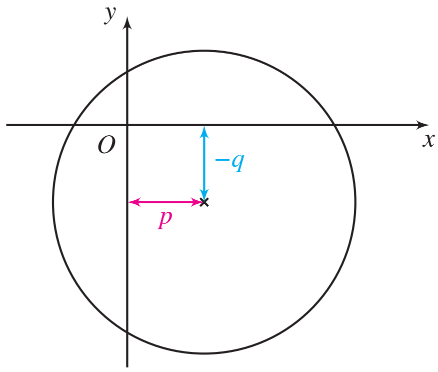
REG-EOC-2223-ASM-SET 4-MATH
Suggested solutions

Multiple Choice Questions

1. $\boxed{\text{D}}$

Centre $(p,\ q)$ lies in quadrant IV. So, $p > 0$ and $q < 0$.



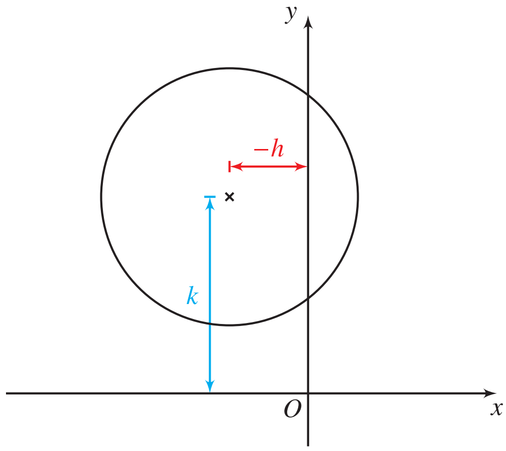
I. ✓.

II. ✓. $p - r < 0$
(length p is shorter than the radius)

III. ✓. $\sqrt{p^2 + q^2} < r$
(distance between origin and centre is smaller than the radius)

2. $\boxed{\text{A}}$

Centre lies in quadrant II. Therefore, $h < 0$ and $k > 0$.



I. ✓. $k + h = k - (-h) > 0$
(distance from x-axis is longer than that from y-axis)

II. ✓. $r - h = r + (-h) > 0$
(positive lengths)

III. ✗. $r - k < 0$
(radius is shorter than distance from x-axis)

3. $\boxed{\text{D}}$

I. Radius $= \sqrt{3^2 + 4^2 - 10} = \sqrt{15}$

II. Radius $= \sqrt{4^2 + 3^2 - 10} = \sqrt{15}$

III. Radius $= \sqrt{5^2 - 10} = \sqrt{15}$

All circles have the same area.
The answer is D.

4. $\boxed{\text{D}}$

C: $x^2 + y^2 + 3x + 4y - \dfrac{25}{2} = 0$

Centre $\left(-\dfrac{3}{2},\ -2\right)$

Radius $= \sqrt{\left(\dfrac{3}{2}\right)^2 + 2^2 + \dfrac{25}{2}} = \dfrac{5\sqrt{3}}{2}$

5. $\boxed{\text{C}}$

Centre $\left(-\dfrac{k}{2},\ -\dfrac{k+1}{2}\right)$

$$-\frac{k}{2} - \frac{k+1}{2} + 1 = 0$$
$$k = \frac{1}{2}$$

6. $\boxed{\text{A}}$

L passes through centre of circle $\left(\dfrac{k}{-2},\ -2\right)$.

$$\frac{3-(-2)}{16-\frac{k}{-2}} = \frac{1}{2}$$
$$k = -12$$

7. $\boxed{\text{D}}$

Diameter passes through centre $(4,\ 3)$.

$$\frac{-5-3}{k-4} = -4$$
$$k = 6$$

8. $\boxed{\text{A}}$

Let the centre of S be G. The coordinates of G are $(1,\ 2)$. Note that $GM \perp AB$.

Slope of $GM = \dfrac{2+2}{1-3} = -2$

Slope of $AB = \dfrac{1}{2}$

Required equation is

$$y + 2 = \frac{1}{2}(x-3)$$
$$x - 2y - 7 = 0$$

9. $\boxed{\text{B}}$

Radius $= \sqrt{1^2 + 2^2 + 4} = 3$

Required area $= 4 \times \dfrac{(3)(3)}{2}$

$= 18$

10. $\boxed{\text{C}}$

Distance between centres $= \sqrt{(3+3)^2 + (7+1)^2}$

$= 10$

$= 8 + 2$

The two circles touch each other externally.

