

REG-MAE-2223-ASM-SET 2-MATH

建議題解

結構式試題

$$1. \quad (a) \quad \left(x + \frac{1}{x}\right)^2 = a^2$$

$$x^2 + 2 + \frac{1}{x^2} = a^2$$

$$x^2 + \frac{1}{x^2} = a^2 - 2$$

1A

$$(b) \quad \text{設 } x + \frac{1}{x} = a。$$

$$6\left(x^2 + \frac{1}{x^2}\right) - 5\left(x + \frac{1}{x}\right) - 38 = 0$$

$$6(a^2 - 2) - 5(a) - 38 = 0$$

1M

$$6a^2 - 5a - 50 = 0$$

$$a = \frac{10}{3} \quad \text{或} \quad -\frac{5}{2}$$

1A

$$\text{當 } a = \frac{10}{3},$$

$$x + \frac{1}{x} = \frac{10}{3}$$

$$x^2 - \frac{10}{3}x + 1 = 0$$

$$x = 3 \quad \text{或} \quad \frac{1}{3}$$

1A

$$\text{當 } a = -\frac{5}{2},$$

$$x + \frac{1}{x} = -\frac{5}{2}$$

$$x^2 + \frac{5}{2}x + 1 = 0$$

$$x = -0.5 \quad \text{或} \quad -2$$

1A

故此， $x = -2$ 或 -0.5 或 $\frac{1}{3}$ 或 3 。

2. (a) $\sqrt{2x} + \frac{4}{\sqrt{2x}} = 4$
 $(\sqrt{2x})^2 - 4\sqrt{2x} + 4 = 0$ 1M
 $\sqrt{2x} = 2$ 1M
 $2x = 2^2$
 $x = 2$ 1A
- (b) $x^5 - 4x = 3x^3$
 $x(x^4 - 3x^2 - 4) = 0$ 1M
 $x(x^2 - 4)(x^2 + 1) = 0$
 $x(x + 2)(x - 2)(x^2 + 1) = 0$ 1M
 $x = 0$ 或 ± 2 1A
3. $\sqrt{a} + \sqrt{2a + 1} = 5$
 $\sqrt{a} = 5 - \sqrt{2a + 1}$
 $a = 25 - 10\sqrt{2a + 1} + 2a + 1$ 1M
 $10\sqrt{2a + 1} = 26 + a$
 $100(2a + 1) = a^2 + 52a + 676$ 1M
 $0 = a^2 - 148a + 576$
 $a = 4$ 或 144 (捨去) 2A
4. $(2 - \sqrt{x})(3 + \sqrt{x}) = 6x$
 $-x - \sqrt{x} + 6 = 6x$
 $-7x - \sqrt{x} + 6 = 0$ 1A
 $\sqrt{x} = \frac{6}{7}$ 或 -1 (捨去) 1M+1A
 $x = \frac{36}{49}$ 1A
5. $6(\sqrt{x} - 3)^2 + 13(\sqrt{x} - 3) - 5 = 0$
 $\sqrt{x} - 3 = \frac{1}{3}$ 或 $-\frac{5}{2}$ 1A+1A
 $\sqrt{x} = \frac{10}{3}$ 或 $\frac{1}{2}$
 $x = \frac{100}{9}$ 或 $\frac{1}{4}$ 1A
6. $x^{\frac{4}{5}} - 13x^{\frac{2}{5}} + 36 = 0$
 $(x^{\frac{2}{5}})^2 - 13x^{\frac{2}{5}} + 36 = 0$ 1M
 $x^{\frac{2}{5}} = 4$ 或 9 1A
 $x = 4^{\frac{5}{2}}$ 或 $9^{\frac{5}{2}}$
 $x = 32$ 或 243 1A

$$\begin{aligned}
7. \quad & \log_4 x + \log_x 16 = 3 \\
& \frac{\log x}{2 \log 2} + \frac{4 \log 2}{\log x} = 3 & 1M \\
& (\log x)^2 + 8(\log 2)^2 = 6(\log 2)(\log x) \\
& (\log x)^2 - 6(\log x)(\log 2) + 8(\log 2)^2 = 0 & 1A \\
& (\log x - 4 \log 2)(\log x - 2 \log 2) = 0 & 1M \\
& \log x = 4 \log 2 \quad \text{或} \quad 2 \log 2 & 1A \\
& x = 16 \quad \text{或} \quad 4 & 1A
\end{aligned}$$

$$\begin{aligned}
8. \quad & \log_2 \sqrt{x+4} = \log_{16} (2x^2 + 12x + 11) \\
& \frac{\frac{1}{2} \log(x+4)}{\log 2} = \frac{\log(2x^2 + 12x + 11)}{\log 16} & 1M \\
& \frac{\log(x+4)}{2 \log 2} = \frac{\log(2x^2 + 12x + 11)}{4 \log 2} & 1M \\
& 2 \log(x+4) = \log(2x^2 + 12x + 11) \\
& (x+4)^2 = 2x^2 + 12x + 11 & 1M \\
& x^2 + 4x - 5 = 0 \\
& (x+5)(x-1) = 0 \\
& x = 1 \quad \text{或} \quad -5 \quad (\text{捨去}) & 1A
\end{aligned}$$

