

REG-LOG-2223-ASM-SET 3-MATH

建議題解

多項選擇題

- | | | | | |
|-------|-------|-------|-------|-------|
| 1. C | 2. C | 3. D | 4. C | 5. C |
| 6. A | 7. A | 8. C | 9. B | 10. C |
| 11. A | 12. D | 13. D | 14. A | 15. C |
| 16. D | 17. D | 18. C | 19. A | 20. B |
| 21. B | 22. D | 23. D | 24. A | 25. C |
| 26. B | 27. B | 28. B | 29. C | 30. D |

1. C

$$a = \log \frac{b+c}{b-c}$$

$$10^a = \frac{b+c}{b-c}$$

$$10^a(b-c) = b+c$$

$$10^a b - b = c + 10^a c$$

$$c = \frac{b(10^a - 1)}{10^a + 1}$$

2. C

$$\log(x+y) = \log x + \log y$$

$$\log(x+y) = \log xy$$

$$x+y = xy$$

$$x = \frac{-y}{1-y}$$

$$= \frac{y}{y-1}$$

3. D

I. ✗。考慮當 $x = 1$ 及 $y = z = 10$ 的情況。可得 $\log x^2 + \log y^2 = \log z^2$ 但 $x^2 + y^2 \neq z^2$ 。

II. ✓。 $\log x^2 + \log y^2 = \log z^2$

$$2 \log x + 2 \log y = 2 \log z$$

$$\log x + \log y = \log z$$

III. ✓。 $\log x^2 + \log y^2 = \log z^2$

$$\log x^2 y^2 = \log z^2$$

$$x^2 y^2 = z^2$$

4. ☐ C

$$\log(1 + x^2) = 1 + \log y$$

$$\log(1 + x^2) = \log 10 + \log y$$

$$\log(1 + x^2) = \log(10y)$$

$$1 + x^2 = 10y$$

$$y = \frac{1}{10}(x^2 + 1)$$

5. ☐ C

$$\log x^2 - 2 = \log 15x$$

$$\log x^2 - \log 15x = 2$$

$$\log \frac{x^2}{15x} = 2$$

$$\frac{x}{15} = 10^2$$

$$x = 1500$$

6. ☐ A

$$\log 10a = 2 \log a + 4 \log b$$

$$\log 10a = \log(a^2 b^4)$$

$$10a = a^2 b^4$$

$$10 = ab^4$$

$$a = \frac{10}{b^4}$$

7. ☐ A

$$\frac{\log m + \log n}{\log \frac{m}{n}} = 3$$

$$\log m + \log n = 3(\log m - \log n)$$

$$-2 \log m = -4 \log n$$

$$\log m = 2 \log n$$

$$m = n^2$$

8. ☐ C

$$\frac{1}{2} \log x + 1 = \log 5$$

$$\log x^{\frac{1}{2}} = \log \frac{5}{10}$$

$$x = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

9. B

$$\begin{aligned}\log x^2 &= 2 \log 2 + \log 9 \\ &= \log(4 \times 9) \\ x^2 &= 36 \\ x &= \pm 6\end{aligned}$$

10. C

$$\begin{aligned}\log(3x + 4) &= \log \frac{x}{2} + 1 \\ \log(3x + 4) &= \log \frac{x}{2} + \log 10 \\ 3x + 4 &= \frac{x}{2} \times 10 \\ x &= 2\end{aligned}$$

11. A

$$\begin{aligned}\log x^4 &= \log 4x^2 + 2 \\ \log x^4 - \log 4x^2 &= 2 \\ \log \frac{x^4}{4x^2} &= 2 \\ \frac{x^2}{4} &= 10^2 \\ x^2 &= 400 \\ x &= 20 \quad \text{或} \quad -20\end{aligned}$$

12. D

$$\begin{aligned}\log_9(x + 2) - 3 \log_9 y &= \frac{1}{2} \\ \log_9 \frac{x + 2}{y^3} &= \frac{1}{2} \\ \frac{x + 2}{y^3} &= 9^{\frac{1}{2}} \\ x &= 3y^3 - 2\end{aligned}$$