11. $\boxed{\text{D}}$

Substitute the coordinates of the points into L.H.S. of the equation.

A. $10^2 + 6^2 - 8(10) + 4(6) - 16 = 64 > 0$. W lies outside the circle.

B. $8^2 + 8^2 - 8(8) + 4(8) - 16 = 80 > 0$. X lies outside the circle.

C. $6^2 + 6^2 - 8(6) + 4(6) - 16 = 32 > 0$. Y lies outside the circle.

D. $9^2 + 0 - 8(9) - 16 = -7 < 0$. Z lies inside the circle.

12. $\boxed{\text{C}}$

I. ✓. The coordinates of the centre are $(-1,\ 0)$.
The centre lies on the x-axis.

II. ✓. Radius $= \sqrt{9} = 3$

III. ✗. When $y = 0$,

$$(x+1)^2 + 0 = 9$$

$$x = 2 \quad \text{or} \quad -4$$

The circle intersects the x-axis at $(2,\ 0)$ and $(-4,\ 0)$.

13. $\boxed{\text{C}}$

C_2: $x^2 + y^2 + 6x - 8y + \dfrac{25}{2} = 0$

I. ✓. $G_1\ (8,\ -6)$, $G_2\ (-3,\ 4)$. $OG_1 = 10 = 2OG_2$

II. ✗. $m_{OG_1} \times m_{OG_2} = \dfrac{-6}{8} \times \dfrac{4}{-3} \neq -1$

III. ✓. Radius of $C_1 = \sqrt{8^2 + 6^2 - 75} = 5$, radius of $C_2 = \sqrt{3^2 + 4^2 - \dfrac{25}{2}} = \dfrac{5}{\sqrt{2}}$

Ratio of area of C_1 to area of C_2 is $5 : \left(\dfrac{5}{\sqrt{2}}\right)^2 = 2 : 1$.

14. $\boxed{\text{C}}$

C_1: $x^2 + y^2 + 2x + 4y - \dfrac{149}{2} = 0$.

I. ✓. $(-1)^2 + (2)^2 - 8(-1) - 20(2) - 53 = 0$. It lies on C_2.

II. ✗. Radius of $C_1 = \sqrt{1^2 + 2^2 + \dfrac{149}{2}} = \sqrt{79.5}$ and radius of $C_2 = \sqrt{4^2 + 10^2 + 53} = 13$.

III. ✓. Distance between centre = 13, which lies between sum and difference of their radii, i.e., $13 - \sqrt{79.5}$ and $13 + \sqrt{79.5}$.

So, they intersect at two distinct points.

15. $\boxed{\text{D}}$

C: $x^2 + y^2 - 3x + y - \dfrac{13}{2} = 0$

I. ✗. Centre $\left(\dfrac{3}{2}, -\dfrac{1}{2}\right)$

II. ✓. $AB = \sqrt{(2-1)^2 + (1+2)^2} = \sqrt{10}$

Radius $= \sqrt{\left(\dfrac{3}{2}\right)^2 + \left(\dfrac{1}{2}\right)^2 + \dfrac{13}{2}} = 3 < AB$

III. ✓. Slope of $AB = \dfrac{1+2}{2-1} = 3$

Slope of $AG = \dfrac{-2 + \frac{1}{2}}{1 - \frac{3}{2}} = 3$, where G is the centre.

Thus, three points are collinear.

16. $\boxed{\text{B}}$

I. ✓. Radius $= \sqrt{3^2 + 6^2 + 4} = 7$

II. ✗. Centre $(3, -6)$ lies in the fourth quadrant.

III. ✓. $0^2 + 0^2 - 6(0) + 12(0) - 4 = -4 < 0$

The origin lies inside the circle.

17. $\boxed{\text{D}}$

I. ✓. $G_1\ (-2, 6)$, $G_2\ (2, 4)$. Slope of $OG_2 \times$ slope of $G_1G_2 = \dfrac{4}{2} \times \dfrac{6-4}{-2-2} = -1$

II. ✓. Distance between centres $= \sqrt{(2+2)^2 + (6-4)^2} = \sqrt{20}$

Radius of $C_1 = \sqrt{2^2 + 6^2 + 40} = \sqrt{80} = 2\sqrt{20}$; radius of $C_2 = \sqrt{2^2 + 4^2} = \sqrt{20}$

Since distance between centres = difference in radii, the circles touch each other internally.