$$\begin{aligned}
9. \quad (a) \quad & \log_{(x+1)} 512 = 3 \\
& (x+1)^3 = 512 & 1M \\
& x+1 = \sqrt[3]{512} \\
& x = 7 & 1A \\
(b) \quad & \log(2x+5) + \log(4-x) - \log(x+4) = \log 5 \\
& \log[(2x+5)(4-x)] = \log[5(x+4)] & 1M \\
& (2x+5)(4-x) = 5(x+4) \\
& -2x^2 + 3x + 20 = 5x + 20 \\
& -2x^2 - 2x = 0 & 1A \\
& -2x(x+1) = 0 \\
& x = -1 \quad \text{或} \quad 0 & 1A
\end{aligned}$$

10. $\log_3(x-3) - \log_9(x-5) = 1$
- $$\frac{\log(x-3)}{\log 3} - \frac{\log(x-5)}{\log 9} = 1 \quad 1M$$
- $$\frac{\log(x-3)}{\log 3} - \frac{\log(x-5)}{2 \log 3} = 1$$
- $$2 \log(x-3) - \log(x-5) = 2 \log 3$$
- $$\log \frac{(x-3)^2}{x-5} = \log 9 \quad 1M$$
- $$\frac{(x-3)^2}{x-5} = 9$$
- $$x^2 - 6x + 9 = 9x - 45$$
- $$x^2 - 15x + 54 = 0 \quad 1M$$
- $$x = 6 \quad \text{或} \quad 9 \quad 1A$$
-
11. $\log(2x+1) + \log(3x-7) = \log(11x+1)$
- $$\log[(2x+1)(3x-7)] = \log(11x+1) \quad 1M$$
- $$(2x+1)(3x-7) = 11x+1 \quad 1M$$
- $$6x^2 - 11x - 7 = 11x + 1$$
- $$6x^2 - 22x - 8 = 0$$
- $$3x^2 - 11x - 4 = 0 \quad 1A$$
- $$(x-4)(3x+1) = 0$$
- $$x = 4 \quad \text{或} \quad -\frac{1}{3} \quad (\text{捨去}) \quad 2A$$
-
12. (a) $z = \frac{5(a+i)}{a-i} \times \frac{a+i}{a+i}$ 1M
- $$= \frac{5(a^2-1) + 10ai}{a^2+1}$$
- $$= \frac{5(a^2-1)}{a^2+1} + \frac{10a}{a^2+1}i \quad 1A$$
- (b) $\frac{5(a^2-1)}{a^2+1} - \frac{10a}{a^2+1} = 1$ 1M
- $$5a^2 - 10a - 5 = a^2 + 1$$
- $$4a^2 - 10a - 6 = 0$$
- $$a = 3 \quad \text{或} \quad -0.5 \quad (\text{捨去}) \quad 1A$$
- $z = 4 + 3i$, 方程的兩根為 $4 \pm 3i$ 。
- $$s = (4+3i)(4-3i) = 25 \quad 1A$$

13. (a) $f(x) = -x^2 + 25x$

$$= -\left[x^2 - 2(x)\left(\frac{25}{2}\right) + \left(\frac{25}{2}\right)^2\right] + \frac{625}{4} \quad 1M$$

$$= -\left(x - \frac{25}{2}\right)^2 + \frac{625}{4} \quad 1A$$

因此， $f(x)$ 的極大值為 $\frac{625}{4}$ 。 1A

(b) (i) $900 = k \times 1.5^2$

$$k = 400 \quad 1A$$

(ii) $1500 = 400 \times 1.5^m$

$$1.5^m = \frac{15}{4}$$

$$m \log 1.5 = \log \frac{15}{4} \quad 1M$$

$$m \approx 3.26$$

該密度在 3.26 小時後達至 $1500 \mu \text{g/m}^3$ 。 1A

(c) (i) $0 = 400 \times 1.5^{n+4} - 81 \times 2.25^n$ 1M

$$0 = 2025^4 \times 1.5^n - 81 \times 1.5^{2n}$$

$$0 = 1.5^n (2025 - 81 \times 1.5^n)$$

$$1.5^n = \frac{2025}{81} \quad \text{或} \quad 0 \quad (\text{捨去})$$

$$n \log 1.5 = \log \frac{2025}{81} \quad 1M$$

$$n \approx 7.94$$

在加入化學物的 7.94 小時後，X 被完全清除。 1A

(ii) $E = 400 \times 1.5^{n+4} - 81 \times 2.25^n$

$$= 400 \times 1.5^4 \times 1.5^n - 81 \times 1.5^{2n}$$

$$= -81y^2 + 2025y \quad 1A$$

(iii) 利用 (c)(ii)， $E = 81(-y^2 + 25y)$ 。

利用 (a)，

$$E = 81 \left[-\left(y - \frac{25}{2}\right)^2 + \frac{625}{4} \right]$$

$$= -81 \left(y - \frac{25}{2}\right)^2 + \frac{50625}{4}$$

因此，污染物 X 的最大密度為 12656 g/m^3 。 1A