13. D

$$\begin{aligned}\log_2(2x + 1) &= \log_4(x + 11) \\ \frac{\log(2x + 1)}{\log 2} &= \frac{\log(x + 11)}{2 \log 2} \\ \log(2x + 1)^2 &= \log(x + 11) \\ (2x + 1)^2 &= x + 11 \\ 4x^2 + 3x - 10 &= 0 \\ x &= \frac{5}{4} \quad \text{或} \quad -2 \quad (\text{捨去})\end{aligned}$$

14. A

$$2 \log y + 1 = 4 \log x$$

$$2 \log y - 4 \log x = -1$$

$$\log \frac{y^2}{x^4} = -1$$

$$\frac{y^2}{x^4} = 10^{-1}$$

$$y^2 = \frac{x^4}{10}$$

$$y = \sqrt{\frac{x^4}{10}}$$

15. C

$$\frac{\log a}{2} = \log \frac{b}{2}$$

$$\log a = \log \left(\frac{b}{2} \right)^2$$

$$a = \frac{b^2}{4}$$

16. D

$$\frac{\log 2x}{\log y} = \frac{2a}{b}$$

$$\log 2x = \frac{2a}{b} \log y$$

$$\log 2x = \log y^{\frac{2a}{b}}$$

$$2x = y^{\frac{2a}{b}}$$

$$x = \frac{1}{2} y^{\frac{2a}{b}}$$

17. D

$$\log \frac{a}{2} = 10 \log b$$

$$\log \frac{a}{2} = \log b^{10}$$

$$\frac{a}{2} = b^{10}$$

$$a = 2b^{10}$$

18. C

$$\log x + \log 2 = 2$$

$$\log 2x = 2$$

$$2x = 10^2$$

$$x = 50$$

19. A

$$\log b - 2 \log a = 1$$

$$\log b = 2 \log a + \log 10$$

$$\log b = \log(10a^2)$$

$$b = 10a^2$$

20. B

$$\log_3(2x - 1) - \log_3(3x - 2) = 2$$

$$\log_3 \frac{2x - 1}{3x - 2} = 2$$

$$\frac{2x - 1}{3x - 2} = 3^2$$

$$2x - 1 = 27x - 18$$

$$x = \frac{17}{25}$$

21. B

$$\log(a - 2x) = 3$$

$$a - 2x = 10^3$$

$$x = \frac{1}{2}(a - 1000)$$

22. D

$$\log_2 y - \log_2 x = 6.3 - 2.3$$

$$\log_2 \frac{y}{x} = 4$$

$$\frac{y}{x} = 2^4$$

$$y = 16x$$

23. D

$$\log(x + 1) = -1$$

$$x + 1 = 10^{-1}$$

$$x = -\frac{9}{10}$$

24. A

$$\log_{2x}(4x - 5) = 1$$

$$4x - 5 = (2x)^1$$

$$x = \frac{5}{2}$$

25. C

$$\log_5(4x + 3) - 2 = 0$$

$$\log_5(4x + 3) = 2$$

$$4x + 3 = 5^2$$

$$x = \frac{11}{2}$$

26. B

$$x = y^n$$

$$\log x = n \log y$$

$$\frac{\log y}{\log x} = \frac{1}{n}$$

$$\log_x y = \frac{1}{n}$$

27. B

$$10^c = ab$$

$$c = \log ab$$

$$= \log a + \log b$$

28. B

$$3^a = 4^b \quad \text{及} \quad 3^a = 12^c$$

$$a \log 3 = b \log 4 \quad a \log 3 = c \log 12$$

$$\frac{a}{b} = \frac{\log 4}{\log 3} \quad \frac{a}{c} = \frac{\log 12}{\log 3}$$

