M2REG-INDINT-2223-ASM-SET 1-MATH
Suggested solutions

1. $\int \frac{6\,dx}{\sqrt[5]{x^3}} = 6\int x^{-\frac{3}{5}}\,dx$

$= 15x^{\frac{2}{5}} + \text{constant}$ 1A

2. $\int (3x^5 + \sqrt{2}x + 1)\,dx = \frac{1}{2}x^6 + \frac{\sqrt{2}}{2}x^2 + x + \text{constant}$ 1A

3. $\int \left(x - \frac{2}{x}\right)^3 dx = \int \left(x^3 - 6x + \frac{12}{x} - \frac{8}{x^3}\right) dx$ 1M

$= \frac{x^4}{4} - 3x^2 + 12\ln|x| + 4x^{-2} + \text{constant}$ 1A

4. $\int (\sqrt{x} + \sqrt[3]{x})^4\,dx = \int (x^{\frac{1}{2}} + x^{\frac{1}{3}})^4\,dx$

$= \int \left(x^2 + 4x^{\frac{11}{6}} + 6x^{\frac{5}{3}} + 4x^{\frac{3}{2}} + x^{\frac{4}{3}}\right) dx$ 1M

$= \frac{1}{3}x^3 + \frac{24}{17}x^{\frac{17}{6}} + \frac{9}{4}x^{\frac{8}{3}} + \frac{8}{5}x^{\frac{5}{2}} + \frac{3}{7}x^{\frac{7}{3}} + \text{constant}$ 1A

5. $\int \frac{\left(\sqrt[3]{x} - \frac{2}{\sqrt{x}}\right)^3}{x^2}\,dx = \int \frac{x - 6x^{\frac{1}{6}} + 6x^{-\frac{2}{3}} - 8x^{-\frac{3}{2}}}{x^2}\,dx$ 1M

$= \int \left(\frac{1}{x} - 6x^{-\frac{11}{6}} + 6x^{-\frac{8}{3}} - 8x^{-\frac{7}{2}}\right) dx$

$= \ln|x| + \frac{36}{5}x^{-\frac{5}{6}} - \frac{36}{5}x^{-\frac{5}{3}} + \frac{16}{5}x^{-\frac{5}{2}} + \text{constant}$ 1A

6. (a) $\frac{d}{dx}[(x^4 - 1)\ln(x^2 + 1)] = 4x^3\ln(x^2 + 1) + (x^4 - 1)\frac{2x}{x^2 + 1}$ 1M

$= 4x^3\ln(x^2 + 1) + (x^2 - 1)(x^2 + 1)\frac{2x}{x^2 + 1}$

$= 2x^3 - 2x + 4x^3\ln(x^2 + 1)$ 1A

(b) $\int [2x^3 - 2x + 4x^3\ln(x^2 + 1)]\,dx = (x^4 - 1)\ln(x^2 + 1) + \text{constant}$ 1M

$2\int (x^3 - x)\,dx + 4\int x^3\ln(x^2 + 1)\,dx = (x^4 - 1)\ln(x^2 + 1) + \text{constant}$

$\int x^3\ln(x^2 + 1)\,dx = \frac{1}{4}(x^4 - 1)\ln(x^2 + 1) - \frac{1}{2}\int (x^3 - x)\,dx + \text{constant}$

$= \frac{1}{4}(x^4 - 1)\ln(x^2 + 1) - \frac{1}{8}x^4 + \frac{1}{4}x^2 + \text{constant}$ 1A

7. $\int (2e^{\frac{x}{3}} + 1)(4e^{\frac{2x}{3}} - 2e^{\frac{x}{3}} + 1)\,dx = \int (8e^x + 1)\,dx$ 1M

$= 8e^x + x + \text{constant}$ 1A

8. $\int \left(\frac{2e^{2x}-3}{e^{3x}}\right)^2 \mathrm{d}x = \int \frac{4e^{4x}-12e^{2x}+9}{e^{6x}} \mathrm{d}x$

$= \int (4e^{-2x} - 12e^{-4x} + 9e^{-6x}) \mathrm{d}x$ 1M

$= -2e^{-2x} + 3e^{-4x} - \frac{3}{2}e^{-6x} + \text{constant}$ 1A

9. $\int 2^{3-5x} \mathrm{d}x = \int e^{(3-5x)\ln 2} \mathrm{d}x$ 1M

$= \frac{1}{-5\ln 2} e^{(3-5x)\ln 2} + \text{constant}$

$= -\frac{1}{5\ln 2} 2^{3-5x} + \text{constant}$ 1A

10. $\int \sin^2 \frac{x}{4} \mathrm{d}x = \frac{1}{2}\int \left(1 - \cos\frac{x}{2}\right) \mathrm{d}x$ 1M

$= \frac{1}{2}x - \sin\frac{x}{2} + \text{constant}$ 1A

11. $\int \sin 5x \cos\frac{3x}{2} \mathrm{d}x = \frac{1}{2}\int \left(\sin\frac{13x}{2} + \sin\frac{7x}{2}\right) \mathrm{d}x$ 1M

$= -\frac{1}{13}\cos\frac{13x}{2} - \frac{1}{7}\cos\frac{7x}{2} + \text{constant}$ 1A

12. $\int (3\tan x + \sec x)(3\tan x - \sec x) \mathrm{d}x = \int (9\tan^2 x - \sec^2 x) \mathrm{d}x$

$= \int [9(\sec^2 x - 1) - \sec^2 x] \mathrm{d}x$ 1M

$= \int (8\sec^2 x - 9) \mathrm{d}x$

$= 8\tan x - 9x + \text{constant}$ 1A

13. $\int (\cos^4 x - \sin^4 x) \mathrm{d}x = \int (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x) \mathrm{d}x$

$= \int (\cos 2x)(1) \mathrm{d}x$ 1M

$= \frac{1}{2}\sin 2x + \text{constant}$ 1A

14. $\int \frac{4}{\cot x + \tan x} \mathrm{d}x = \int \frac{4}{\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}} \mathrm{d}x$

$= \int \frac{4\sin x \cos x}{\cos^2 x + \sin^2 x} \mathrm{d}x$

$= \int 4\sin x \cos x \mathrm{d}x$ 1A

$= 2\int \sin 2x \mathrm{d}x$ 1M

$= -\cos 2x + \text{constant}$ 1A

15. $\int \sin 2x \sin 3x \sin 5x \, dx = -\frac{1}{2} \int (\cos 5x - \cos x) \sin 5x \, dx$ 1M

$= -\frac{1}{2} \int \cos 5x \sin 5x \, dx + \frac{1}{2} \int \cos x \sin 5x \, dx$

$= -\frac{1}{4} \int \sin 10x \, dx + \frac{1}{4} \int (\sin 6x + \sin 4x) \, dx$ 1M

$= \frac{1}{40} \cos 10x - \frac{1}{24} \cos 6x - \frac{1}{16} \cos 4x + \text{constant}$ 1A