III. ✓. Area ratio $= \left(\dfrac{\sqrt{80}}{\sqrt{20}}\right)^2 = 4$

18. $\boxed{\text{A}}$

$x^2 + y^2 - x - \dfrac{y}{2} + \dfrac{1}{4} = 0$

I. ✗. Centre $\left(\dfrac{1}{2}, \dfrac{1}{4}\right)$

II. ✗. Radius $= \sqrt{\left(\dfrac{1}{2}\right)^2 + \left(\dfrac{1}{4}\right)^2 - \dfrac{1}{4}} = \dfrac{1}{4} \neq 2$

III. ✓. Radius $= \dfrac{1}{4} = y$-coordinate of centre.
So, the circle touches the x-axis.

19. $\boxed{\text{C}}$

C_2: $x^2 + y^2 + 6x - 8y + \dfrac{33}{2} = 0$
G_1 $(4, 3)$ and G_2 $(-3, 4)$.

I. ✓. Slope of $G_1O\times$ slope of $G_2O = \dfrac{3}{4} \times \dfrac{4}{-3} = -1$

II. ✗. Radius of $C_1 = \sqrt{4^2 + 3^2 - 20} = \sqrt{5}$ and radius of $C_2 = \sqrt{3^2 + 4^2 \dfrac{33}{2}} = \sqrt{8.5} > \sqrt{5}$
So, area of C_1 is smaller than the area of C_2.

III. ✓. $OG_1 = OG_2 = \sqrt{3^2 + 4^2} = 5$

20. $\boxed{\text{A}}$

$x^2 + y^2 - 9x + 8y - \dfrac{1}{2} = 0$

I. ✓. $0 + y^2 - 0 + 8y - \dfrac{1}{2} = 0$

$y \approx 0.0620$ or -8.06

The circle intersect y-axis at two points.

II. ✗. Coordinates of centre are $\left(\dfrac{9}{2}, -4\right)$.

III. ✗. Sub $(0, 0)$, L.H.S. $= -\dfrac{1}{2} < 0$. Origin lies inside the circle.

21. $\boxed{\text{C}}$

I. ✗. The coordinates of the centres of C_1 and C_2 are $(-4, 3)$ and $(4, -3)$ respectively.
They are not concentric circle.

II. ✓. Radius of both circles $= \sqrt{4^3 + 3^2 + 25} = \sqrt{50}$
The lengths of diameters are the same.

III. ✓. Distance between the centre and the y-axis is 4, which is smaller than the radius.
Both C_1 and C_2 cut the y-axis at two distinct points.

22. $\boxed{\text{A}}$

I. ✓. Radius of $C_1 = \sqrt{3^2 + 4^2} = 5$; radius of $C_2 = \sqrt{25} = 5$

II. ✓. Distance between centres $= \sqrt{(3-0)^2 + (0+4)^2} = 5$

III. ✗. $(0,\ 0)$ does not satisfy the equation of C_2. C_2 does not pass through the origin.

23. $\boxed{\text{D}}$

I. ✓. $(0,\ 0)$ satisfies the equation.

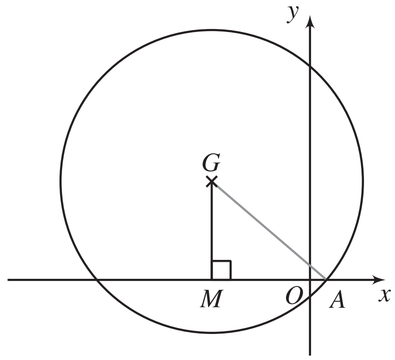
II. ✗. The coordinates of the centre are $(0,\ -4)$.
It does not lie on the x-axis.

III. ✓. Radius $= \sqrt{0^2 + 4^2 - 0} = 4$
Distance between centre and x-axis is equal to the radius.
C touches the x-axis.

24. $\boxed{\text{C}}$

Denote the centre by G.

Let M be a point on the x-axis such that GM is perpendicular to the x-axis.
Let A be the intersection of the circle and the positive x-axis.
The coordinates of M are $(-3,\ 0)$.
$AM = \dfrac{8}{2} = 4$ and the coordinates of A are $(1,\ 0)$.
Radius of circle $= \sqrt{(1+3)^2 + (0-3)^2} = 5$
Required equation is $(x+3)^2 + (y-3)^2 = 25$.



