

M2REG-INDINT-2223-ASM-SET 2-MATH**Suggested solutions**

1. Let $u = x^6 - 2$. Then $du = 6x^5 dx$. 1M

$$\begin{aligned}\int x^5(x^6 - 2)^5 dx &= \frac{1}{6} \int u^5 du & 1M \\ &= \frac{u^6}{36} + \text{constant} \\ &= \frac{(x^6 - 2)^6}{36} + \text{constant} & 1A\end{aligned}$$

2. Let $u = 3x + 1$. Then $du = 3 dx$. 1M

$$\begin{aligned}\int x^2(3x + 1)^8 dx &= \frac{1}{3} \int \left(\frac{u-1}{3}\right)^2 u^8 du & 1M \\ &= \frac{1}{27} \int (u^{10} - 2u^9 + u^8) du & 1M \\ &= \frac{u^{11}}{297} - \frac{u^{10}}{135} + \frac{u^9}{243} + \text{constant} \\ &= \frac{(3x+1)^{11}}{297} - \frac{(3x+1)^{10}}{135} + \frac{(3x+1)^9}{243} + \text{constant} & 1A\end{aligned}$$

3. Let $u = \sqrt{x} - 2$. Then $du = \frac{dx}{2\sqrt{x}}$. 1M

$$\begin{aligned}\int \frac{5^{\sqrt{x}-2}}{\sqrt{x}} dx &= 2 \int 5^u du & 1M \\ &= 2 \int e^{u \ln 5} du & 1M \\ &= \frac{2e^{u \ln 5}}{\ln 5} + \text{constant} \\ &= \frac{2}{\ln 5} (5^{\sqrt{x}-2}) + \text{constant} & 1A\end{aligned}$$

4. Let $u = \frac{3}{2} - \frac{2}{x}$. Then $du = \frac{2}{x^2} dx$. 1M

$$\begin{aligned}\int \frac{dx}{x^2 \sqrt{\frac{3}{2} - \frac{2}{x}}} &= \frac{1}{2} \int \frac{du}{\sqrt{u}} & 1M \\ &= u^{\frac{1}{2}} + \text{constant} \\ &= \left(\frac{3}{2} - \frac{2}{x}\right)^{\frac{1}{2}} + \text{constant} & 1A\end{aligned}$$

5. Let $u = \frac{1}{x}$. Then $du = -\frac{1}{x^2} dx$

$$\int \frac{\sqrt[3]{4x^3 - 2}}{x^5} dx = \int \sqrt[3]{4 - \frac{2}{x^3}} \cdot \frac{1}{x^4} dx \quad 1M$$

$$= - \int \sqrt[3]{4 - 2u^3} u^2 du \quad 1M$$

Let $w = 4 - 2u^3$. Then $dw = -6u^2 du$. 1M

$$\int \frac{\sqrt[3]{4x^3 - 2}}{x^5} dx = \frac{1}{6} \int \sqrt[3]{w} dw \quad 1M$$

$$= \frac{1}{8} w^{\frac{4}{3}} + \text{constant}$$

$$= \frac{1}{8} \left(4 - \frac{2}{x^3} \right)^{\frac{4}{3}} + \text{constant} \quad 1A$$

6. $\int \sqrt{x^4 + 2x^2} dx = \int x \sqrt{x^2 + 2} dx$

Let $u = x^2 + 2$. Then $du = 2x dx$. 1M

$$\int \sqrt{x^4 + 2x^2} dx = \frac{1}{2} \int \sqrt{u} du \quad 1M$$

$$= \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + \text{constant}$$

$$= \frac{(x^2 + 2)^{\frac{3}{2}}}{\frac{3}{2}} + \text{constant} \quad 1A$$

7. Let $u = 3 + x^3$. Then $du = 3x^2 dx$. 1M

$$\int x^2 \sqrt{3 + x^3} dx = \frac{1}{3} \int \sqrt{u} du \quad 1M$$

$$= \frac{2u^{\frac{3}{2}}}{\frac{3}{2}} + \text{constant}$$

$$= \frac{2(3 + x^3)^{\frac{3}{2}}}{\frac{3}{2}} + \text{constant} \quad 1A$$

8. Let $u = 5 - 2x$. Then $du = -2 dx$. 1M

$$\int \frac{x^2 dx}{\sqrt{5 - 2x}} = -\frac{1}{2} \int \left(\frac{5 - u}{2} \right)^2 \cdot \frac{1}{\sqrt{u}} du \quad 1M$$

$$= -\frac{1}{8} \int (25u^{-\frac{1}{2}} - 10u^{\frac{1}{2}} + u^{\frac{3}{2}}) du \quad 1M$$

$$= -\frac{25}{4} u^{\frac{1}{2}} + \frac{5}{6} u^{\frac{3}{2}} - \frac{1}{20} u^{\frac{5}{2}} + \text{constant}$$

$$= -\frac{25}{4} (5 - 2x)^{\frac{1}{2}} + \frac{5}{6} (5 - 2x)^{\frac{3}{2}} - \frac{1}{20} (5 - 2x)^{\frac{5}{2}} + \text{constant} \quad 1A$$

