

REG-LOG-2223-ASM-SET 2-MATH**建議題解****多項選擇題**

- | | | | | |
|-------|-------|-------|-------|-------|
| 1. A | 2. C | 3. C | 4. A | 5. C |
| 6. B | 7. C | 8. A | 9. B | 10. D |
| 11. B | 12. D | 13. D | 14. C | 15. B |
| 16. B | 17. D | 18. B | 19. B | 20. C |
| 21. D | 22. A | 23. A | 24. C | 25. C |
| 26. A | 27. C | 28. B | 29. B | 30. B |

1. A

- A. $\log_4 10 \approx 1.66$
B. $\log_8 10 \approx 1.11$
C. $\log_6 15 \approx 1.51$
D. $\log_{15} 4 \approx 0.512$

答案為 A。

2. C

$$\begin{aligned}\log_{2\sqrt{2}} 64 &= \frac{\log 64}{\log 2\sqrt{2}} \\ &= \frac{\log 2^6}{\log 2^{1+\frac{1}{2}}} \\ &= \frac{6 \log 2}{\frac{3}{2} \log 2} \\ &= 4\end{aligned}$$

3. C

$$\begin{aligned}\log_2 32 \times \log_2 \sqrt{128} &= \log_2 2^5 \times \log_2 2^{\frac{7}{2}} \\ &= 5 \times \frac{7}{2} \\ &= 17.5\end{aligned}$$

4. A

$$\begin{aligned}\log_{\frac{1}{\sqrt{5}}} 25 &= \frac{\log 25}{\log \frac{1}{\sqrt{5}}} \\ &= \frac{2 \log 5}{-\frac{1}{2} \log 5} \\ &= -4\end{aligned}$$

5. C

$$\begin{aligned}(\log_4 27)(\log_3 2) &= \frac{3 \log 3}{2 \log 2} \times \frac{\log 2}{\log 3} \\&= \frac{3}{2}\end{aligned}$$

6. B

$$\begin{aligned}9^{\log_3 a} &= (3^2)^{\log_3 a} \\&= (3^{\log_3 a})^2 \\&= a^2\end{aligned}$$

7. C

可得 $\log b > \log a > 0$ 。
故此， $\log_a b = \frac{\log b}{\log a} > 1$ 。

8. A

$$\begin{aligned}(\log_b a^3)(\log_a b)^2 &= \frac{3 \log a}{\log b} \times \left(\frac{\log b}{\log a}\right)^2 \\&= 3 \times \frac{\log b}{\log a} \\&= 3 \log_a b\end{aligned}$$

9. B

$$\begin{aligned}\frac{\log_a b}{\log_b a} &= \frac{\log b}{\log a} \times \frac{\log a}{\log b} \\&= 1\end{aligned}$$

10. D

$$\begin{aligned}\frac{\log_3 \frac{16x^2}{5}}{2 \log_3 (4x) - \log_3 5} &= \frac{\log_3 16 + 2 \log_3 x - \log_3 5}{2(\log_3 4 + \log_3 x) - \log_3 5} \\&= \frac{2 \log_3 4 + 2 \log_3 x - \log_3 5}{2 \log_3 4 + 2 \log_3 x - \log_3 5} \\&= 1\end{aligned}$$

11. B

I. ✗。當 $x = 5$ ， $\log_5 x^2 = \log_5 5^2 = 2$ 及 $\frac{1}{2 \log_x 5} = \frac{1}{2 \log_5 5} = \frac{1}{2} \neq 2$ 。

$$\begin{aligned}\text{II. } \checkmark。 \frac{\log_3 x}{3 \log_3 2} &= \frac{\log x}{\log 3} \div \frac{3 \log 2}{\log 3} \\&= \frac{\log x}{3 \log 2} \\&= \frac{\log x}{\log 8} \\&= \log_8 x\end{aligned}$$

III. ✗。當 $x = 10$ ， $\log_x 10 = \log 10 = 1$ 及 $\log x^{-1} = \log 10^{-1} = -1 \neq 1$ 。

12. D

$$\begin{aligned} a &= \log_3 \frac{1}{b} \\ &= -\log_3 b \\ \frac{1}{a} &= -\frac{1}{\log_3 b} \end{aligned}$$

13. D

$$\begin{aligned} \log_a b^3 \cdot \log_b c^3 &= \frac{3 \log b}{\log a} \times \frac{3 \log c}{\log b} \\ &= 9 \times \frac{\log c}{\log a} \\ &= 9 \log_a c \end{aligned}$$

14. C

$$\begin{aligned} \frac{1}{y} &= \log_2 x \\ &= \frac{\log x}{\log 2} \\ y &= \frac{\log 2}{\log x} \\ &= \log_x 2 \end{aligned}$$

15. B

$$\begin{aligned} \frac{\log_x \sqrt{x^3} + 2 \log_y \sqrt[5]{y}}{3 \log_x \sqrt{x} - \log_y \sqrt{y^5}} &= \frac{\log_x x^{\frac{3}{2}} + 2 \log_y y^{\frac{1}{5}}}{3 \log_x x^{\frac{1}{2}} - \log_y y^{\frac{5}{2}}} \\ &= \frac{\frac{3}{2} + 2 \times \frac{1}{5}}{3 \times \frac{1}{2} - \frac{5}{2}} \\ &= -\frac{19}{10} \end{aligned}$$

