-axis.

Reduce along x-axis to $\frac{1}{2}$ times the original.

The answer is B.

17. $\boxed{\text{A}}$

The graph $y = -f(x-1)$ is obtained by

translating the graph of $y = f(x)$ rightwards by 1 unit;

reflect the resulting graph with respect to the x-axis.

Only option A satisfies this.

18. $\boxed{\text{C}}$

From $y = f(x)$ to $y = g(x)$:

Translate leftwards by 4 units and downwards by 3 units.

So, $g(x) = f(x+4) - 3$

19. $\boxed{\text{D}}$

From $\boxed{y = f(x)}$ to $\boxed{y = f\left(\frac{x}{2}\right)}$, the graph is enlarged along the x-axis to 2 times the original.

The x-intercepts are doubled and the y-intercept remains unchanged.

20. $\boxed{\text{A}}$

Note that $f(2) = f(5) = 0$.

A. ✓.

B. ✗. When $x = -1$, $y = f(1-1) = f(0) \neq 0$.

C. ✗. When $x = -1$, $y = -f(-1) - 1 \neq 0$.

D. ✗. When $x = -1$, $y = -f(-1) + 1 \neq 0$.

21. B

$y = \frac{4}{x} \quad \longrightarrow \quad y = \frac{4}{x+4}$

I. ✓.

II. ✗. When $x = -3.5$, $y = \frac{4}{-3.5+4} = 8$

III. ✓. When $x = 0$, $y = \frac{4}{4} = 1$, only one y-intercept.

22. D

Let $f(x) = (x-h)^2 + k$. The resulting graph is

$y = -f(x-h)$

$= -(x-2h)^2 - k$

23. C

It shows the graphs of $y = f(x)$ and $y = -f(x-5)$.

The graph of $y = f(x)$ is reflected about the x-axis and is translated leftwards by 5 units.

24. B

$y = f(x) \to y = f(-x)$: reflect about y-axis

$y = f(-x) \to y = f(-x) - 5$: translate downwards by 5 units

25. A

From $y = f(x)$ to $y = f(2x+1)$:

Translate 1 unit leftwards $\to$ reduce along x-axis to $\frac{1}{2}$ the original $\Rightarrow$ Option A

Alternative transformation:

Reduce along x-axis to $\frac{1}{2}$ the original $\to$ translate $\frac{1}{2}$ units leftwards $\Rightarrow$ Option A

Alternative solution:

Note that $f(0) = f(5) = 0$.

$g\left(-\frac{1}{2}\right) = f\left[2\left(-\frac{1}{2}\right)+1\right] = f(0) = 0$ and $g(2) = f[2(2)+1] = f(5) = 0$.

The graph of $y = g(x)$ has x-intercepts $-\frac{1}{2}$ and 5 $\Rightarrow$ Option A.

26. A

Note that $f(2) = 1$ corresponds to $g(5) = -1$.

A. ✓. $g(5) = f(2) - 2 = 1 - 2 = -1$

B. ✗. $g(5) = f(2) + 2 = 1 + 2 = 3 \neq -1$

C. ✗. $g(5) = f(8) - 2 = ?$

D. ✗. $g(5) = f(8) + 2 = ?$

27. A

From $y = f(x)$ to $y = 1 - f(x)$, the graph is
reflected about the x-axis and then translated upwards by 1 unit.
Their vertex should lie on the same vertical line and the answer is A.

28. B

From $y = f(x)$,
reflect about x-axis $\rightarrow$ $y = -f(x)$
translate leftwards by 2 units $\rightarrow$ $y = -f(x+2)$

29. D

We have $f(3) = 2$ and $g(3) = 1$.

A. ✗. $g(3) = -2f(3) + 2 = -2(2) + 2 = -2 \neq 1$

B. ✗. $g(3) = -2f(3) + 3 = -2(2) + 3 = -1 \neq 1$

C. ✗. $g(3) = -\frac{1}{2}f(3) + 3 = -1 + 3 = 2 \neq 1$

D. ✓. $g(3) = -\frac{1}{2}f(3) + 2 = -1 + 2 = 1$

30. B

2 units upwards $\Rightarrow$ $y = \log x + 2$
Enlarged along y-axis to 3 times the original $\Rightarrow$ $y = 3(\log x + 2) = 3\log x + 6$

END OF PAPER