REG-2223-MOCK-SET 6-MATH-CP 2

Answers:

1. B	2. C	3. C	4. A	5. B	6. B	7. C	8. D	9. D	10. B
11. B	12. D	13. A	14. D	15. C	16. D	17. A	18. B	19. A	20. A
21. A	22. D	23. A	24. C	25. C	26. C	27. A	28. C	29. B	30. D
31. A	32. B	33. A	34. D	35. C	36. A	37. B	38. D	39. B	40. D
41. C	42. B	43. C	44. D	45. C					

Suggested Solutions:

1. B

$$\frac{9^{3x+1}}{27^{2x+1}} = \frac{3^{6x+2}}{3^{6x+3}}$$
$$= \frac{1}{3}$$

2. C

Check the coefficient of each term.

	$-2b$	$-4a$
A.	✗	
B.	✓	✗
C.	✓	✓
D.	✗	

3. C

Solve $\begin{cases} 2x + y = 5 \\ 3x - 2y + 1 = 5 \end{cases}$, we have $x = 2$ and $y = 1$.

4. A

Put $x = -1$,

$$0 - 3 = (-1 + 2)^2 + \beta$$
$$\beta = -4$$

5. B

$$x = 2 - \frac{y+1}{y}$$
$$xy = 2y - (y+1)$$
$$y(x-1) = -1$$
$$y = \frac{1}{1-x}$$

6. B

A. ✗. 0.001 is of 3 decimal places.

B. ✓.

C. ✗. $x = 0.001$ (correct to 3 decimal places)

D. ✗. 0.0012 has only 2 significant figures.

7. C

The inequalities become $x \le 7$ or $x < -6$.
Thus, $x \le 7$.
The greatest integer is 7.

8. D

$$\begin{aligned} f(\alpha) - f(\alpha - 1) &= 5[(\alpha)^2 - (\alpha - 1)^2] - (1 - 1) \\ &= 5(2\alpha - 1) \\ &= 10\alpha - 5 \end{aligned}$$

9. D

$$g\left(\frac{1}{2}\right) = \frac{k}{8} - \frac{5}{4} - k + 3 = 0$$
$$k = 2$$
$$g(-2) = 2(-8) - 5(4) - 4(-2) + 3 = -25$$

10. B

I. ✗. When $x = 3$, $y = (-3+1)^2 + 2 = 6 \neq -2$.

II. ✓. Coefficient of $x^2 = (-1)^2 = 1 > 0$. The graph opens upwards.

III. ✗. y-intercept $= (0+1)^2 + 2 = 3 \neq 2$

11. B

Let the cost of the handbag be $\$x$.

$$\text{Percentage profit} = \frac{x(1+50\%)(1-20\%)-x}{x} \times 100\%$$
$$= 20\%$$

12. D

Let $a = 6$, then $b = \frac{2a}{3} = 4$ and $c = \frac{2a}{4} = 3$.

Thus, $a : b : c = 6 : 4 : 3$.

13. A

Let $p = \frac{kr}{q^2}$, where k is a non-zero constant.

$$\frac{p_2}{p_1} = \frac{1-10\%}{(1+20\%)^2}$$
$$= 0.625$$

p is decreased by 37.5%.

14. D

The numbers are formed by +2, +4, +6, . . .

The sequence is 4, 6, 10, 16, 24, 34, 46, 60.

Required number is 60.

15. C

$$(2q)^2(3p) = 648$$
$$pq^2 = 54$$
$$\text{Required volume} = \frac{1}{3}(3q)^2(2p)$$
$$= 6pq^2$$
$$= 324\,\text{cm}^3$$

16. D

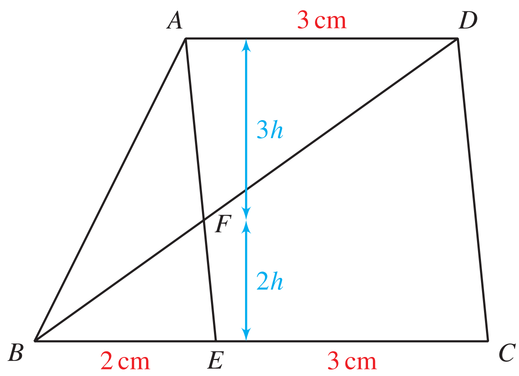
Let $BE = 2\,\text{cm}$. Then $CE = AD = 3\,\text{cm}$.

