REG-LOG-2223-ASM-SET 1-MATH
Suggested solutions

Multiple Choice Questions

1. A	2. A	3. A	4. B	5. B
6. B	7. D	8. A	9. C	10. A
11. C	12. A	13. D	14. D	15. B
16. D	17. D	18. B	19. A	20. C
21. C	22. C	23. D	24. D	25. C
26. D	27. C	28. D	29. C	30. A

1. A

$$\begin{aligned} 100^{\log a} &= 10^{2\log a} \\ &= (10^{\log a})^2 \\ &= a^2 \end{aligned}$$

2. A

$$10^{\log a} + \log 10^b = a + b$$

3. A

$$\begin{aligned} 1000^{\log(2a)} &= 10^{3\log(2a)} \\ &= 10^{\log(2a)^3} \\ &= (2a)^3 \\ &= 8a^3 \end{aligned}$$

4. B

$$\begin{aligned} 10^{2\log a} &= (10^{\log a})^2 \\ &= a^2 \end{aligned}$$

5. B

$$\log(2x - a) = 3$$

$$2x - a = 10^3$$

$$x = \frac{1000 + a}{2}$$

6. B

I. ✗. $(\log a)^2 = (\log a)(\log a) \neq 2\log a$

II. ✗. Take $a = 100$ and $b = 10$, then $\dfrac{\log a}{\log b} = \dfrac{2}{1} \neq \dfrac{a}{b}$.

III. ✓. $\log \dfrac{a}{b} + \log b = \log\left(\dfrac{a}{b} \times b\right) = \log a$

7. D

I. ✗. Take $a = \dfrac{1}{2}$ and $b = 2$. Then $\log(ab) = \log 1 = 0$.

II. ✓. $\log b > 0 > \log a$ and $\log \dfrac{b}{a} = \log b - \log a > 0$.

III. ✓.

8. A

$$\log \sqrt[3]{x} = k$$
$$\frac{1}{3}\log x = k$$
$$-2\log x = -6k$$
$$\log \frac{1}{x^2} = -6k$$

9. C

$$\begin{aligned}\log \sqrt[3]{x^2 + y^2} &= \frac{1}{3}\log\left(x^2 + y^2\right)\\ &= \frac{1}{3}\log[(x+y)^2 - 2xy]\\ &= \frac{1}{3}\log[28^2 - 2(-108)]\\ &= \frac{1}{3}\log 1000\\ &= 1\end{aligned}$$

10. A

$$\log \sqrt{x} = k$$
$$\frac{1}{2}\log x = k$$
$$\log x = 2k$$
$$\begin{aligned}\log\left(\frac{1}{x}\right) &= -\log x\\ &= -2k\end{aligned}$$

11. C

$$\frac{\log a^6 - \log \sqrt[3]{a^2}}{2\log a^4} = \frac{6\log a - \frac{2}{3}\log a}{8\log a}$$
$$= \frac{\frac{16}{3}\log a}{8\log a}$$
$$= \frac{2}{3}$$

12. A

$$\frac{\log(1000x^3)}{\log \frac{1}{10x}} = \frac{\log 1000 + 3\log x}{-\log 10 - \log x}$$
$$= \frac{3 + 3\log x}{-1 - \log x}$$
$$= \frac{3(1 + \log x)}{-(1 + \log x)}$$
$$= -3$$

13. D

$$\log\left(a^a b^b\right) = \log a^a + \log b^b$$
$$= a\log a + b\log b$$

14. D

$$\log x^2 + (\log x)^2 = 2\log x + (\log x)^2$$
$$= (\log x)(2 + \log x)$$

15. B

$$\frac{2\log x^2}{\log(3x) + \log x - \log 3} = \frac{4\log x}{\log 3 + \log x + \log x - \log 3}$$
$$= \frac{4\log x}{2\log x}$$
$$= 2$$

16. D

L.H.S. $= \log q + p\log x$ and R.H.S. $= 3\log x - 3\log 2 + 2\log x = 5\log x - 3\log 2 = 5\log x + \log 2^{-3}$
So, $p = 5$ and $q = 2^{-3} = \frac{1}{8}$.