16. B

$$\begin{aligned} \log_2 a + \log_{\frac{1}{2}} \sqrt{a} &= \frac{\log a}{\log 2} + \frac{\frac{1}{2} \log a}{\log \frac{1}{2}} \\ &= \frac{\log a}{\log 2} + \frac{\log a}{-2 \log 2} \\ &= \frac{\log a}{2 \log 2} \\ &= \frac{1}{2} \log_2 a \end{aligned}$$

17. D

$$\begin{aligned} \frac{\log_8 x^2}{\log_2 x} &= \frac{2 \log x}{\log 8} \div \frac{\log x}{\log 2} \\ &= \frac{2 \log x}{3 \log 2} \times \frac{\log 2}{\log x} \\ &= \frac{2}{3} \end{aligned}$$

18. B

$$\begin{aligned}\log(10a^b b^a) &= \log 10 + \log a^b + \log b^a \\ &= 1 + b \log a + a \log b\end{aligned}$$

19. B

$$\begin{aligned}\frac{\log_x x^5 + \log_y \sqrt[3]{y^2}}{\log_x \sqrt{x^5} - \log_y y^3} &= \frac{5 + \log_y y^{\frac{2}{3}}}{\log_x x^{\frac{5}{2}} - 3} \\ &= \frac{5 + \frac{2}{3}}{\frac{5}{2} - 3} \\ &= -\frac{34}{3}\end{aligned}$$

20. C

可得 $a = \log 3$ 及 $b = \log 4 = 2 \log 2$ 。

$$\begin{aligned}\log \frac{5}{6} &= \log \frac{10}{2^2 \times 3} \\ &= \log 10 - 2 \log 2 - \log 3 \\ &= 1 - a - b\end{aligned}$$

21. D

可得 $a = \log_3 4$ 及 $b = \log_3 6$ 。

$$\begin{aligned}\log_3 \frac{1}{8} &= \log_3 \frac{3}{4 \times 6} \\ &= \log_3 3 - \log_3 4 - \log_3 6 \\ &= 1 - a - b\end{aligned}$$

另一題解

$$\begin{aligned}\log_3 \frac{1}{8} &= -3 \log_3 2 \\ &= -3 \times \frac{a}{2} \\ &= -\frac{3a}{2}\end{aligned}$$

22. A

$$\begin{aligned}\frac{1}{y} &= \log_5 x \\ \frac{1}{y} &= \frac{\log x}{\log 5} \\ y &= \frac{\log 5}{\log x} \\ &= \log_x 5\end{aligned}$$

23. A

$$x = 8^n$$

$$n = \log_8 x$$

$$\log_8 \frac{x^3}{64} = 3 \log_8 x - \log_8 64$$

$$= 3n - 2$$

24. C

繪畫直線 $x = 1$ 。交點為 $(1, b)$ 及 $(1, c)$ 。

I. ✗。 $b > c$

II. ✓。 $b > c > 1 \Rightarrow bc > 1$

III. ✓。設 $L: y = k$ 。

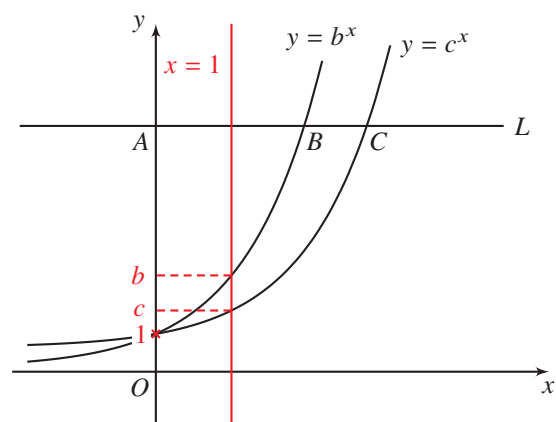
$B(\log_b k, k)$ 及 $C(\log_c k, k)$

$$\frac{AB}{AC} = \frac{\log_b k}{\log_c k}$$

$$= \frac{\log k}{\log b} \div \frac{\log k}{\log c}$$

$$= \frac{\log c}{\log b}$$

$$= \log_b c$$



25. C

$$y = \log_7 2x$$

$$2x = 7^y$$

$$x = \frac{1}{2}(7^y)$$

26. A

$$\log_2(x - y) = 2$$

$$x - y = 2^2$$

$$x - y = 4$$

$$\text{解} \begin{cases} x + y = 2 \\ x - y = 4 \end{cases}, \text{ 可得 } x = 3 \text{ 及 } y = -1。$$

27. C

$$x = 3^y \cdot 9^a$$

$$\log x = y \log 3 + a \log 9$$

$$= y \log 3 + 2a \log 3$$

$$y \log 3 = \log x - 2a \log 3$$

$$y = \frac{\log x}{\log 3} - 2a$$

$$= \log_x 3 - 2a$$

28. B

$$x = \log_4 \frac{1}{7}$$

$$4^x = \frac{1}{7}$$

29. B

$$\log_a 8 = 3$$

$$a^3 = 8$$

$$a = 2$$

30. B

$$5 \log_2 \alpha + 5 \log_2 \beta = \frac{10}{3}$$

$$\log_2 \alpha \beta = \frac{2}{3}$$

$$\alpha \beta = 2^{\frac{2}{3}}$$