$\triangle ADF \sim \triangle EBF$ (ratio = 3 : 2)

Required ratio

$$= \frac{(3)(5h)}{2} : \left[(3)(5h) - \frac{(3)(3h)}{2}\right]$$

$$= 5 : 7$$



17. A

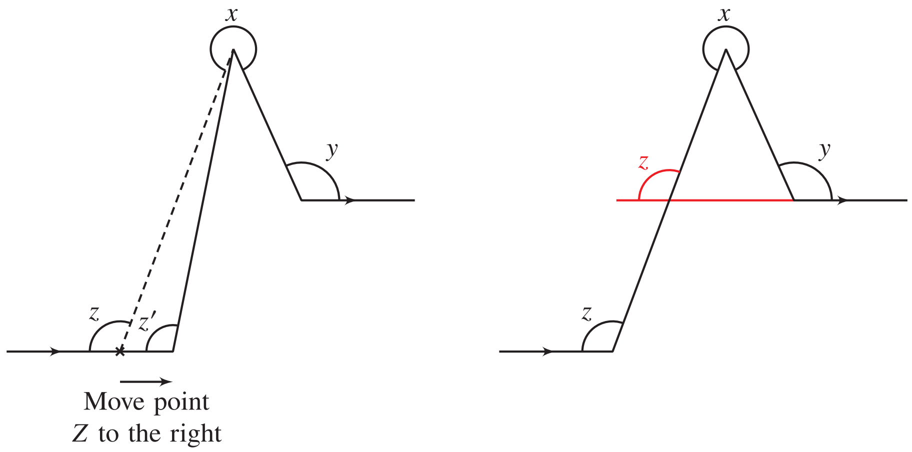
$\angle AOB = 2\angle ACB = 40°$ and $\angle BOC = 140° - 40° = 100°$

Required area $= \pi(6)^2 \times \frac{100°}{360°} - \frac{1}{2}(6)^2 \sin 100°$

$\approx 14\,\text{cm}^2$

18. B

Move point Z to the right as shown in the figure.



Angle z becomes smaller and angle x becomes larger.

I. ✗. z decreases while y remains unchanged. They cannot be always equal.

II. ✗. Since z decreases and x increases, the size of $y + z - x$ decreases upon the movement of point Z.
So, it cannot be always equal to 180°.

III. ✓. $(180° - z) + (360° - x) = y$

$$x + y + z = 540°$$

19. A

$\angle BAC = 32° + 60° = 92°$
Since $AB = AC$, $\angle ACB = \dfrac{180° - 92°}{2} = 44°$
$\angle DEC = \angle ADE - \angle ECF = 60° - 44° = 16°$

20. A

Let $AB = 1$. Then $CD = AB = 1$.

$$\frac{BF}{CE} = \frac{1}{\sin\beta} \div \frac{1}{\sin\alpha} = \frac{\sin\alpha}{\sin\beta}$$

21. A

Let $BD = x$ cm.
Then $DE = \dfrac{42}{x}$ cm.
Since $\triangle EFC \sim \triangle ABC$,

$$\frac{x}{\left(20 - \frac{42}{x}\right)} = \frac{AB}{BC}$$

$$2x = 20 - \frac{42}{x}$$

$$2x^2 - 20x + 42 = 0$$

$$x = 3 \quad \text{or} \quad 7$$

Required length = 3 cm

22. D

I. ✓. Since $ABCD$ is a parallelogram, $\angle BAD = \angle BCD$.

$$360° - \angle BCD = 2\angle BAD$$

$$360° - \angle BAD = 2\angle BAD$$

$$\angle BAD = 120°$$

II. ✓. Since $ABCD$ is a parallelogram, $AB = CD$ and $AD = BC$.
Since $BC = CD$, the four sides of $ABCD$ are equal and it is a rhombus.