17. D

$$y = ab^x$$
$$\log y = \log a + \log b^x$$
$$\log y = \log a + x\log b$$

18. B

$$\frac{\log a^2}{\log b} \div \frac{\log a^3}{\log \frac{1}{b}} = \frac{2\log a}{\log b} \times \frac{-\log b}{3\log a}$$
$$= -\frac{2}{3}$$

19. A

$$f(x+1) - f(x) = [\log(2x+2) - \log 2x] - [(x+1) - x]$$
$$= \log \frac{2x+2}{2x} - 1$$
$$= \log \frac{x+1}{x} - \log 10$$
$$= \log \frac{x+1}{10x}$$

20. C

$$\log \frac{45}{4} = \log \frac{3^2 \times 5}{2^2}$$
$$= \log \frac{3^2 \times 10}{2^3}$$
$$= 2\log 3 + \log 10 - 3\log 2$$
$$= 2y - 3x + 1$$

21. C

$10^x = 2$ and $100^y = 3$

$x = \log 2$ $10^{2y} = 3$

$2y = \log 3$

$$\log 6 = \log 2 + \log 3$$
$$= x + 2y$$

22. C

$$\log 135 = \log\left(3^3 \times 5\right)$$
$$= 3\log 3 + \log \frac{10}{2}$$
$$= 3\log 3 + \log 10 - \log 2$$
$$= 3y - x + 1$$

23. D

$$\log 9 = n$$
$$2\log 3 = n$$
$$\log 3 = \frac{n}{2}$$
$$\log 24 = \log\left(2^3 \times 3\right)$$
$$= 3\log 2 + \log 3$$
$$= 3m + \frac{n}{2}$$

24. D

$$\log 180 = \log\left(2^2 \times 3^2 \times 5\right)$$
$$= 2\log 2 + 2\log 3 + \log 5$$
$$= 2\log \frac{10}{5} + 2\log 3 + \log 5$$
$$= 2(1 - \log 5) + 2\log 3 + \log 5$$
$$= 2 - \log 5 + 2\log 3$$
$$= 2 - n + 2m$$

25. C

$$\log 8 = a$$
$$3\log 2 = a$$
$$\log 2 = \frac{a}{3}$$
$$\log \frac{1}{16} = \log \frac{1}{2^4}$$
$$= -4\log 2$$
$$= -\frac{4a}{3}$$

26. D

$p = \log 2$ and $q = \log 3$.

$$\log \frac{5}{3} = \log 10 - \log 2 - \log 3 = 1 - p - q$$

27. C

$$\log 4.5 = \log \frac{3^2}{2}$$
$$= \log 3^2 - \log \frac{10}{5}$$
$$= 2\log 3 - \log 10 + \log 5$$
$$= 2x + y - 1$$

28. D

We have $a = \log 3$ and $b = \log 5$.

$$\begin{aligned}\log\sqrt{6} &= \frac{1}{2}(\log 2 + \log 3)\\ &= \frac{1}{2}\left(\log\frac{10}{5} + \log 3\right)\\ &= \frac{1}{2}(\log 10 - \log 5 + \log 3)\\ &= \frac{1}{2}(a - b + 1)\end{aligned}$$

29. C

$$\begin{aligned}\log 280 &= \log\left(2^2 \times 7 \times 10\right)\\ &= 2\log 2 + \log 7 + \log 10\\ &= 2a + b + 1\end{aligned}$$

30. A

$$3^a = 12^m \quad \text{and} \quad 4^b = 12^m$$

$$a\log 3 = m\log 12 \qquad b\log 4 = m\log 12$$

$$\frac{m}{a} = \frac{\log 3}{\log 12} \qquad \frac{m}{b} = \frac{\log 4}{\log 12}$$

$$\begin{aligned}\frac{m}{a} + \frac{m}{b} &= \frac{\log 3}{\log 12} + \frac{\log 4}{\log 12}\\ &= \frac{\log(3 \times 4)}{\log 12}\\ &= 1\end{aligned}$$