Conventional Questions

25. (a) $(x-6)^2+(y+5)^2=6^2+5^2$ 1M

$(x-6)^2+(y+5)^2=61$ 1A

(b) (i) $H=(12,\ 0)$ and $K=(0,\ -10)$ 1A+1A

(ii) O, P and Q are collinear. 1A

(iii) Required area $=12\times 10$ 1M

$=120$ 1A

26. (a) Since $FB=FE$, $\angle FBE=\angle FEB$.

$\angle FCA=\frac{1}{2}\angle FEA$ 1M

In $\triangle ABC$,

$$\angle ABC+\angle BCA=\angle CAE$$

$$\angle ABC+\frac{1}{2}\angle ABC=\theta \quad \text{1M}$$

$$\angle ABC=\frac{2\theta}{3} \quad \text{1A}$$

(b) (i) $\angle ABC=\frac{2}{3}(45°)=30°$ 1M

$BE=\frac{CE}{\tan 30°}=\sqrt{3}CE$ and $AE=\frac{CE}{\tan 45°}=CE$

$$AB=BE-AE$$

$$0-(1-\sqrt{3})=CE(\sqrt{3}-1) \quad \text{1M}$$

$$CE=1 \quad \text{1A}$$

Coordinates of $C=(0+1,\ 0+1)=(1,\ 1)$ 1A

Coordinates of $D=(0+1+1,\ 0)=(2,\ 0)$ 1A

(ii) Equation of circle $ADCF$ is $(x-1)^2+y^2=1$ 1A

27. (a) $y^2 - 12y + 32 = 0$

$y = 4 \quad \text{or} \quad 8$

Coordinates of A are $(0,\ 4)$. 1A

(b) $c = 4$ 1A

Coordinates of P are $(6,\ 6)$. 1M

Slope of $AP = \dfrac{6-4}{6-0} = \dfrac{1}{3}$

Slope of $L = m = -3$ 1A

(c) Let the x-coordinate of B be b.

Since B lies on $y = -3x + 4$, the coordinates of B are $(b,\ -3b+4)$.

$\sqrt{b^2 + (-3b+4-4)^2} = \sqrt{(0-6)^2 + (4-6)^2}$ 1M

$b^2 + 9b^2 = 40$

$b = 2 \quad \text{or} \quad -2 \text{ (rejected)}$

The coordinates of B are $(2,\ -2)$. 1A

The equation of C_2 is

$(x+10)^2 + (y+6)^2 = (2+10)^2 + (-2+6)^2$ 1M

$(x+10)^2 + (y+6)^2 = 160$ 1A

28. (a) $OQ = OP = r$ 1A

$AP = AQ = 4 - r$ and $BP = BR = 3 - r$ 1M+1A

(b) $(3-r) + (4-r) = \sqrt{3^2 + 4^2}$ 1M

$r = 1$

The coordinates of C are $(1,\ 1)$. 1A

(c) The equation of the circle is

$(x-1)^2 + (y-1)^2 = 1^2$

$x^2 + y^2 - 2x - 2y + 1 = 0$ 1A

29. (a) Let the equation of circle be $x^2 + y^2 + Dx + Ey + F = 0$, where D, E and F are constants. 1A

$$\begin{cases} 1 + 4 + D + 2E + F = 0 & (1) \\ 9 + 3E + F = 0 & (2) \\ 16 + 4D + F = 0 & (3) \end{cases}$$ 1M

Consider (2) − (1) and (3) − (2).

$$\begin{cases} -D + E = -4 \\ 4D - 3E = -7 \end{cases}$$ 1M

Solving, we have $D = -19$, $E = -23$. 1A

When $D = -19$, $E = -23$, $F = -9 - 3(-23) = 60$.

The equation of the circle is $x^2 + y^2 - 19x - 23y + 60 = 0$. 1A

(b) Centre of the circle $= \left(\frac{19}{2}, \frac{23}{2}\right)$ 1A

Radius of the cirlce $= \sqrt{\left(\frac{19}{2}\right)^2 + \left(\frac{23}{2}\right)^2 - 60} = \frac{5\sqrt{26}}{2}$ 1A

(c) If two points on the circle form a diameter, then the midpoint of them must be at the centre of circle.

Midpoint of $AB = \left(\frac{1}{2}, \frac{5}{2}\right)$ 1M

Midpoint of $BC = \left(2, \frac{3}{2}\right)$

Midpoint of $CA = \left(\frac{5}{2}, 1\right)$

None of the above is at the centre of the circle.

Thus, the claim is incorrect. 1A

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