III. ✓. Since $AB = AD$, $\overset{\frown}{AB} : \overset{\frown}{AD} = 1 : 1$.

23. A

x-intercept = $\dfrac{5}{b}$ and y-intercept = $\dfrac{5}{a}$.
Note that a and b are positive.

$$\frac{5}{b} < 2 \quad \text{and} \quad \frac{1}{2} \times \frac{5}{b} \times \frac{5}{a} > 4$$

$$b > \frac{5}{2} \qquad\qquad ab < \frac{25}{8}$$

I. ✓.

II. ✓.

III. ✗. Take $a = 1$ and $b = 3$. These values of a and b satisfy all the conditions above but $2a < b$.

24. C

Two lines are parallel.

$$\frac{-2}{3} = \frac{-6}{k}$$
$$k = 9$$

y-intercept $= \frac{k}{3} = 3$

25. C

$(-2,\ -5) \longleftarrow B(-2,\ 5) \longleftarrow A(-2,\ 2) = \left(2\sqrt{2},\ 135°\right)$

26. C

The locus of P is a pair of straight lines, $y = -1$ and $y = 11$.

27. A

$x^2 + y^2 + 2x + 4y + \frac{4}{3} = 0$

I. ✗. x-coordinate of centre $= -1$

II. ✓. $0^2 + 0^2 + 0 + 0 + \frac{4}{3} > 0$. The origin lies outside C.

III. ✗. Radius $= \sqrt{1^2 + 2^2 - \frac{4}{3}} \neq 1$

28. C

$$\text{Required probability} = 1 - \left(\frac{4}{7}\right)^2 = \frac{33}{49}$$

29. B

Let the ratio of number of boys to girls be $1 : \beta$, where $0 < \beta < 1$.

Mean $= \frac{60(1) + 70(\beta)}{1 + \beta} = 60 + \frac{10\beta}{1 + \beta} = 65 + \frac{5\beta - 5}{1 + \beta}$

Since $\frac{10}{1 + \beta} > 0 > \frac{5\beta - 5}{1 + \beta}$, the mean of the test marks lies between 60 and 65.

Only option B satisfies this.

30. D

A. ✗. Mode = 3

B. ✗. Median = 3

C. ✗. Lower quartile = 2.5

D. ✓.

31. A

Slope of the graph = −4 and $y^3 = -4\log_5 x + 12$.

$$2^3 = -4\log_5 x + 12$$
$$\log_5 x = 1$$
$$x = 5$$

32. B

$y = f(x) \longrightarrow y = f(-x) \longrightarrow y = f(-2x)$

Reflect about y-axis.

Reduce along x-axis to $\frac{1}{2}$ times the original.

The answer is B.

33. A

$$\pi^{2x} - 9\pi^x + 20 < 2$$
$$(\pi^x)^2 - 9\pi^x + 18 < 0$$
$$3 < \pi^x < 6$$
$$\log 3 < x\log\pi < \log 6$$
$$\frac{\log 3}{\log\pi} < x < \frac{\log 6}{\log\pi}$$
$$\log_\pi 3 < x < \log_\pi 6$$

34. D

$512 = 200_{16}$

$11 \times 8^{16} = 11 \times 2^{48} = 11 \times 16^{12} = \text{B000000000000}_{16}$

The answer is D.

35. C

Put $k = 2$.

$$\frac{i^{2020}}{k + i^{2019}} = \frac{1}{2 + i^3}$$
$$= \frac{1}{2 - i}$$
$$= \frac{2}{5} + \frac{1}{5}i$$

Imaginary part $= \frac{1}{5}$

Only option C gives $\frac{1}{5}$ when $k = 2$.