Conventional Questions

31. $\dfrac{\log\sqrt{3}+\log 2-\log\sqrt{10}}{\log 6-\log 5}=\dfrac{\frac{1}{2}\log 3+\log 2-\frac{1}{2}\log 10}{\log 6-\log 5}$ 1M

$=\dfrac{\frac{1}{2}\log 3+\log 2-\frac{1}{2}\log 2-\frac{1}{2}\log 5}{\log 2+\log 3-\log 5}$ 1M

$=\dfrac{\frac{1}{2}(\log 3+\log 2-\log 5)}{\log 2+\log 3-\log 5}$

$=\dfrac{1}{2}$ 1A

32. $\dfrac{\log\sqrt[4]{x}-\log x^2}{\log\frac{1}{x^3}}=\dfrac{\frac{1}{4}\log x-2\log x}{-3\log x}$ 1M

$=\dfrac{-\frac{7}{4}\log x}{-3\log x}$

$=\dfrac{7}{12}$ 1A

33. $\dfrac{3\log x^4+6\log x^2}{2\log\frac{1}{x}+\log x^6}=\dfrac{12\log x+12\log x}{-2\log x+6\log x}$ 1M

$=\dfrac{24\log x}{4\log x}$

$=6$ 1A

34. $\dfrac{\log(xy^3)-\log(\sqrt{x}y^6)}{\log\sqrt{x}-\log y^3}=\dfrac{\log x+\log y^3-(\log x^{\frac{1}{2}}+\log y^6)}{\log x^{\frac{1}{2}}-\log y^3}$ 1M

$=\dfrac{\log x+3\log y-\frac{1}{2}\log x-6\log y}{\frac{1}{2}\log x-3\log y}$ 1M

$=\dfrac{\frac{1}{2}\log x-3\log y}{\frac{1}{2}\log x-3\log y}$

$=1$ 1A

35. $\dfrac{\log x^2+\log x^3}{\log x-\log x^2}=\dfrac{2\log x+3\log x}{\log x-2\log x}$ 1M

$=\dfrac{5\log x}{-\log x}$

$=-5$ 1A

36. $\dfrac{\log\sqrt{x^3}}{\log x^2-\log\frac{1}{x^{\frac{1}{2}}}}=\dfrac{\frac{3}{2}\log x}{2\log x+\frac{1}{2}\log x}$ 1M

$=\dfrac{\frac{3}{2}\log x}{\frac{5}{2}\log x}$

$=\dfrac{3}{5}$ 1A

37. $\dfrac{\log\sqrt{xy^3}-\log(xy)}{-\frac{3}{2}\log(xy)+\log\left(x^2y\right)} = \dfrac{\frac{1}{2}(\log x+\log y^3)-\log x-\log y}{-\frac{3}{2}(\log x+\log y)+(\log x^2+\log y)}$ 1M

$= \dfrac{\frac{1}{2}\log x+\frac{3}{2}\log y-\log x-\log y}{-\frac{3}{2}\log x-\frac{3}{2}\log y+2\log x+\log y}$ 1M

$= \dfrac{-\frac{1}{2}\log x+\frac{1}{2}\log y}{\frac{1}{2}\log x-\frac{1}{2}\log y}$

$= -1$ 1A

38. $\dfrac{\log 54+\log 2}{3\log\sqrt{3}-\frac{1}{3}\log\frac{1}{8}} = \dfrac{\log\left(2\times 3^3\right)+\log 2}{\frac{3}{2}\log 3+\log 2}$ 1M

$= \dfrac{2\log 2+3\log 3}{\frac{1}{2}(3\log 3+2\log 2)}$ 1M

$= 2$ 1A

 END OF PAPER