9. Let $u = x^3 - 3x^2 + 1$. Then $du = (3x^2 - 6x) dx$. 1M

$$\begin{aligned}\int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} dx &= \frac{1}{3} \int \frac{du}{u} & 1M \\ &= \frac{\ln |u|}{3} + \text{constant} \\ &= \frac{\ln |x^3 - 3x^2 + 1|}{3} + \text{constant} & 1A\end{aligned}$$

10. Let $y = 1 - t^5$. Then $du = -5t^4 dt$. 1M

$$\begin{aligned}\int \frac{t^4}{(1 - t^5)^5} dt &= \frac{1}{-5} \int \frac{du}{u^5} & 1M \\ &= \frac{1}{20u^4} + \text{constant} \\ &= \frac{1}{20(1 - t^5)^4} + \text{constant} & 1A\end{aligned}$$

11. Let $u = e^{2x} - e^{-2x}$. Then $du = (2e^{2x} + 2e^{-2x}) dx$. 1M

$$\begin{aligned}\int \frac{e^{2x} + e^{-2x}}{e^{2x} - e^{-2x}} dx &= \frac{1}{2} \int \frac{du}{u} & 1M \\ &= \frac{\ln |u|}{2} + \text{constant} \\ &= \frac{\ln |e^{2x} - e^{-2x}|}{2} + \text{constant} & 1A\end{aligned}$$

12. Let $u = 1 - e^{-3x}$. Then $du = 3e^{-3x} dx$.

$$\begin{aligned}\int \frac{dx}{e^{3x} - 1} &= \int \frac{e^{-3x}}{1 - e^{-3x}} dx & 1M \\ &= \frac{1}{3} \int \frac{du}{u} & 1M \\ &= \frac{\ln |u|}{3} + \text{constant} \\ &= \frac{\ln |1 - e^{-3x}|}{3} + \text{constant} & 1A\end{aligned}$$

13. Let $u = \frac{x+1}{x-2}$. Then $du = -\frac{3}{(x-2)^2} dx$. 1A

$$\begin{aligned}\int \frac{dx}{x^2 - x - 2} &= \int \frac{(x-2) dx}{(x-2)^2(x+1)} & 1M \\ &= -\frac{1}{3} \int \frac{du}{u} & 1M \\ &= -\frac{\ln |u|}{3} + \text{constant} \\ &= \frac{1}{3} \ln \left| \frac{x-2}{x+1} \right| + \text{constant} & 1A\end{aligned}$$

14. Let $u = 1 + \sin x$. Then $du = \cos x \, dx$. 1M

$$\int \frac{\cos^3 x}{1 + \sin x} \, dx = \int \frac{(1 - \sin^2 x) \cos x}{1 + \sin x} \, dx$$

$$= \int \frac{1 - (u - 1)^2}{u} \, du \quad 1M$$

$$= \int (2 - u) \, du \quad 1M$$

$$= 2u - \frac{u^2}{2} + \text{constant}$$

$$= 2(1 + \sin x) - \frac{(1 + \sin x)^2}{2} + \text{constant} \quad 1A$$

Alternative answer: $\sin x - \frac{\sin^2 x}{2} + \text{constant}$

15. Let $u = \cos x$. Then $du = -\sin x \, dx$. 1M

$$\int \frac{\sin x}{\cos^6 x} \, dx = - \int \frac{du}{u^6} \quad 1M$$

$$= \frac{1}{5u^5} + \text{constant}$$

$$= \frac{1}{5 \cos^5 x} + \text{constant} \quad 1A$$

16. Let $u = 4\sqrt{x} - 1$. Then $du = \frac{2}{\sqrt{x}} \, dx$. 1M

$$\int \frac{dx}{\sqrt{x}(4\sqrt{x} - 1)} = \frac{1}{2} \int \frac{du}{u} \quad 1M$$

$$= \frac{\ln |u|}{2} + \text{constant}$$

$$= \frac{\ln |4\sqrt{x} - 1|}{2} + \text{constant} \quad 1A$$