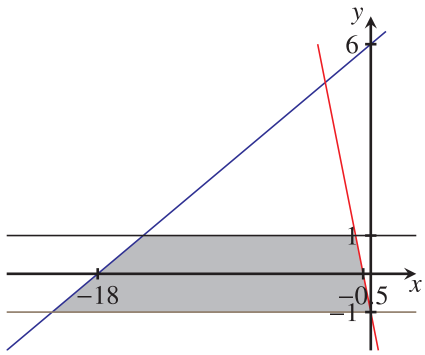
36. A

Line	x**-intercept**	y**-intercept**
$x - 3y + 18 = 0$	-18	6
$2x + y + 1 = 0$	$-\frac{1}{2}$	-1
$y = -1$		-1
$y = 1$		1

Sketch the graph using the intercepts.

Value of o5$x - 2y + k$ is greater when x is larger and y is smaller, i.e., the bottom right corner.
The coordinates of the bottom right corner are $(0, -1)$.

$$0 - 2(-1) + k = 12$$
$$k = 10$$



37. B

I. ✓. General term $= (1 - 2^{-n}) - (1 - 2^{-(n-1)})$

$= 2^{-n}(-1 + 2)$

$= 2^{-n}$

We have $2^{-n} < 1$ for all positive integers n.

II. ✗. The nth term $= \dfrac{1}{2^n}$ is a rational number for all positive integers n.

III. ✓. $\log T_{n+1} - \log T_n = \log 2^{-n-1} - \log 2^{-n}$

$= (-n - 1)\log 2 + n \log 2$

$= -\log 2 = \text{constant}$

Thus, it is an arithmetic sequence.

38. D

Solve the system $\begin{cases} x - y + m = 0 \\ x^2 + y^2 + 2x - 4y - 13 = 0 \end{cases}$ using the calculator program.

Value of m	Number of intersections	Sign of Δ
-9	0	$-$

Required range does not contain $m = -9$ and -9 is not a boundary value of the required range.

The answer is D.

39. B

Since $DQ = AQ$, $\angle ADQ = \dfrac{180° - 50°}{2} = 65°$.

$\angle ACD = \angle ADQ = 65°$

$\angle ADC = 180° - 100° = 80°$ and $\angle CAD = 180° - 65° - 80° = 35°$

Since $BC = CD$, $\angle BAC = \angle CAD = 35°$.

$\angle DCE = \angle BAD = 35° + 35° = 70°$

$\angle CED = \angle CDQ - \angle DCE = 80° + 65° - 70° = 75°$

$\angle PEB = 180° - 75° = 105°$

40. D

$$2\cos^2\theta = 2 - \sin\theta$$
$$2(1-\sin^2\theta) = 2 - \sin\theta$$
$$-2\sin^2\theta + \sin\theta = 0$$
$$\sin\theta = 0 \quad \text{or} \quad \frac{1}{2}$$

When $\sin\theta = 0$, $\theta = 0°$ or $180°$ or $360°$.
When $\sin\theta = \frac{1}{2}$, $\theta = 30°$ or $150°$.
There are 5 roots.

41. C

The straight line $x - 2y + 10 = 0$ is perpendicular to the straight line $2x + y + a = 0$.
The triangle formed is a right-angled triangle. The orthocentre lies at the right-angled vertex.
When $x = -6$,

$$(-6) - 2y + 10 = 0$$
$$y = 2$$

Substitute $(-6,\ 2)$ into $2x + y + a = 0$,

$$2(-6) + (2) + a = 0$$
$$a = 10$$

42. B

$$\text{Required probability} = \frac{1}{8} + \frac{1}{8} - \frac{6!}{8!} = \frac{13}{56}$$

43. C

$$\text{Required number} = C_2^3C_1^5C_1^2 + C_1^3C_2^5C_1^2 + C_1^3C_1^5C_2^2 = 105$$

44. D

Let the mean and standard deviation be $\overline{x}$ marks and σ marks respectively.

$$\begin{cases} \dfrac{26-\overline{x}}{\sigma} = -1 \\ \dfrac{92-\overline{x}}{\sigma} = 0.5 \end{cases}$$

Solving, we have $\overline{x} = 70$ and $\sigma = 44$.

45. C

Median = $15 \times 2 + 3 = 33$

Interquartile range = $10 \times 2 = 20$

Variance = $40 \times 2^2 = 160$