$$\frac{a}{b} - \frac{a}{c} = \frac{\log 4}{\log 3} - \frac{\log 12}{\log 3}$$

$$= \frac{1}{\log 3} \log \frac{4}{12}$$

$$= \frac{-\log 3}{\log 3}$$

$$= -1$$

29. C

$$2^x = 10^z \quad \text{及} \quad 5^y = 10^z$$

$$x \log 2 = z \quad z = y \log 5$$

$$\frac{x}{z} = \frac{1}{\log 2} \quad \frac{y}{z} = \frac{1}{\log 5}$$

$$\frac{x+y}{z} = \frac{1}{\log 2} + \frac{1}{\log 5}$$

$$= \frac{\log 5 + \log 2}{(\log 2)(\log 5)}$$

$$= \frac{1}{(\log 2)(\log 5)}$$

30. D

$$4^{x+5} + 1 = 11$$

$$4^{x+5} = 10$$

$$(x + 5) \log 4 = \log 10$$

$$x = \frac{\log 10}{\log 4} - 5$$

$$= \frac{1}{\log 4} - 5$$

結構式試題

31. (a) $5 \cdot 4^x - 6 \cdot 3^{2x+1} = 0$

$$5 \cdot 4^x = 6 \cdot 3^{2x+1}$$

$$\log 5 + x \log 4 = \log 6 + (2x + 1) \log 3 \quad 1M$$

$$x(\log 4 - 2 \log 3) = \log 6 + \log 3 - \log 5 \quad 1M$$

$$x = \frac{\log 6 + \log 3 - \log 5}{\log 4 - 2 \log 3}$$

$$\approx -1.58 \quad 1A$$

(b) $3^{5x-1} = 5$

$$(5x - 1) \log 3 = \log 5 \quad 1M$$

$$5x \log 3 = \log 5 + \log 3$$

$$x = \frac{\log 5 + \log 3}{5 \log 3}$$

$$\approx 0.493 \quad 1A$$

32. (a) $3^{x-1} = 7^{4x-3}$

$$(x - 1) \log 3 = (4x - 3) \log 7 \quad 1M$$

$$x(\log 3 - 4 \log 7) = -3 \log 7 + \log 3 \quad 1M$$

$$x = \frac{\log 3 - 3 \log 7}{\log 3 - 4 \log 7}$$

$$\approx 0.709 \quad 1A$$

(b) $2^{2x+3} - 4^{x-2} = 10$

$$4^x(2^3 - 4^{-2}) = 10 \quad 1M$$

$$4^x = \frac{160}{127}$$

$$x \log 4 = \log \frac{160}{127} \quad 1M$$

$$x = \frac{\log \frac{160}{127}}{\log 4}$$

$$\approx 0.167 \quad 1A$$

33. (a) $\log_3(x-1) + \log_5(x-1) = 2.5$
 $\frac{\log(x-1)}{\log 3} + \frac{\log(x-1)}{\log 5} = 2.5$ 1M
 $\log(x-1) \left(\frac{1}{\log 3} + \frac{1}{\log 5} \right) = 2.5$
 $\log(x-1) \approx 0.708\,902\,142$ 1M
 $x-1 \approx 10^{0.708\,902\,142}$
 $x \approx 6.12$ 1A
- (b) $7^{x+2} - 7^{x-1} = 342$
 $7^x(7^2 - 7^{-1}) = 342$
 $7^x = 7$ 1M
 $x = 1$ 1A
34. (a) $(\sqrt{3})^{x+2} + (\sqrt{3})^{x+6} = 810$
 $(\sqrt{3})^x(3 + 3^3) = 810$ 1M
 $(\sqrt{3})^x = 27$
 $x \log \sqrt{3} = \log 27$ 1M
 $x = \frac{\log 27}{\log \sqrt{3}}$
 $= 6$ 1A
- (b) $(\sqrt[3]{7})^{x+6} - (\sqrt[3]{7})^{x+3} - 14\,406 = 0$
 $(\sqrt[3]{7})^x(7^2 - 7) = 14\,406$ 1M
 $(\sqrt[3]{7})^x = 343$
 $x \log \sqrt[3]{7} = \log 343$ 1M
 $x = \frac{\log 343}{\log \sqrt[3]{7}}$
 $= 9$ 1A

35. (a) $\log_x(2-x) = 2$

$$2-x = x^2$$

1M

$$0 = x^2 + x - 2$$

1A

$$= (x+2)(x-1)$$

$$x = -2 \text{ (捨去)} \quad \text{或} \quad 1 \text{ (捨去)}$$

1A

方程無解。

(b) $\log_{5\sqrt{2}}(2x^2 - 10x) = 2$

$$2x^2 - 10x = (5\sqrt{2})^2$$

1M

$$2x^2 - 10x - 50 = 0$$

$$x = \frac{10 \pm \sqrt{10^2 - 4(2)(-50)}}{2(2)}$$

1M

$$= \frac{5 \pm 5\sqrt{5}}{2}$$

1A